1. Evaluate

$$I = \int_0^{\sqrt{2}} \int_x^{\sqrt{4-x^2}} (x^2+y^2)\, dydx$$

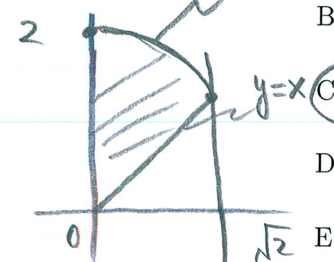
by converting to polar coordinates.

$$I = \int_0^2 \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} r^2\, r\, d\theta\, dr$$

$$\begin{cases} x \le y \le \sqrt{4-x^2} \\ 0 \le x \le \sqrt{2} \end{cases} \quad x^2+y^2=4$$

$$= \frac{\pi}{4} \cdot \frac{1}{4} r^4 \Big|_0^2 \qquad \begin{cases} 0 \le r \le 2 \\ \frac{\pi}{4} \le \theta \le \frac{\pi}{2} \end{cases}$$

$$= \frac{\pi}{4} \cdot \frac{1}{4} \cdot 2^4 = \pi$$



$$y = \sqrt{4-x^2} \Rightarrow x^2+y^2=4$$

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. π

D. $\frac{2\pi}{3}$

E. 2π

2. Find the area of the surface given by $z = x^2 + y$ for (x, y) in

$$D = \{(x,y) : 0 \le y \le 4x,\ 0 \le x \le 1\}.$$

$$A = \iint_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dxdy$$

$$= \int_0^1 \int_0^{4x} \sqrt{1+4x^2+1}\, dy\, dx$$

$$= \sqrt{2} \int_0^1 4x\sqrt{1+2x^2}\, dx$$

$$\underset{du = 4x\,dx}{\overset{u = 1+2x^2}{=\!=\!=}} \sqrt{2}\int_1^3 u^{\frac{1}{2}}\, du = \sqrt{2} \cdot \frac{2}{3} u^{\frac{3}{2}} \Big|_1^3$$

$$= \frac{2\sqrt{2}}{3}\left[3\sqrt{3} - 1\right] = 2\sqrt{6} - \frac{2}{3}\sqrt{2}$$

A. $12\sqrt{6} - 4\sqrt{2}$

B. $2\sqrt{6} - \frac{2}{3}\sqrt{2}$

C. $2\sqrt{6}$

D. $10\sqrt{5} - 2$

E. $\frac{5\sqrt{5}-1}{3}$

3. Let E be the solid region in the first octant that is above the xy plane and below the plane $2x + y + z = 4$. Then the iterated integral satisfying

$$\iiint_E f(x,y,z)dV = \int_0^a \int_0^b \int_0^c f(x,y,z)\; dzdydx$$

must have

A. $a = 4,\ b = 4 - 2x - y$

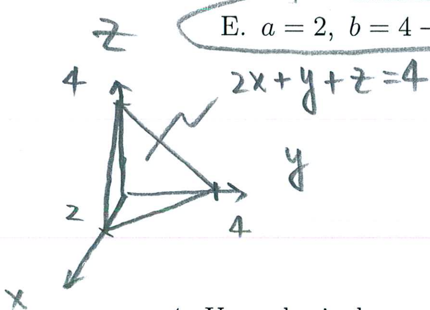
B. $a = 4,\ b = 4 - 2x$

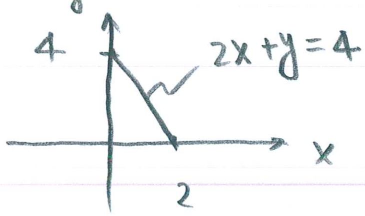
C. $a = 2,\ b = 4 - 2x - y$

D. $a = 2,\ b = 4 - z - 2x$

E. $a = 2,\ b = 4 - 2x$ y

$z = 4 - 2x - y$





$\Rightarrow y = 4 - 2x$

$a = 2$

4. Use spherical coordinates to compute $\iiint_\Omega \sqrt{x^2+y^2+z^2}\; dV$ where Ω is the region above the cone $\sqrt{3}z = \sqrt{x^2+y^2}$ and below the sphere $x^2 + y^2 + z^2 = 10$.

$3z^2 = x^2 + y^2 \Rightarrow 4z^2 = x^2 + y^2 + z^2 = \rho^2$

$\Rightarrow \rho = 2z = 2\rho\cos\varphi \Rightarrow \cos\varphi = \frac{1}{2} \Rightarrow \varphi = \frac{\pi}{3}$

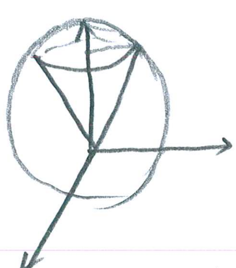
A. 5π

B. 25π

C. $50\pi(1 - \frac{\sqrt{3}}{2})$

D. $50\pi(1 - \frac{1}{\sqrt{2}})$

E. $\frac{5\pi^2}{6} - \frac{5\pi\sqrt{3}}{8}$



$$\begin{cases} 0 \le \rho \le \sqrt{10} \\ 0 \le \varphi \le \frac{\pi}{3} \\ 0 \le \theta \le 2\pi \end{cases}$$

$$I = \int_0^{\sqrt{10}} \int_0^{\frac{\pi}{3}} \int_0^{2\pi} \rho \cdot \rho^2 \sin\varphi \, d\theta\, d\varphi\, d\rho$$

$$= 2\pi \left[-\cos\varphi\right]_0^{\frac{\pi}{3}} \frac{1}{4}\rho^4 \Big|_0^{\sqrt{10}} = 2\pi\left[1 - \frac{1}{2}\right] \cdot \frac{1}{4}\, 100 = 25\pi$$

5. A solid material occupies the region enclosed by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $x + z = 8$. If its mass density per unit volume is $\rho(x, y, z) = y^2$, which of the following integrals equals its mass?

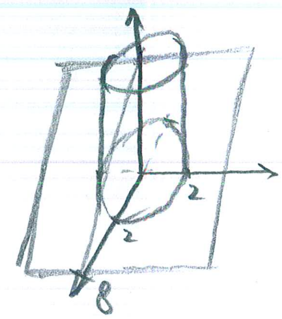
A. $\int_0^{2\pi} \int_0^{2} \int_0^{8-2\cos\theta} r^3 \sin^2\theta \, dz dr d\theta$

B. $\int_0^{2\pi} \int_0^{4} \int_0^{8-2\cos\theta} r^2 \sin^2\theta \, dz dr d\theta$

C. $\int_0^{2\pi} \int_0^{2} \int_0^{8-2\cos\theta} r^2 \sin^2\theta \, dz dr d\theta$

D. $\int_0^{2\pi} \int_0^{2} \int_0^{8-r\cos\theta} r^3 \sin^2\theta \, dz dr d\theta$

E. $\int_0^{2\pi} \int_0^{2} \int_0^{8-r\cos\theta} r^2 \sin^2\theta \, dz dr d\theta$

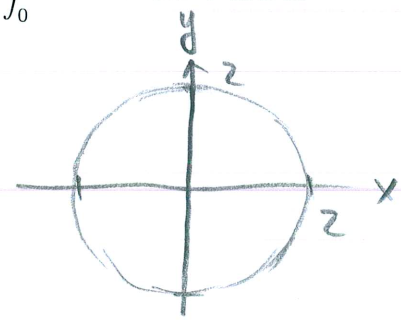


$0 \le z \le 8 - x = 8 - r\cos\theta$

$0 \le r \le 2$

$0 \le \theta \le 2\pi$

$y^2 = r^2 \sin^2\theta$



6. Evaluate the line integral $\int_C 12x \cos z \, ds$, where the curve C is parameterized by $\vec{r}(t) = \langle t, t^2 + 3, 0\rangle$ for $0 \le t \le 1$.

A. 1

B. $5^{\frac{3}{2}}$

C. $\frac{3}{2}\left(5^{\frac{3}{2}} - 1\right)$

D. $\frac{1}{2}\left(5^{\frac{3}{2}} - 1\right)$

E. $5^{\frac{3}{2}} - 1$

$$\int_C 12x\cos z \, ds = \int_0^1 12t \cdot \cos 0 \cdot \sqrt{1^2 + (2t)^2 + 0^2}\, dt$$

$$= \int_0^1 12t\sqrt{1+4t^2}\, dt$$

$u = 1 + 4t^2$, $du = 8t\,dt$

$$\int_1^5 \frac{12}{8} u^{\frac{1}{2}} du = \frac{3}{2}\cdot\frac{2}{3} u^{\frac{3}{2}}\Big|_1^5$$

$$= 5^{\frac{3}{2}} - 1$$

7. Let $\vec{r}(t) = \langle \cos t,\, \sin t,\, t\rangle$, $0 \le t \le \pi$, be the parametrization of the curve C, and let $\vec{F} = \nabla f$ with $f(x, y, z) = y^3 + 2xz$. The line integral $\int_C \vec{F} \cdot d\vec{r}$ is

$$\int_C \vec{F}\cdot d\vec{r} = f(\vec{r}(\pi)) - f(\vec{r}(0))$$

$$= f(-1, 0, \pi) - f(1, 0, 0)$$

$$= [0 + 2\cdot(-1)\cdot\pi] - [0 + 0]$$

$$= -2\pi$$

A. -2π

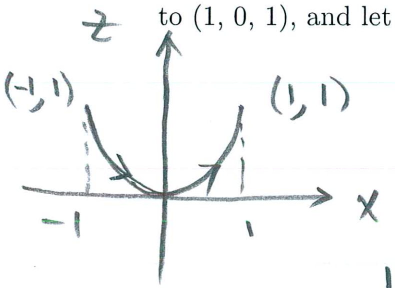
B. 3π

C. 6π

D. π

E. 4

8. Let C be the oriented curve described by the intersection of $z = x^2$ and $y = 0$ from $(-1, 0, 1)$ to $(1, 0, 1)$, and let $\vec{F} = \langle 3x^2, -e^x y, 1\rangle$. Then the line integral $\int_C \vec{F} \cdot d\vec{r}$ is



$\vec{r}(t) = \langle t, 0, t^2\rangle$

$t: -1 \to 1$

$$\int_C \vec{F}\cdot d\vec{r} = \int_{-1}^{1} \vec{F}(\vec{r}(t))\cdot \vec{r}\,'(t)\,dt$$

$$= \int_{-1}^{1} \vec{F}(t, 0, t^2)\cdot\langle 1, 0, 2t\rangle\,dt$$

$$= \int_{-1}^{1} \langle 3t^2, 0, 1\rangle\cdot\langle 1, 0, 2t\rangle\,dt$$

$$= \int_{-1}^{1}(3t^2 + 2t)\,dt = 2\int_0^1 3t^2\,dt = 2t^3\Big|_0^1 = 2$$

A. -4

B. -2

C. 2

D. 4

E. 0

9. A potential function for the vector field $\vec{F}(x, y) = \langle 2e^y, 2xe^y + y\rangle$ is

A. There is no potential function for this vector field

B. $2xe^y + xy + C$

C. $2xe^y + x^2y + C$

D. $2xe^y + \frac{y^2}{2} + C$

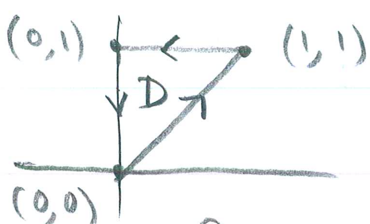
E. $2xe^y + C$

$$\left\{\begin{array}{l} \frac{\partial f}{\partial x} = 2e^y \Rightarrow f(x,y) = 2xe^y + g(y) \\ \frac{\partial f}{\partial y} = 2xe^y + y = 2xe^y + g'(y) \end{array}\right.$$

$$\Rightarrow g'(y) = y \Rightarrow g(y) = \frac{1}{2}y^2 + C$$

$$\Rightarrow f(x,y) = 2xe^y + \frac{1}{2}y^2 + C$$

10. Let C be the boundary of the triangle with vertices $(0, 0)$, $(1, 1)$, and $(0, 1)$, and C is oriented counterclockwise, then the line integral $\int_C \sin y\, dx + (x\cos y + 4x)\, dy$ is



A. 1

B. 2

C. 4

D. -1

E. -2

$$\int_C \sin y\, dx + (x\cos y + 4x)\, dy$$

$$= \iint_D (\cos y + 4 - \cos y)\, dx\, dy = 4|D| = 4 \cdot \frac{1}{2} = 2$$