MA261 EXAM I FALL 2013
OCTOBER 10, 2013
EXAM TYPE I

INSTRUCTIONS:
1. There are 7 different test pages (including this cover page). Make sure you have a complete test.
2. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet.
3. No books, notes, calculators or any electronic devices may be used on this exam.
4. The number of points each problem is worth is stated next to it. The maximum possible score is 100 points. No partial credit.
5. Use a # 2 pencil to fill in the required information in your scantron and fill in the circles.
6. Make sure the color of your scantron matches the color of the cover page of your exam.
7. After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

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1)Students taking this exam are not allowed to seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only your instructor.
2)You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3) You may not consult notes, books, calculators, phones or cameras, or any electronic device until after you have finished your exam, handed it in to your instructor and left the room.
4) Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course.
5) All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

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SECTION NUMBER: ______________________________

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Solutions

1. (**8 points**) Compute $D_u f(3,2,1)$ where u is in the direction of $< 1,1,1 >$ where $f(x,y,z) = x^2 + yz$.

A. $\frac{5}{\sqrt{3}}$

B. $\frac{3}{\sqrt{3}}$

C. $\frac{9}{\sqrt{3}}$

D. $\frac{7}{\sqrt{3}}$

E. $\frac{1}{\sqrt{3}}$

Solution:

$\vec{u} = \frac{1}{\sqrt{3}}(1,1,1)$.

$\nabla f(x,y,z) = (2x, z, y)$

$\nabla f(3,2,1) = (6, 1, 2)$

$\nabla f(3,2,1) \cdot \vec{u} = \frac{1}{\sqrt{3}}(6+1+2) = 9/\sqrt{3}$

2. (**8 points**) The equation $x^2 + y^2 + z^2 + 4x - 8y + 6z = 0$ represents

A. A sphere centered at $(2,3,4)$ and radius $\sqrt{29}$

B. A sphere centered at $(2,4,-3)$ and radius 29

C. A sphere centered at $(2,4,-3)$ and radius $\sqrt{29}$

D. A sphere centered at $(-2,4,-3)$ and radius $\sqrt{29}$

E. A sphere centered at $(-2,4,-3)$ and radius 29

Solution:

$$x^2+y^2+z^2+4x-8y+6z =$$

$$= (x+2)^2 - 4 + (y-4)^2 - 16 + (z+3)^2 - 9$$

$$= (x+2)^2 + (y-4)^2 + (z+3)^2 - 29.$$

So: $x^2+y^2+z^2+4x-8y+6z = 0$ is equivalent to

$$\boxed{(x+2)^2 + (y-4)^2 + (z+3)^2 = 29}$$

Center $(-2,4,-3)$

Radius $\sqrt{29}$.

3. (**8 points**) Find the arc-length of the curve given by

$$\vec{r}(t) = 4t\vec{i} + \frac{4\sqrt{2}}{3}t^{\frac{3}{2}}\vec{j} + \frac{1}{2}t^2\vec{k}, \quad 1 \leq t \leq 2.$$

A. $\frac{3}{2}$

B. $\frac{11}{2}$

C. $\frac{2}{3}$

D. 3

E. $\frac{12}{5}$

$L = \int_1^2 |\vec{r}\,'(t)|\,dt$; $\vec{r}\,'(t) = 4\vec{i} + 2\sqrt{2}t^{1/2}\vec{j} + t\vec{k}$

$|\vec{r}\,'(t)|^2 = 16 + 8t + t^2 = (t+4)^2.$

$|\vec{r}\,'(t)| = t + 4$

$L = \int_1^2 (t+4)\,dt = \frac{t^2}{2} + 4t \Big|_1^2 = \frac{3}{2} + 4 = \underline{\underline{\frac{11}{2}}}.$

4. (**8 points**) Consider the curve defined by the vector equation $\vec{r}(t) = \langle 2t, 3t^3, 3t^2 \rangle$. Find the unit tangent vector $\vec{T}(t)$ at the point where $t = 1$.

A. $\vec{T}(1) = \langle \frac{2}{11}, \frac{9}{11}, \frac{6}{11} \rangle$

B. $\vec{T}(1) = \langle 2, 9, 6 \rangle$

C. $\vec{T}(1) = \langle \frac{2}{3}, \frac{1}{3}, \frac{2}{3} \rangle$

D. $\vec{T}(1) = \langle 2, 3, 3 \rangle$

E. $\vec{T}(1) = \langle \frac{2}{\sqrt{22}}, \frac{3}{\sqrt{22}}, \frac{3}{\sqrt{22}} \rangle$

$\vec{T}(t) = \frac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|}$

$\vec{r}\,'(t) = \langle 2, 9t^2, 6t \rangle$

$|\vec{r}\,'(t)|^2 = 4 + 81t^4 + 36t^2.$

$\vec{r}\,'(1) = \langle 2, 9, 6 \rangle$ $\quad |\vec{r}\,'(1)|^2 = 4 + 81 + 36 = 121$

$|\vec{r}\,'(1)| = 11.$

$$\boxed{\vec{T}(1) = \frac{1}{11}\langle 2, 9, 6 \rangle}$$

5. (**8 points**) How long does it take for a particle with position $\vec{r}(t) = \langle 1+t,\, 2+t,\, 1+3t \rangle$ to travel a distance d from the initial position (1, 2, 1)?

A. $\dfrac{d}{\sqrt{11}}$

B. $\dfrac{d}{\sqrt{10}}$

C. $d\sqrt{11}$

D. $\dfrac{d}{4}$

E. $\dfrac{d}{3}$

Distance = Velocity × time.

Velocity $= |\vec{r}\,'(t)|$

$\vec{r}\,'(t) = \langle 1, 1, 3 \rangle$; $|\vec{r}\,'(t)| = \sqrt{11}$.

$d = \sqrt{11} \cdot t$; $\boxed{t = d/\sqrt{11}}$

6. (**8 points**) Which is the equation of the tangent plane to z = sin(xy + 2y) at $(0, \pi)$?

A. $z = \pi x + 2y - 2\pi$

B. $z = 2\pi x + 2y - 2\pi$

C. $z = \pi x + y - 2\pi$

D. $z = \pi x + 2y - \pi$

E. $z = 2\pi x + 2y$

$f(x,y,z) = z - \sin(xy+2y)$

$\nabla f(x,y,z) = \langle -y\cos(xy+2y), -(x+2)\cos(2y+xy), 1 \rangle$

When $x=0$, $y=\pi$, $z = \sin 2\pi = 0$

$\nabla f(0,\pi,0) = \langle -\pi\cos 2\pi, -2\cos 2\pi, 1 \rangle$

$= \langle -\pi, -2, 1 \rangle$

Plane: $\langle x-0, y-\pi, z-0 \rangle \cdot \langle -\pi, -2, 1 \rangle = 0$

$-\pi x - 2y + 2\pi + z = 0$

$\boxed{z = 2y + \pi x - 2\pi}$

7. (**8 points**) Compute $\frac{\partial z}{\partial t}$ where $z = \ln(\sin(x+y))$, $x = t + 2s$, $y = st^2$.

A. $\cos(t + 2s + st^2)$

B. $\cot(t + 2s + st^2)(1 + 2st)$

C. $\cot(t + 2s)(1 + st)$

D. $\tan(t + 2s)(1 + st)$

E. $\ln(t + 2s + st^2)(st)$

$$\frac{\partial z}{\partial t} = \frac{\cos(x+y)}{\sin(x+y)} \cdot \frac{\partial x}{\partial t} + \frac{\cos(x+y)}{\sin(x+y)} \cdot \frac{\partial y}{\partial t}$$

$$= \frac{\cos(x+y)}{\sin(x+y)}(1 + 2st)$$

when $x = t+2s$, $y = st^2$

$$\boxed{\frac{\partial z}{\partial t} = \operatorname{Cotan}(\cancel{t}+ 2s + st^2) \cdot (1+2st)}$$

8. (**8 points**) Let

$$f(x,y) = \frac{x^2 + y^2 - \ln(1 + 2x^2 + 2y^2)}{x^2 + y^2}.$$

$x^2+y^2 = r^2$

Then $\lim_{(x,y)\to(0,0)} f(x,y) =$

A. 3

B. 2

C. -1

D. 0

E. It does not exist

$$f(x,y) = \frac{r^2 - \ln(1+2r^2)}{r^2}.$$

$$\lim_{(x,y)\to 0} f(x,y) = \lim_{r\to 0} \frac{r^2 - \ln(1+2r^2)}{r^2}$$

$$= \lim_{r\to 0} \frac{2r - \frac{4r}{1+r^2}}{2r}$$

L'Hospital Rule

$$= \lim_{r\to 0}\left[1 - \frac{2}{1+r^2}\right] = -1.$$

9. (**8 points**) Let $f(x,y) = \dfrac{1}{8x} + \dfrac{1}{y} + xy$ for $x > 0,\ y > 0$. The critical point for f is:

A. $(4,2)$
B. $(1/4,2)$
C. $(2,4)$
D. $(4,1/4)$
E. $(2,1/4)$

Critical points : $\nabla f(x,y) = 0$

$\frac{\partial f}{\partial x} = 0$, $\frac{\partial f}{\partial y} = 0$

$\frac{\partial f}{\partial x} = y - 1/8x^2$; $\frac{\partial f}{\partial y} = x - 1/y^2$.

$\frac{\partial f}{\partial x} = 0 \quad y = 1/8x^2$

$\frac{\partial f}{\partial y} = 0 \quad x = 1/y^2$

$y = \frac{1}{8}\left(1/y^4\right)$ $\quad y = \frac{y^4}{8}$; $y^3 = 8$

Since $y > 0$ $\quad y = 2$

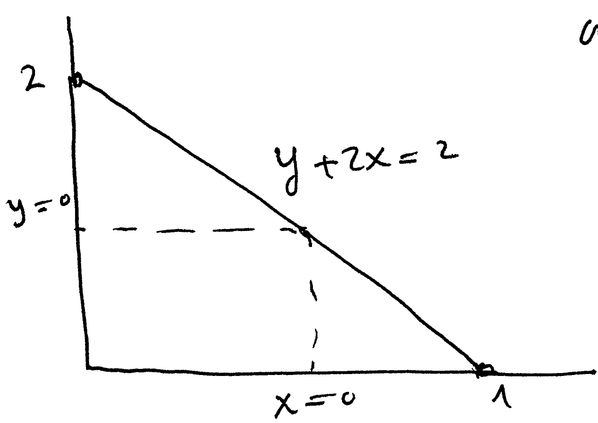
If $y = 2$, $x = 1/4$: $(1/4, 2)$.

10. (**8 points**) Let $f(x,y) = x^2 y$ for (x,y) in the closed triangle T bounded by the lines

$$x = 0,\ y = 0,\ 2x + y = 2.$$

f takes its absolute maximum on the boundary of T. Find this value.

A. $4/3$
B. $4/9$
C. $8/27$
D. $2/3$
E. $8/9$



on $x=0$ or $y=0$
$f(x,y) = 0$

on $y+2x=2$ $\quad f(x,y) = x^2(2-2x) = 2x^2 - 2x^3$.

Look for the max of $g(x) = x^2 - x^3$ in $[0,1]$.

$g(0) = g(1) = 0$. $\quad g'(x) = 2x - 3x^2 = 0 \quad x(2-3x) = 0$

$x = 2/3$; $y = 2 - 4/3 = 2/3$

$x = 2/3$, $y = 2/3$ $\quad \boxed{f(2/3, 2/3) = (2/3)^3 = 8/27}$

11. (**10 points**) Let $f(x,y) = 2x^3 + 6xy + 3y^2$. Which of the following is true?

A. $(1,-1)$ corresponds to a local maximum and $(0,0)$ to a saddle point.

B. $(1,-1)$ corresponds to a local minimum and $(0,0)$ to a saddle point.

C. $(1,-1)$ corresponds to a saddle point and $(0,0)$ to a local minimum.

D. both $(1,-1)$ and $(0,0)$ correspond to saddle points.

E. $(1,-1)$ corresponds to a local minimum and $(0,0)$ is not a critical point.

$\frac{\partial f}{\partial x} = 6x^2 + 6y$; $\frac{\partial f}{\partial y} = 6x + 6y$ so $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0$

$x = -y$; $y^2 = -y$ the solutions are $x = y = 0$ or $x = 1, y = -1$.

Critical points: $(0,0)$; $(1,-1)$

$f_{xx} = \frac{\partial^2 f}{\partial x^2} = 12x$; $f_{yy} = \frac{\partial^2 f}{\partial y^2} = 6$; $\frac{\partial^2 f}{\partial x \partial y} = 6 = f_{xy}$

$D = f_{xx} f_{yy} - (f_{xy})^2 = 72x - 36$

If $x=0$ $D = -36$ Saddle

If $x=1$ $D = 36$ Min

$f_{xx} = 12$

12. (**10 points**) If $f(x,y,z) = 3x + 4y + 12z$ is maximized subject to the constraint

$$x^2 + y^2 + z^2 = 169$$

at (x_0, y_0, z_0) then y_0 equals:

A. 4

B. 3

C. 12

D. -4

E. -12

$g(x,y,z) = x^2 + y^2 + z^2$

$\nabla f = \lambda \nabla g$. $(3, 4, 12) = \lambda(2x, 2y, 2z)$

$2\lambda x = 3$; $2\lambda y = 4$; $2\lambda z = 12$.

$y/x = 4/3$; $z/y = 3$ so $z = 3y$

$x = 3/4\, y$

$x^2 + y^2 + z^2 = 169$; $9y^2 + y^2 + \frac{9}{16} y^2 = 169$

$\frac{169}{16} y^2 = 169$, $y^2 = 16$

$y = 4$ or $y = -4$

when $y = -4$ $f < 0$

when $y = 4$; $f > 0$

So $\boxed{y = 4}$ max