MA262 — EXAM I — SPRING 2016 — FEBRUARY 25, 2016
ANSWER KEYS

Test 01: 1-B, 2-C, 3-A, 4-E, 5-B, 6-D, 7-B, 8-B, 9-A, 10-B

Test 02: 1-A, 2-B, 3-C, 4-B, 5-D, 6-B, 7-D, 8-A, 9-D, 10-C

Test 03: 1-C, 2-D, 3-D, 4-A, 5-A, 6-C, 7-E, 8-E, 8-B, 10-A

Test 04: 1-D, 2-E, 3-E, 4-D, 5-C, 6-E, 7-A, 8-C, 9-D, 10-E

EXAM TYPE 01 TEST 01

1. The general solution of the differential equation $y' = x(1+y^2)$ is:

A. $\ln(1+y^2) = \frac{1}{2}x^2 + C$.

B. $\boxed{y = \tan(\frac{1}{2}x^2 + C)}$.

C. $y + \frac{1}{3}y^3 = \ln x + C$.

D. $\ln(1+y^2) = \frac{1}{2}(x+C)^2$.

E. $y = \arctan(\frac{1}{2}x^2 + C)$.

$\frac{dy}{dx} = x(1+y^2)$; Separable

$\frac{dy}{1+y^2} = x\,dx$; Integrate

$\tan^{-1} y = \frac{x^2}{2} + C$

$$\boxed{y = \tan\left(\frac{x^2}{2} + c\right)}$$

2. An integrating factor of the differential equation $xy' + 3y = \ln x$ for $x > 0$ is:

A. $I(x) = x$.

B. $I(x) = x^2$.

C. $\boxed{I(x) = x^3}$.

D. $I(x) = \ln x$.

E. $I(x) = x \ln x$.

$xy' + 3y = \ln x$, $x > 0$.

Divide by x.

$y' + \frac{3}{x}y = \frac{1}{x}\ln x$

For an equation $y' + f(x)y = g(x)$

an ~~the~~ integrating factor is

$I(x) = e^{\int f(x)\,dx}$

In this case $f(x) = 3/x$; $I(x) = e^{3\int \frac{dx}{x}}$

$I(x) = e^{3\ln x} = x^3$

TEST 01

3. The solution of

$$y^{-2}dx - (2xy^{-3} + y)dy = 0, \quad y(1) = 1$$

satisfies the following equation:

$M\,dx + N\,dy = 0$

$\boxed{\text{A. } y^4 + y^2 - 2x = 0}$

$M = y^{-2}$; $N = -2xy^{-3} - y$.

B. $y^4 - 2y^2 + x = 0$

$\frac{\partial N}{\partial x} = -2y^{-3}$; $\frac{\partial M}{\partial y} = -2y^{-3}$

C. $y^3 + y^2 - 2x = 0$

D. $y^4 + 2y^2 - 3x = 0$

Equation is exact

E. $y^3 + y - 2x = 0$

Find $F(x,y)$ such that $\frac{\partial F}{\partial x} = M$, $\frac{\partial F}{\partial y} =$

$\frac{\partial F}{\partial x} = y^{-2}$; $F(x,y) = xy^{-2} + \psi(y)$;

$\frac{\partial F}{\partial y} = -2xy^{-3} + \psi'(y) = -2xy^{-3} - y$; $\psi'(y) = -y$; $\psi(y) =-$

So; $F(x,y) = xy^{-2} - \frac{y^2}{2} = C$; multiply by y^{-2}

$2x - y^4 = 2Cy^2$; $y(1)=1$; $2C=1$; $\boxed{y^2 + y^4 - 2x =}$

4. Let $y(x)$ be the solution of the following initial value problem

$$y'' + 2y^{-1}(y')^2 = y', \quad y(0) = 1, \quad y'(0) = \frac{1}{3}.$$

Find $y(3)$.

Set $V = \frac{dy}{dx}$; $\frac{dV}{dx} = \frac{dV}{dy}\frac{dy}{dx} = V\frac{dV}{dy}$

A. $y(3) = e^3 + 1$

B. $y(3) = e^3$

$V\frac{dV}{dy} + 2y^{-1}V^2 = V$; Since $V(0) = 1/3$, divide by V

C. $y(3) = 2e^2 + 1$

D. $y(3) = e + 1$

$\frac{dV}{dy} + 2y^{-1}V = 1$. Integrating factor y^2

$\boxed{\text{E. } y(3) = e}$

$\frac{d}{dy}(y^2 V) = y^2$; $y^2 V = \frac{1}{3}y^3 + C$.

$y(0)=1$, $V(0) = 1/3$; $\underline{C=0}$

$V = \frac{1}{3}y = \frac{dy}{dx}$; $\boxed{y = e^{x/3}}$

$y(3) = e$

TEST 01.

5. The rank of $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 3 & 1 & 4 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 2 & -1 & 0 \end{bmatrix}$ is equal to

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 3 & 1 & 4 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 2 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & - \\ 0 & 1 & 0 & - \\ 0 & 2 & -1 & \end{bmatrix}$$

A. 3

$\boxed{\text{B. } 4}$

C. 1

D. 2

E. 0

$$\sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & -3 & 4 \end{bmatrix}$$

4 linearly independ rows : Rank 4

$$\sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -4/3 \end{bmatrix}$$

No zero rows In R-E. Form Rank 4

6. Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & -1 & 1 \\ 0 & 2 & -1 \end{bmatrix}$ and let the inverse of A be denoted by $A^{-1} = [b_{jk}]$. Find b_{12}, which is the element in the first row and second column of A^{-1}.

$$b_{12} = \frac{M_{21}}{\det A}$$

A. $b_{12} = 2$

B. $b_{12} = 3$

C. $b_{12} = 4$

$\boxed{\text{D. } b_{12} = 1}$

E. $b_{12} = 5$

$$M_{21} = (-1) \det \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} = +1 .$$

$$\det A = \det \begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix} - \det \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$$

$$= -1 + 2 = 1$$

$$\boxed{b_{12} = 1}$$

TEST 01

7. Let A be the 3×3 matrix of coefficients of the system $A\mathbf{x}=\mathbf{b}$. Given that the reduced row echelon form of the augmented matrix $(A|\mathbf{b})$ is equal to $\begin{pmatrix}1&0&1&1\\0&1&-2&1\\0&0&0&0\end{pmatrix}$. We can say that the solution set of the system $A\mathbf{x}=\mathbf{b}$ is given by

A. $\{(2-t,\ 2t,\ t),\ t\in\mathbb{R}\}$

B. $\boxed{\{(1-t,\ 1+2t,\ t),\ t\in\mathbb{R}\}}$

C. $\{(1+t,\ 1+2t,\ -t),\ t\in\mathbb{R}\}$

D. $\{(1,1,0)\}$

E. $\{(1+t,\ 2+3t,\ t+1)\}$

The ~~co~~ corresponding system is

$x_1+x_3=1$

$x_2-2x_3=1.$

$x_3=t$; $x_1=1-t$

x_2 $\quad x_2=1+2t.$

$\{(1-t,\ 1+2t,\ t)\}$

8. Given that $\det(A)=-8$ for $A=\begin{pmatrix}2&3&0&1\\1&3&2&0\\0&1&0&1\\2&1&0&1\end{pmatrix}$ and $\alpha\in\mathbb{R}$, find the determinant of $\begin{pmatrix}2&3&0&1+\alpha\\1&3&2&0\\0&1&0&1\\2&1&0&1\end{pmatrix}$.

A. $8\alpha-8$

B. $\boxed{4\alpha-8}$

C. $-4\alpha-8$

D. $2\alpha-8$

E. $-2\alpha-8$

$$\det\begin{bmatrix}2&3&0&1+\alpha\\1&3&2&0\\0&1&0&1\\2&1&0&1\end{bmatrix}=$$

$$\det A+\det\begin{bmatrix}0&0&0&\alpha\\1&3&2&0\\0&1&0&1\\2&1&0&1\end{bmatrix}$$

$$\det A+(-1)^5\,\alpha\,\det\begin{bmatrix}1&3&2\\0&1&0\\2&1&0\end{bmatrix}=-\alpha\,(-1)^4\det\begin{bmatrix}1&2\\2&0\end{bmatrix}$$

$=+4\alpha-8$

TEST 01

9. Which of the following statements are correct?

I) The set of solutions to the differential equation $\frac{d^2y}{dx^2} + (1+x^2)\frac{dy}{dx} + xy = 0$ is a subspace of $C^2(\mathbb{R})$.
II) The set $\{p(x) = x^2 + bx + c, \quad b, c \in \mathbb{R}\}$ is a subspace of the space of all polynomials of degree two.
III) Let $\mathbf{x}$ and $\mathbf{y}$ be 3×1 column vectors. Let A be a 2×3 matrix, and suppose that $A\mathbf{x} = 0$ and $A\mathbf{y} = 0$. If $\mathbf{w} = 3\mathbf{x} - 2\mathbf{y}$, then $A\mathbf{w} = 0$.

A. I and III are true, but II is false

B. II and III are true, but I is false

C. I and II are true, but III is false

D. I, II and III are true

E. I, II and III are false

10. Let α and β be such that the vector $(3, \alpha, \beta)$ is in the span of $\vec{v_1} = (5, 2, 1)$ and $\vec{v_2} = (1, 0, 0)$. Then

A. $\alpha = -\beta$

B. $\alpha = 2\beta$

C. $\alpha = -3\beta$

D. $\alpha = 4\beta$

E. $\alpha = \beta$

$(3, \alpha, \beta)$ is in the span of $\vec{v_1}$ and $\vec{v_2}$

if

$$\det \begin{bmatrix} 1 & 0 & 0 \\ 5 & 2 & 1 \\ 3 & \alpha & \beta \end{bmatrix} = 0$$

$2\beta - \alpha = 0$; $\boxed{\alpha = 2\beta}$