Answers worked out

261 Test 1 FORM A

1. Find a vector function $\mathbf{r}(t)$ that traces the line which contains the point $(3, 4, 0)$ and is perpendicular to the plane $z = 2x - 5y + 7$.

A. $\mathbf{r}(t) = \langle 2 + 3t, -5 + 4t, 1 \rangle$

B. $\mathbf{r}(t) = \langle 3 - t, 2 + 4t, -t \rangle$

C. $\mathbf{r}(t) = \langle 1 + t, 2 - 3t, 7 + t \rangle$

D. $\mathbf{r}(t) = \langle 3 + t, 4 + 5t, t \rangle$

E. $\mathbf{r}(t) = \langle 3 + 2t, 4 - 5t, -t \rangle$

The plane is $2x - 5y - z + 7 = 0$

The line is $(3, 4, 0) + t(n_1, n_2, n_3)$

normal to the plane

And $(n_1, n_2, n_3) = (2, -5, -1)$

from the equation of the plane

2. The approximate change of $z = \sqrt{1+x+y^2}$ as (x, y) changes from $(2,1)$ to $(1.9, 1.2)$ is

$$\left.\frac{\partial z}{\partial x}\right|_{(2,1)} = \left.\frac{1}{2\sqrt{1+x+y^2}}\right|_{x=2,\, y=1} = \frac{1}{4}$$

$$\left.\frac{\partial z}{\partial y}\right|_{(2,1)} = \left.\frac{2y}{2\sqrt{1+x^2+y^2}}\right|_{(2,1)} = \frac{1}{2}$$

Answer : $\frac{1}{4} \cdot (1.9 - 2) + \frac{1}{2}(1.2 - 1) =$

$$-\frac{1}{40} + \frac{4}{40} = \frac{3}{40}$$

A. $\frac{1}{10}$

B. $\frac{1}{\sqrt{10}}$

C. $\frac{3}{40}$

D. $-\frac{1}{40}$

E. $-\frac{1}{20}$

$x(t) \quad y(t) \quad z(t)$

3. The length of the path traced out by $\mathbf{r}(t) = 2t^{3/2}\,\mathbf{i} + \cos 2t\,\mathbf{j} + \sin 2t\,\mathbf{k}$ over the interval $0 \le t \le 2$ is

A. $\int_0^2 \sqrt{4t^3+4}\,dt$

B. $\int_0^2 4t^3+4\,dt$

C. $\int_0^2 \sqrt{9t+4}\,dt$

D. $\int_0^2 9t+4\,dt$

E. $\int_0^2 \frac{1}{\sqrt{4t^3+4}}\,dt$

$$\int_0^2 \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}\,dt$$

$$= \int_0^2 \sqrt{\underbrace{(3t^{1/2})^2}_{9t} + \underbrace{(-2\sin 2t)^2 + (2\cos 2t)^2}_{4}}$$

4. Suppose $f(7,8) = 5, f(7.1,8) = 5.1, f(7,8.2) = 5.4$, and $f(7.1,8.2) = 5.5$. The best estimates for $f_x(7,8)$ and $f_y(7,8)$ based on this data are

A. $f_x(7,8) = 2$ and $f_y(7,8) = 1$

B. $f_x(7,8) = 2$ and $f_y(7,8) = 2$

C. $f_x(7,8) = 1$ and $f_y(7,8) = 1$

D. $f_x(7,8) = 1$ and $f_y(7,8) = 2$

E. $f_x(7,8) = 3$ and $f_y(7,8) = 1$

(7, 8.2)

.2

(7, 8) (7.1, 8)

.1

f_x estimate.

$$f_x(7,8) \approx \frac{f(7.1,8) - f(7,8)}{7.1 - 7} = \frac{.1}{.1} = 1$$

$$f_y(7,8) \approx \frac{f(7,8.2) - f(7,8)}{8.2 - 8} = \frac{.4}{.2} = 2$$

5. Find the equation of the tangent plane to $z = e^{xy}$ at the point $(1, 1, e)$

A. $z = ex + ey + 1$

B. $z = x + y + e - 2$

C. $z = ex + ey + e$

D. $z = ex + ey - e$

E. $z = x + y + 1$

$$\frac{dz}{dx}\Big|_{(1,1)} = ye^{x}\Big|_{(1,1)} = 1e^{1} = e$$

$$\frac{dz}{dy}\Big|_{(1,1)} = xe^{y}\Big|_{(1,1)} \quad 1\cdot e^{1} = e$$

equation

$p(x,y) = Ax + By + C \quad = ex + ey + C$

to find C plug in $\quad p(1,1) = e$

so $\quad e \cdot 1 + e \cdot 1 + C = e \quad , \quad C = -e$

Or $\quad z - z_0 = A(x - x_0) + B(y - y_0)$

to get $z = ex + ey - e$

6. The vector projection of $4\mathbf{j}$ onto $4\mathbf{i} + 4\mathbf{j}$, that is, $\text{proj}_{4\mathbf{i}+4\mathbf{j}}\, 4\mathbf{j}$, equals

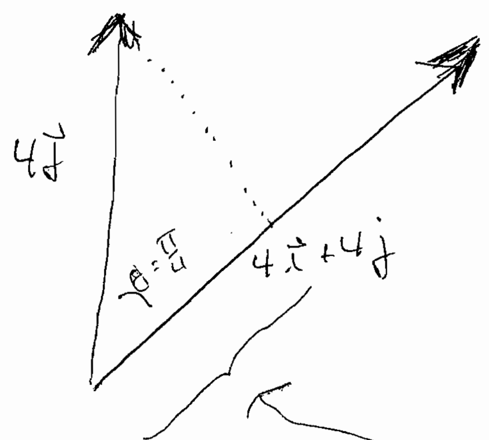
A. $\mathbf{i} + \mathbf{j}$

B. $2\mathbf{i} + 2\mathbf{j}$

C. $3\mathbf{i} + 3\mathbf{j}$

D. $4\mathbf{i} + 4\mathbf{j}$

E. $4\mathbf{j}$



the projection is long and points in the direction of $4i + 4j$, ie of the unit vector $i+j$

so answer is : $(\text{Length of } 4j) \cos\theta \quad \frac{\vec{i}+\vec{j}}{\sqrt{2}}$

$$= (4)\frac{1}{\sqrt{2}} \frac{\vec{i}+\vec{j}}{\sqrt{2}} = \frac{4}{2}(\vec{i}+\vec{j})$$

B

7. Find b and c so that $\mathbf{v} = \langle 4, b, c\rangle$ is parallel to the planes $x+y+z=3$ and $2x+z=0$.

A. $b=-8, c=4$

B. $b=8, c=4$

C. $b=12, c=4$

D. $b=-4, c=-8$

E. $b=4, c=-8$

The easiest way is to note that $\overrightarrow{(4,b,c)}$ must be perpendicular to both $\vec{\imath}+\vec{\jmath}+\vec{k}$, the normal vector for the first plane, and $2\vec{\imath}+\vec{k}$, the normal vector for the second

So $\begin{cases} (4\vec{\imath}+b\vec{\jmath}+c\vec{k})\cdot(\vec{\imath}+\vec{\jmath}+\vec{k})=0 \\ (4\vec{\imath}+b\vec{\jmath}+c\vec{k})\cdot(2\vec{\imath}+\vec{k})=0 \end{cases}$

$\begin{cases} 4+b+c=0 \\ 4\cdot 2+c\cdot 1=0 \end{cases}$ works only for $b=4$, $c=-8$, i.e. E

Alternatively, the answer must be parallel to $(\vec{\imath}+\vec{\jmath}+\vec{k})\times(2\vec{\imath}+\vec{k})$

8. The graph of $x^2 - 2y^2 + 3z^2 - 4 = 0$ is

A. A hyperboloid of one sheet which **does not** intersect the x axis

B. A hyperboloid of one sheet which **does not** intersect the y axis

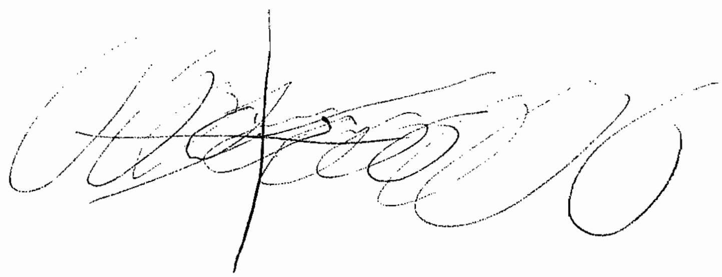
C. A hyperboloid of one sheet which **does not** intersect the z axis

D. A hyperboloid of two sheets which **does** intersect the y axis

E. A hyperboloid of two sheets which **does** intersect the z axis

The sketch is the best way to see this

$x^2 + 3z^2 = 4 + 2y^2$



9. Let $\mathbf{r}(0) = \mathbf{i} + \mathbf{j}$, $\mathbf{v}(0) = 2\mathbf{i} + 3\mathbf{j}$, $\mathbf{a}(t) = e^{2t}\mathbf{j}$, where $\mathbf{r}''(t) = \mathbf{a}(t)$ and $\mathbf{r}'(t) = \mathbf{v}(t)$. Find $\mathbf{r}(1)$.

A. $3\mathbf{i} + (\frac{5}{2} + \frac{1}{2}e^2)\,\mathbf{j}$

B. $2\mathbf{i} + (3 + e)\,\mathbf{j}$

C. $3\mathbf{i} + (\frac{5}{4} + e)\,\mathbf{j}$

D. $3\mathbf{i} + (\frac{13}{4} + \frac{1}{4}e^2)\,\mathbf{j}$

E. $2\mathbf{i} + (4 + \frac{1}{2}e)\,\mathbf{j}$

$$\vec{v(t)} = \int_0^t a(s)\,ds + v(0)$$

$$= \left[\int_0^t 0\,ds\right]\vec{i} + \left[\int_0^t e^{2s}\,ds\right]\vec{j} + 2\vec{i} + 3\vec{j}$$

$$= 2\vec{i} + \left[\tfrac{1}{2}e^{2s}\Big|_0^t + 3\right]\vec{j}$$

$$= 2\vec{i} + \left[\tfrac{1}{2}e^{2t} + 2\tfrac{1}{2}\right]\vec{j}$$

$$\vec{r(t)} = \vec{r(0)} + \int_0^t v(s)\,ds =$$

$$\vec{i} + \vec{j} + \left[\int_0^t 2\,ds\right]\vec{i} + \left[\int_0^t \left(\tfrac{1}{2}e^{2s} + 2\tfrac{1}{2}\right)ds\right]\vec{j}$$

$$\underbrace{= \vec{i}\,[1+2] + \vec{j}\,\Big[1 + \underbrace{\tfrac{1}{4}e^{2s}\Big|_0^t + 2\tfrac{1}{2}\Big|_0^t}_{1 + 2\frac{1}{4} + \frac{1}{2}e^{2t}}}_{}$$

Plug in $t = 1$ ⇒ D

10. If E is the region defined by $y > 0, y - x < 0$, and $x^2 + y^2 + z^2 < 4$, then describe E in spherical coordinates

A. $0 < \rho < 4,\ 0 < \theta < \frac{\pi}{2}, 0 < \phi < \pi$

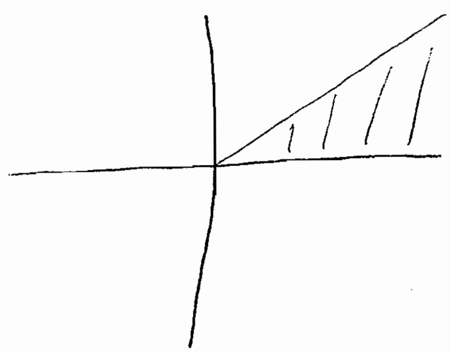
B. $0 < \rho < 2,\ 0 < \theta < \frac{\pi}{4}, 0 < \phi < \frac{\pi}{2}$

C. $0 < \rho < 2,\ \frac{\pi}{4} < \theta < \pi,\ 0 < \phi < \frac{\pi}{2}$

D. $0 < \rho < 2,\ 0 < \theta < \frac{\pi}{2}, 0 < \phi < \frac{\pi}{4}$

$\boxed{\text{E.}}$ $0 < \rho < 2,\ 0 < \theta < \frac{\pi}{4}, 0 < \phi < \pi$

$\{y > 0, y - x < 0\}$ in the xy plane looks like this



since $y - x < 0$ is the part of the plane on the $+x$ side of the line $y - x = 0$

In polar coordinates this is $0 < \theta < \frac{\pi}{4}$.

In 3d, $\{y > 0, y - x < 0\}$ is everything which projects to the region sketched above, a wedge. $x^2 + y^2 + z^2 < 4 = \rho < 2$, the sphere of radius 2 about the origin.

So the region is a wedge out of a sphere with the sharp edge of the wedge along the z axis from -2 to 2. So every ϕ from 0 to π is the ϕ of a point in the region: E

11. The tangent line to the curve traced out by $\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle$ at the point $(0, 1, \frac{\pi}{2})$ hits the xy plane at the point where

t must be $\frac{\pi}{2}$

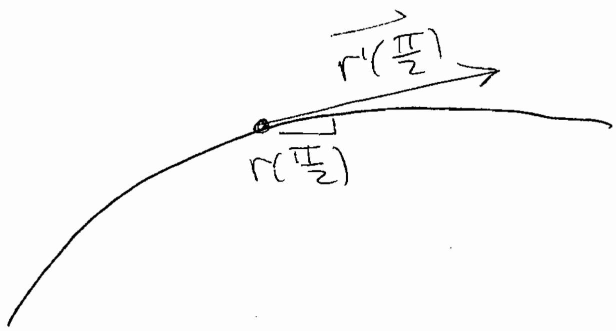
A. $x = 1,\ y = \pi$

B. $x = \frac{\pi}{2},\ y = 1$

C. $x = \pi,\ y = \frac{\pi}{2}$

D. $x = -\frac{\pi}{2},\ y = 1$

E. $x = -1,\ y = \pi/2$



To find the line use $r(\frac{\pi}{2}) = (0, 1, \frac{\pi}{2})$ as the point

and $r'(\frac{\pi}{2}) = (-\sin t, \cos t, 1)\Big|_{t=\frac{\pi}{2}} = (-1, 0, 1)$

as a parallel vector

$(0, 1, \frac{\pi}{2}) + t(-1, 0, 1)$ gives tangent line

This hits the x y plane when the z coordinate $= 0$: $\frac{\pi}{2} + t = 0$, $t = -\frac{\pi}{2}$

The answer is the first two coordinates of $(0, 1, \frac{\pi}{2}) + (-\frac{\pi}{2})(-1, 0, 1)$:

$x = 0 + (-\frac{\pi}{2}) \cdot (-1)$

$y = 1 + (-\frac{\pi}{2}) \cdot 0$