MA 26100
FALL 2017
EXAM 2

1. Find the maximum value of $f(x,y,z) = 2x+y+4z$ subject to the constraint $x^2+y+z^2 = 6$. (You may assume that this function has an absolute maximum and no absolute minimum.)

A. 6

B. 7

C. $\boxed{11}$

D. $4\sqrt{6}$

E. $6\sqrt{6}$

$\nabla f = \lambda \nabla g$ $\quad \nabla g = \langle 2x, 1, 2z \rangle \neq \vec{0}$

$2 = 2x\lambda \rightarrow x = 1$

$1 = \lambda$

$4 = 2z\lambda \rightarrow z = 2$

$1^2 + y + 2^2 = 6$

$y = 1$

$f(1,1,2) = 2+1+8$

2. Compute the following double integral by changing the order of integration:

$$\int_0^4 \int_{\sqrt{x}}^2 y\cos(y^4)\, dy\, dx$$

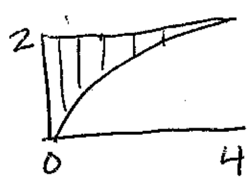
A. $\boxed{\dfrac{\sin 16}{4}}$

B. $\dfrac{8\sin 16}{3}$

C. $2\sin 16$

D. $\dfrac{3\sin 16}{4}$

E. $\dfrac{\sin 16}{8}$



$\int_0^2 \int_0^{y^2} y\cos(y^4)\, dx\, dy$

$= \int_0^2 y^3 \cos(y^4)\, dy$

$= \frac{1}{4}\sin(y^4)\Big|_0^2$

3. D is the region between the circles $x^2+y^2=1$ and $x^2+y^2=2$. Find $\iint_D e^{x^2+y^2}\,dA$.

A. $\boxed{\pi e^2 - \pi e}$

B. $2\pi e^2 - 2\pi e$

C. $\pi e^4 - \pi e$

D. $2\pi e^2$

E. $6\pi e^4$

$$\int_0^{2\pi}\int_1^{\sqrt{2}} e^{r^2}\, r\,dr\,d\theta$$

$$= 2\pi \cdot \frac{1}{2} e^{r^2} \Big|_{r=1}^{r=\sqrt{2}}$$

4. Find the volume of the part of the unit ball (radius 1, center at the origin) that lies between the cones $\phi = \pi/6$ and $\phi = \pi/3$.

A. $\dfrac{\pi(\sqrt{2}-1)}{3}$

B. $\dfrac{\pi^2}{3}$

C. $\pi(\sqrt{3}-1)$

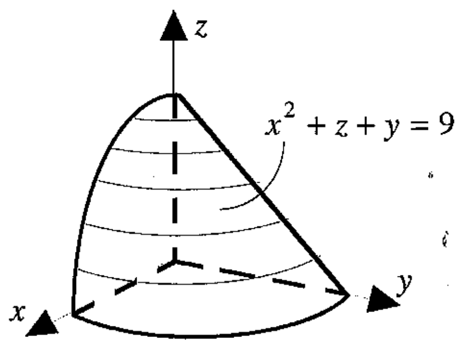
D. $\boxed{\dfrac{\pi(\sqrt{3}-1)}{3}}$

E. $\pi(\sqrt{2}-1)$

$$\int_0^{2\pi}\int_{\pi/6}^{\pi/3}\int_0^1 \rho^2 \sin\phi \,d\rho\,d\phi\,d\theta$$

$$= 2\pi \left[\frac{1}{3}\rho^3\right]_0^1 \left[\cos\phi\right]_{\pi/3}^{\pi/6}$$

5. Which of the following is an **INCORRECT** setup for $\iiint_E f(x,y,z)\ dV$, where E is the solid region in the first octant bounded by the surface $x^2+z+y=9$ and the three planes $x=0$, $y=0$, and $z=0$? (see figure below:)



$\sqrt{9-z}$

A. $\int_0^3 \int_0^{9-x^2} \int_0^{9-y-x^2} f(x,y,z)\ dz\ dy\ dx$

B. $\boxed{\int_0^9 \int_0^{9-z} \int_0^{9-z-x^2} f(x,y,z)\ dy\ dx\ dz}$

C. $\int_0^3 \int_0^{9-x^2} \int_0^{9-z-x^2} f(x,y,z)\ dy\ dz\ dx$

D. $\int_0^9 \int_0^{9-z} \int_0^{\sqrt{9-z-y}} f(x,y,z)\ dx\ dy\ dz$

E. $\int_0^9 \int_0^{\sqrt{9-y}} \int_0^{9-y-x^2} f(x,y,z)\ dz\ dx\ dy$

6. Do NOT evaluate. Rewrite the integral in cylindrical coordinates.

$$\int_0^2 \int_0^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} \sqrt{x^2+y^2}\ dz\ dx\ dy$$

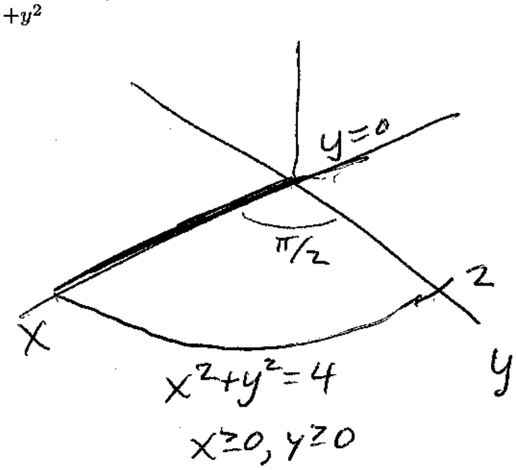
A. $\boxed{\int_0^{\pi/2} \int_0^2 \int_r^{\sqrt{8-r^2}} r^2\ dz\ dr\ d\theta}$

B. $\int_0^{\pi/2} \int_0^2 \int_r^{\sqrt{8-r^2}} r\ dz\ dr\ d\theta$

C. $\int_0^{\pi} \int_0^2 \int_r^{\sqrt{8-r^2}} r^2\ dz\ dr\ d\theta$

D. $\int_0^{\pi} \int_0^2 \int_r^{\sqrt{8-r^2}} r\ dz\ dr\ d\theta$

E. None of the above.



7. Find the surface area of the part of the paraboloid $z = 2 - x^2 - y^2$ that lies above the xy-plane.

A. $\dfrac{(3\sqrt{3}-1)\pi}{2}$

B. $\dfrac{17\sqrt{17}\pi}{6}$

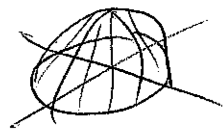
C. $\boxed{\dfrac{13\pi}{3}}$

D. $2\sqrt{2}\pi$

E. $\dfrac{11\pi}{6}$

$z=0=2-x^2-y^2$

$r^2=2$

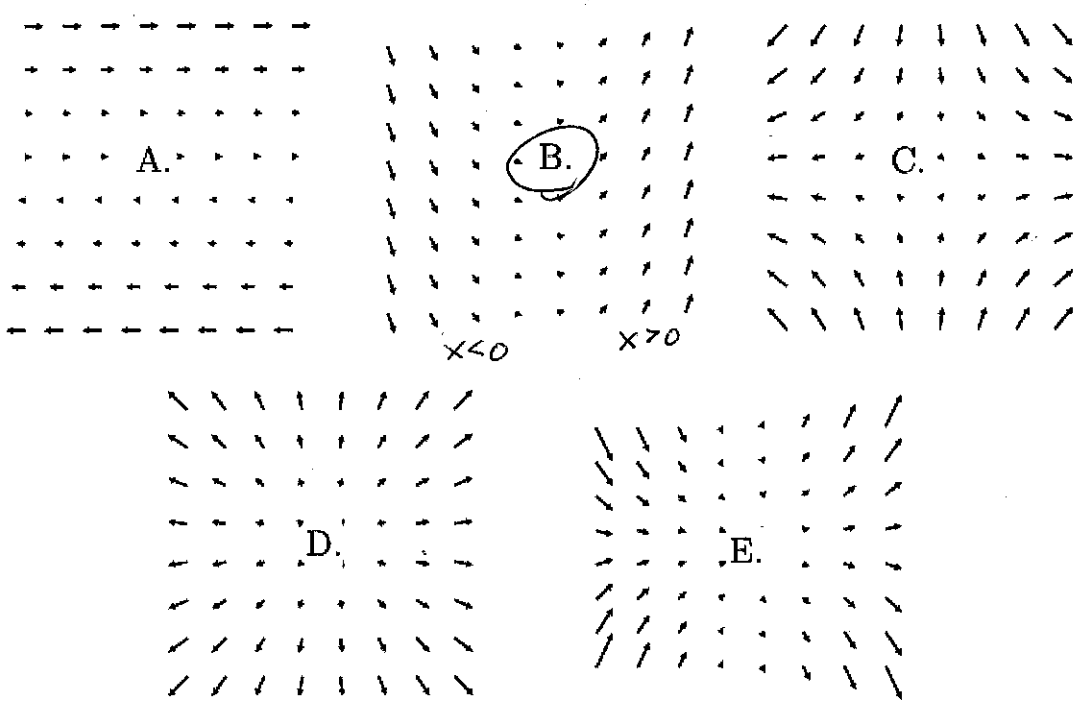


$$\iint \sqrt{1+f_x^2+f_y^2}\,dA = \iint \sqrt{1+(-2x)^2+(-2y)^2}\,dA = \iint \sqrt{1+4(x^2+y^2)}\,dA$$

$$= \int_0^{2\pi}\int_0^{\sqrt{2}} \sqrt{1+4r^2}\,r\,dr\,d\theta = \left(\int_0^{2\pi} d\theta\right)\left(\int_1^9 \sqrt{u}\left(\tfrac{1}{8}\right)du\right)$$

$$= \frac{2\pi}{8}\cdot\frac{2}{3}u^{3/2}\Big|_1^9$$

8. Select the correct plot for the vector field $\mathbf{F}(x,y) = \mathbf{i} + x\mathbf{j}$.



9. Evaluate the line integral

$$\int_C (x + 2y + z)\, ds$$

where C is the line segment from $(1, 1, 1)$ to $(3, 2, 3)$.

A. 5/3

B. 7

C. 11

D. 14

E. $\boxed{21}$

$ds = \sqrt{2^2 + 1^2 + 2^2}\,dt = 3dt$

$\int_0^1 [(1+2t) + 2(1+t) + (1+2t)]\, 3\, dt$

$= 3\int_0^1 (4+6t)\, dt$

$= 21$

10. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y) = xy\mathbf{i} - y\mathbf{j}$ and C is given by $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j}$, $0 \le t \le 1$.

A. 1

B. -1

C. $\frac{1}{4}$

D. 0

E. $\boxed{-\frac{1}{4}}$

$\vec{F}(\vec{r}(t)) = (t)(t^2)\vec{i} - (t^2)\vec{j}$

$\vec{r}\,'(t) = \vec{i} + 2t\vec{j}$

$\vec{F} \cdot \vec{r} = t^3 - 2t^3 = -t^3$

$\int_0^1 -t^3\, dt = -\frac{1}{4}t^4 \Big|_0^1$