1. The two values of x for which the vectors $\langle x^2, 1, 3\rangle$ and $\langle 1, -5x, 2\rangle$ are perpendicular are

$0 = \langle x^2, 1, 3\rangle \cdot \langle 1, -5x, 2\rangle$

$= x^2 - 5x + 6 = (x-2)(x-3)$

$\Rightarrow \underline{x = 2, 3}$

A. 2, −3

B. −2, 3

C. 0, 2

D. 0, 3

E. 2, 3

2. The plane passing through the point (0, 1, 0) and parallel to the plane $x + y - 2z = 3$ intersects the x-axis at the point:

$0 = \langle x, y-1, z\rangle \cdot \langle 1, 1, -2\rangle$

$= x + y - 1 - 2z$

$= x + y - 2z - 1$

$\underline{x\text{-axis}}$ $y = 0, z = 0$

$\Rightarrow 0 = x - 1 \Rightarrow x = 1$

$(1, 0, 0)$

A. (−1, 0, 0)

B. (1, 0, 0)

C. (−2, 0, 0)

D. (2, 0, 0)

E. (−3, 0, 0)

3. The arc-length of the curve defined by $\vec{r}(t) = \langle t^3, \frac{\sqrt{6}}{2} t^2, t \rangle$ for $-1 \le t \le 1$ is

$$L = \int_{-1}^{1} |\vec{r}\,'(t)|\,dt$$

$$= \int_{-1}^{1} \sqrt{(3t^2)^2 + (\sqrt{6}t)^2 + 1^2}\,dt$$

$$= \int_{-1}^{1} \sqrt{(3t^2)^2 + 6t^2 + 1}\,dt$$

$$= \int_{-1}^{1} (3t^2 + 1)\,dt = \left[t^3 + t\right]_{-1}^{1} = 2 + 2 = 4$$

A. 2

B. 3

C. 4

D. 5

E. 6

4. $\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2} =$

$\underline{y = mx}$ $\lim_{x\to 0} \frac{mx^3}{x^4+m^2x^2} = \lim_{x\to 0} \frac{mx}{x^2+m^2} = \begin{cases} 0 & m \neq 0 \\ 0 & m = 0 \end{cases}$

$\underline{y = mx^2}$ $\lim_{x\to 0} \frac{mx^4}{x^4+m^2x^4} = \frac{m}{1+m^2}$

A. 0

B. 1/2

C. 1

D. ∞

E. Does not exist

5. Find the equation of the line that ~~contains~~ passes through the point (1, 2, 1) and that is parallel to the vector tangent to the curve $\vec{r}(t) = \langle t^2 + 3t + 2,\ e^t \cos t,\ \ln(t+1)\rangle$ at (2, 1, 0).

A. $x = 1 + 3t,\ y = 2 + t,\ z = 1 + t$

B. $x = 3 + 2t,\ y = e^t(\cos t - \sin t),\ z = \frac{1}{1+t}$

C. $x = 3 + 2t,\ y = 1 + t,\ z = 1$

D. $x = 2 + 3t,\ y = 1 + t,\ z = t$

E. $x = 3t,\ y = 2t,\ z = 3 - 3t$

$\vec{r}'(t) = \langle 2t+3,\ e^t(\cos t - \sin t),\ \frac{1}{t+1}\rangle$

$\vec{r}(t) = \langle t^2+3t+2,\ e^t\cos t,\ \ln(t+1)\rangle = \langle 2,1,0\rangle$

$\ln(t+1) = 0 \Rightarrow t = 0$

$\vec{r}(0) = \langle 2, 1, 0\rangle$

$\vec{r}'(0) = \langle 3, 1, 1\rangle$

$x = 1 + 3t$
$y = 2 + t$
$z = 1 + t$

6. A particle starts at the origin with initial velocity $\vec{i} - \vec{j}$. Its acceleration is $\vec{a}(t) = t\vec{i} + t\vec{j} + \vec{k}$. Find its position at $t = 1$.

A. $\vec{i} + \vec{j} + \vec{k}$

B. $\vec{i} + \vec{j}$

C. $\frac{3}{2}\vec{i} - \frac{1}{2}\vec{j} + \vec{k}$

D. $\frac{7}{6}\vec{i} - \frac{5}{6}\vec{j} + \frac{1}{2}\vec{k}$

E. $\frac{4}{3}\vec{i} + \frac{2}{3}\vec{j} + \frac{1}{2}\vec{k}$

$\vec{v}(t) = \int \vec{a}(t)\,dt = \langle \frac{1}{2}t^2, \frac{1}{2}t^2, t\rangle + \vec{c}$

$\langle 1, -1, 0\rangle = \vec{v}(0) = \vec{c}$

$\vec{v}(t) = \langle \frac{1}{2}t^2 + 1, \frac{1}{2}t^2 - 1, t\rangle$

$\vec{r}(t) = \int \vec{v}(t)\,dt = \langle \frac{1}{6}t^3 + t, \frac{1}{6}t^3 - t, \frac{1}{2}t^2\rangle + \vec{c}$

$\langle 0,0,0\rangle = \vec{r}(0) = \vec{c}$

$\vec{r}(t) = \langle \frac{1}{6}t^3 + t, \frac{1}{6}t^3 - t, \frac{1}{2}t^2\rangle$

$\vec{r}(1) = \langle \frac{7}{6}, -\frac{5}{6}, \frac{1}{2}\rangle$

7. Using the linear approximation to $f(x,y)=\sqrt{x^2+3y}$ at the point $(4,3)$, the approximate value of $\sqrt{(4.02)^2+3(2.97)}$ is:

$f(4,3)=\sqrt{4^2+3^2}=5$

$\frac{\partial f}{\partial x}(4,3)=\frac{1}{2}(x^2+3y)^{-\frac{1}{2}}\cdot 2x\Big|_{(4,3)}=\frac{4}{5}$

$\frac{\partial f}{\partial y}(4,3)=\frac{1}{2}(x^2+3y)^{-\frac{1}{2}}\cdot 3\Big|_{(4,3)}=\frac{3}{10}$

$f(4.02,2.97)\approx f(4,3)+\langle\frac{4}{5},\frac{3}{10}\rangle\cdot\langle 0.02,-0.03\rangle$

$=5+0.016-0.009$

$=5+0.007=5.007$

A. 5.004

B. 5.05

C. 4.95

D. 4.093

E. 5.007

8. If $z=\sin(xy)$, $x=\pi t^2$, and $y=h(t)$ with $h(1)=\frac{1}{3}$ and $h'(1)=2$, what is $\frac{dz}{dt}$ when $t=1$?

$x(1)=\pi$

$y(1)=h(1)=\frac{1}{3}$

$\frac{dz}{dt}\Big|_{t=1}=\frac{\partial z}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dt}\Big|_{t=1}$

$=\left[y\cos(xy)\cdot 2\pi t+x\cos(xy)\,h'(t)\right]_{t=1}$

$=\frac{2}{3}\pi\cos\left(\frac{\pi}{3}\right)+2\pi\cos\left(\frac{\pi}{3}\right)$

$=\frac{8\pi}{3}\cos\frac{\pi}{3}=\frac{8\pi}{3}\cdot\frac{1}{2}=\frac{4\pi}{3}$

A. $\frac{4\pi\sqrt{3}}{3}$

B. $\frac{4\pi}{3}$

C. $\pi+1$

D. $(\pi+1)\sqrt{3}$

E. $\sqrt{3}\pi+1$

9. The directional derivative of $f(x,y,z) = y^2 e^{x-z}$ at the point $(3,1,2)$ in the direction $\vec{u} = 2\vec{i} + 5\vec{j} + \vec{k}$ is:

$|\vec{u}| = \sqrt{2^2+5^2+1^2} = \sqrt{30}$

$\frac{\vec{u}}{|\vec{u}|} = \langle 2,5,1\rangle / \sqrt{30}$

$\nabla f\Big|_{(3,1,2)} = \langle y^2 e^{x-z}, 2ye^{x-z}, -y^2e^{x-z}\rangle\Big|_{(3,1,2)}$

$= \langle e, 2e, -e\rangle = e\langle 1,2,-1\rangle$

$D_{\vec{u}} f(3,1,2) = e\langle 1,2,-1\rangle \cdot \langle 2,5,1\rangle / \sqrt{30}$

$= \frac{e[2+10-1]}{\sqrt{30}} = \frac{11}{\sqrt{30}}e$

A. $\frac{13e}{\sqrt{30}}$

B. $13e$

C. $\frac{35e}{\sqrt{14}}$

D. $\frac{11e}{\sqrt{30}}$

E. $11e$

10. The function $f(x,y) = 2x^3 + 6xy + 3y^2$ has:

A. one local maximum and one local minimum

B. one local maximum and one saddle point

C. one local minimum and one saddle point

D. 2 saddle points

E. 2 local maxima

$\nabla f = \langle 6x^2+6y, 6x+6y\rangle$

$\begin{cases} x^2+y=0 \\ x+y=0 \end{cases} \Rightarrow 0 = x^2 - x = x(x-1)$

$\Rightarrow x = 0$ or 1

$\Rightarrow y = 0$ or -1

$(0,0)$ and $(1,-1)$

$f_{xx} = 12x$, $f_{yy} = 6$, $f_{xy} = 6$

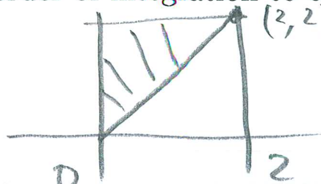
$D(0,0) = \begin{vmatrix} 0 & 6 \\ 6 & 6 \end{vmatrix} = -36 < 0 \Longrightarrow$ saddle pt.

$D(1,-1) = \begin{vmatrix} 12 & 6 \\ 6 & 6 \end{vmatrix} = 6\times 6 = 36 > 0$

$f_{xx}(1,-1) = 12 > 0 \Rightarrow$ l. min

11. Reverse the order of integration to compute $\int_0^2 \int_x^2 e^{y^2}\, dy dx. = I$

$x \le y \le 2$
$0 \le x \le 2$



$\begin{cases} 0 \le x \le y \\ 0 \le y \le 2 \end{cases}$

$I = \int_0^2 dy \left(\int_0^y e^{y^2} dx \right) = \int_0^2 y e^{y^2} dy$

$= \frac{1}{2} e^{y^2} \Big|_0^2 = \frac{1}{2}\left[e^4 - 1\right]$

A. $\frac{1}{2}(e^4 - 1)$

B. $(e^2 - 1)$

C. $\frac{1}{3}(e^3 - 1)$

D. $(e^2 - e)$

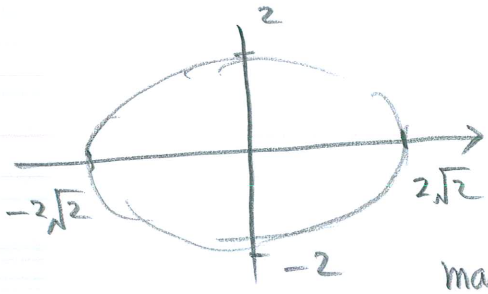
E. $\frac{1}{2}(e^2 - e)$

12. Find the absolute maximum and absolute minimum values of $f(x, y) = x^2 + y^2 + 2y$ on the ellipse $x^2 + 2y^2 = 8$.

$x^2 = 8 - 2y^2$ $\quad f(x,y) = 8 - 2y^2 + y^2 + 2y$

$= -\left[y^2 - 2y\right] + 8$

$= -(y-1)^2 + 9 = g(y)$



max/min $\left[9 - (y-1)^2\right]$

$-2 \le y \le 2$ $\qquad -3 \le y - 1 \le 1$

max $= 9 = g(1)$ and min $= 9 - 9 = 0 = g(-2)$

A. max is 8, min is 0

B. max is 9, min is 0

C. max is 8, min is -2

D. max is 9, min is -2

E. max is 9, min is -4

$\nabla f = \langle 2x, 2y + 2 \rangle = \lambda \nabla g = \langle 2\lambda x, 4\lambda y \rangle \Rightarrow \begin{cases} x = \lambda x \Rightarrow x(1-\lambda) = 0 \\ y + 1 = 2\lambda y \\ x^2 + 2y^2 = 8 \end{cases}$

$x = 0 \xrightarrow{x^2+2y^2=8} 2y^2 = 8 \Rightarrow y = \pm 2$

$(2\lambda - 1) y = 1 \Rightarrow 2\lambda - 1 = \frac{1}{y} = \pm\frac{1}{2} \Rightarrow \lambda = \frac{1}{2}\left(\pm\frac{1}{2} + 1\right) = \frac{3}{4}$ or $\frac{1}{4}$

min $0 = f(0, -2)$ and $f(0, 2) = 8$, $\quad \lambda = 1 \Rightarrow y + 1 = 2y \Rightarrow y = 1 \xrightarrow{x^2+2y^2=8} x^2 = 8 - 2 = 6$

$f(\pm\sqrt{6}, 1) = 6 + 1 + 2 = 9$ max