B D A C D A B A E D

Math 26100 Exam 2

11/12/15

$\boxed{\text{Version01}}$

Name:

Section number:

10 digit PU-ID Number:

Signature:

(1) **Do Not Open** until instructed to do so.
(2) You have to be in your section and in your assigned seat.
(3) When time is called: **Remain in Your Seat.**
(4) You may not use any electronic devices or have them out.
(5) You may not have anything else out, like paper, extra pens etc. Just the exam, scantron, one pencil and eraser (optional).
(6) Use No 2 pencil.
(7) **Mark Your Test With The Version Number!**
(8) Sign the exam policies on the next page.
(9) There are 10 problems each worth 10 points.
(10) If you take the exam apart, mark each page with your name.

General Exam Policies

(1) Students may not open the exam until instructed to do so.
(2) Students must obey the orders and requests by all proctors, TAs, and lecturers.
(3) No student may leave in the first 20 min or in the last 10 min of the exam.
(4) Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
(5) After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
(6) Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: ______________________________

STUDENT SIGNATURE: ______________________________

PROBLEM 1: Find the minimum value of the function $f(x,y,z) = x^2+y^2+z^2$ on the plane $x+2y+2z=9$. You may assume that this function has a global minimum.

A. 10

B. 9

C. $\frac{25}{2}$

D. $\frac{5}{2}$

E. 5

$f(x,y,z) = x^2+y^2+z^2$, $g(x,y,z) = x+2y+2z=9$

(1) $\nabla f = \lambda g$ $\quad$ (2) $g = 9$

$$\left.\begin{array}{l} 2x=\lambda \\ 2y = 2\lambda \\ 2z = 2\lambda \end{array}\right\} \rightsquigarrow \lambda = z = y = 2x \quad (*)$$

$$x+2y+2z \overset{(*)}{=} x+4x+4y = 9x \overset{(2)}{=} 9$$

$x=1$, $y=z=2$

$f(1,2,2) = 1+4+4 = 9$

PROBLEM 2: Evaluate the integral $\int\int_R \frac{2xy^2}{x^2+1}\,dA$ where $R=[0,1]\times[0,2]$

A. $\frac{1}{3}\ln 5$

B. $4\ln 2$

C. $\frac{1}{6}\ln 5$

D. $\frac{8}{3}\ln 2$

E. $\frac{4}{3}\ln 2$

$$\int_0^1\int_0^2 \frac{2xy^2}{x^2+1}\,dy\,dx$$

$$= \int_0^1 \frac{2}{3}\frac{xy^3}{x^2+1}\Big|_0^2 dx$$

$$= \frac{2}{3}\int_0^1 \frac{8x}{x^2+1}\,dx$$

$$= \frac{8}{3}\ln(x^2+1)\Big|_0^1 = \frac{8}{3}\ln 2$$

PROBLEM 3: Calculate the integral

$$\int_0^2 \int_{\sqrt{\frac{y}{2}}}^1 xe^y \,dxdy$$

by reversing the order of integration.

$x = \sqrt{\frac{y}{2}}$

$y = 2x^2$

A. $\frac{1}{4}e^2 - \frac{3}{4}$

B. $\frac{1}{2}e^2 - \frac{3}{2}$

C. $\frac{1}{2}e^{2x}$

D. $\frac{1}{2}$

E. 2

$D = \{(x,y) \mid 0 \le x \le 1,\ 0 \le y \le 2x^2\}$

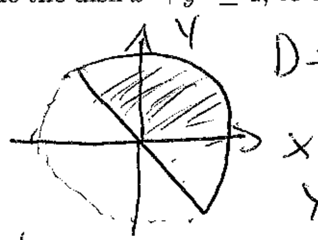
$\int_0^1 \int_0^{2x^2} x e^y \,dy\,dx = \int_0^1 x e^y \Big|_0^{2x^2} dx$

$= \int_0^1 x e^{2x^2} - x \,dx = \frac{1}{4} e^{2x^2} - \frac{1}{2}x \Big|_0^1 = \frac{1}{4}e^2 - \frac{3}{4}$

PROBLEM 4: Evaluate

$$\iint_D e^{-x^2-y^2} dA$$

where D is the region inside the disk $x^2+y^2 \le 1$, to the right of $y=-x$, and above the x axis.



$y = -x$

$D = \{(r,\theta) \mid 0 \le r \le 1,\ 0 \le \theta \le \frac{3}{4}\pi\}$

A. $\frac{\pi}{2}(1-\frac{1}{e})$

B. $\frac{5\pi}{8}(1-\frac{1}{e})$

C. $\frac{3\pi}{8}(1-\frac{1}{e})$

D. $\frac{3\pi}{8}(1-e)$

E. $\frac{\pi}{2}(1-e)$

$\int_0^{\frac{3}{4}\pi}\int_0^1 e^{-r^2} r \,dr\,d\theta$

$= \frac{3}{4}\pi \left(-\frac{1}{2}e^{-r^2}\right)\Big|_0^1$

$= \frac{3}{4}\pi \left(-\frac{1}{2}e^{-1} + \frac{1}{2}\right)$

$= \frac{3}{8}\pi\left(1 - \frac{1}{e}\right)$

PROBLEM 5: Find the area of the surface $z = 2x + 3y$ that lies above the region in the 1st quadrant between $y = 1$ and $y = x^2$.

A. $2\sqrt{13}/3$

B. $\sqrt{14}/3$

C. $\sqrt{13}/3$

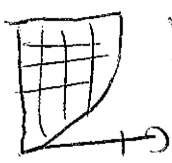
D. $2\sqrt{14}/3$

E. $\sqrt{13}/2$

$f(x,y) = 2x+3y$

$f_x = 2$

$f_y = 3$



$y = x^2$

$D = \{(x,y) \mid 0 \le x \le 1,\ x^2 \le y \le 1\}$

$$A = \int_0^1 \int_{x^2}^1 \sqrt{f_x^2 + f_y^2 + 1}\, dy\, dx$$

$$= \int_0^1 \int_{\cancel{x}^2}^1 \sqrt{4+9+1}\, dy\, dx = \sqrt{14} \int_0^1 1 - x^2\, dx$$

$$= \sqrt{14}\left(x - \tfrac{1}{3}x^3\right)\Big|_0^1 = \sqrt{14}\left(1 - \tfrac{1}{3}\right) = +\tfrac{2}{3}\sqrt{14}$$

PROBLEM 6: A solid occupies the region in the first octant bounded by the surfaces

$$x^2 + z^2 = 9,\ y = 2x,\ y = 0,\ \text{and } z = 0.$$

Its mass density is $\rho(x,y,z) = yz$. Which integral gives the mass of the solid?

A. $\displaystyle\int_0^3 \int_0^{2x} \int_0^{\sqrt{9-x^2}} yz\,dz\,dy\,dx$

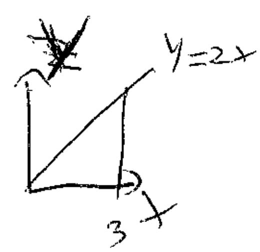
B. $\displaystyle\int_0^3 \int_0^{y/2} \int_0^{\sqrt{9-y^2/4}} yz\,dz\,dy\,dx$

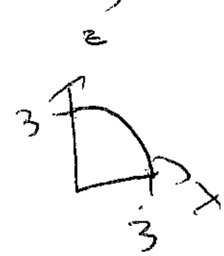
C. $\displaystyle\int_0^6 \int_0^{2x} \int_0^{x^2+z^2} yz\,dz\,dy\,dx$

D. $\displaystyle\int_0^6 \int_0^{x/2} \int_0^{\sqrt{9-x^2}} yz\,dz\,dy\,dx$

E. $\displaystyle\int_0^3 \int_0^{2x} \int_0^{x^2+z^2} yz\,dz\,dy\,dx$

$E = \{(x,y,z) \mid 0 \le x \le 3,\ 0 \le y \le 2x,\ 0 \le z \le \sqrt{9-x^2}\}$





PROBLEM 7: Calculate the volume of the solid inside $x^2+y^2+z^2=4$, below $z=\sqrt{x^2+y^2}$ and above the $x-y$ plane.

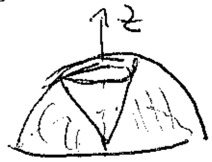
A. $\frac{8}{3}\pi^2$

B. $\frac{8}{3}\sqrt{2}\pi$

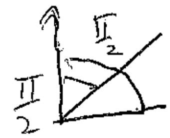
C. $\frac{8}{3}\pi$

D. $8\pi^2$

E. $\frac{16}{3}\sqrt{2}\pi$



$$E = \{(\rho,\theta,\varphi) \mid 0 \le \rho \le 2,\ 0 \le \theta \le 2\pi,\ \frac{\pi}{4} \le \varphi \le \frac{\pi}{2}\}$$



$$\int_{\pi/4}^{\pi/2}\int_0^{2\pi}\int_0^2 \rho^2 \sin\varphi \, d\rho\, d\theta\, d\varphi$$

$$= \frac{1}{3}\rho^3\Big|_0^2 \; 2\pi \; (-\cos\varphi)\Big|_{\pi/4}^{\pi/2} = \frac{8}{3}\cdot 2\pi \cdot \frac{1}{\sqrt{2}} = \frac{8}{3}\sqrt{2}\pi$$

PROBLEM 8: Consider the curve C parameterized by $\mathbf{r}(t) = \langle t, \frac{2\sqrt{2}}{3}t^{\frac{3}{2}}, \frac{1}{2}t^2\rangle$ where $t \in [0,1]$. Calculate

$$\int_C (x+z)ds$$

A. $\frac{9}{8}$

B. $\frac{9}{4}$

C. $\frac{5}{6}$

D. $2\sqrt{2}+1$

E. $\frac{3}{2}$

$$r'(t) = \langle 1, \sqrt{2}t^{\frac{1}{2}}, t\rangle$$

$$|r'(t)| = \sqrt{1+2t+t^2} = 1+t$$

$$\int_C x+z \, ds = \int_0^1 \left(t+\frac{1}{2}t^2\right)(1+t)\,dt$$

$$= \int_0^1 t + \frac{3}{2}t^2 + \frac{1}{2}t^3 \, dt$$

$$= \frac{1}{2}t^2 + \frac{1}{2}t^3 + \frac{1}{8}t^4\Big|_0^1$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{1}{8} = \frac{9}{8}$$

Name:

PROBLEM 9: Consider the vector field $\mathbf{F} = \langle \sqrt{5+y^2}, 3x \rangle$ and let C be the curve which is given by $x = y^2$ for $0 \leq y \leq 2$

Calculate

$$\int_C \mathbf{F} \cdot d\mathbf{r} \qquad r(t) = \langle t^2, t \rangle \quad 0 \leq t \leq 2$$

A. 26

$$r'(t) = \langle 2t, 1 \rangle$$

B. $13 - \frac{5}{3}\sqrt{5}$

$$\int_0^2 F \cdot r'(t)\,dt = \int_0^2 \sqrt{5+t^2} \cdot 2t + 3t^2 \cdot 1 \, dt$$

C. 13

D. $\frac{1}{\sqrt{2}}$

$$= \frac{2}{3}(5+t^2)^{\frac{3}{2}} + t^3 \Big|_0^2 = \frac{2}{3} 9^{\frac{3}{2}} - \frac{2}{3} 5\sqrt{5}$$

E. $26 - \frac{10}{3}\sqrt{5}$

$$= 26 - \frac{10}{3} 5\sqrt{5}$$

PROBLEM 10: Let C be an oriented curve in $\mathbb{R}^3$, $f = f(x,y,z)$ a function and $\mathbf{F}$ a vector field. Which of the following are true

✓ (1) $\int_C \mathbf{F} \cdot d\mathbf{r}$ is independent of parametrization.

✓ (2) $\int_C \mathbf{F} \cdot d\mathbf{r}$ changes sign if the direction (i.e. orientation) of the curve is changed.

— (3) $\int_C f ds$ changes sign if the direction (i.e. orientation) of the curve is changed.

— (4) $\int_C \mathbf{F} \cdot d\mathbf{r}$ is independent of path.

✓ (5) $\int_C \nabla f \cdot d\mathbf{r}$ is independent of path.

A. 1,3,5

B. 2,4,5

C. all

D. 1,2,5

E. 1,2,4