MATH 261 – FALL 2000 – FIRST EXAM
September 26, 2000

STUDENT NAME GRADING KEY

STUDENT ID

RECITATION INSTRUCTOR

RECITATION HOUR

INSTRUCTIONS:
1. This test booklet has 5 pages including this one.
2. Fill in your name, your student ID number, your recitation hour and your recitation instructor's name above.
3. There are 9 questions, each worth 11 points.
4. Questions 1 to 6 are multiple choice. Circle the letter of your choice for the correct answer. No partial credit will be given.
5. Question 7 to 9 are partial credit. You should carefully explain your solution. No points will be given to solutions without explanations.
6. No books, notes or calculators may be used.

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1) Which of the following is an equation of the plane that contains the point $(-1,2,1)$ and is perpendicular to the line with vector equation $\vec{r}(t) = (-3+t)\vec{i} + (1-t)\vec{j} + (4+2t)\vec{k}$.

A) $x - y + z = -2$

B) $2x - y + 2z = -2$

C) $x - y + z = 3$

D) $x - y + 2z = -1$

E) $2x - y + 2z = 1$

2) The length of the curve $\vec{r}(t) = t\vec{i} + \frac{t^2}{2}\vec{j} + \left(\frac{2}{3}\sqrt{2}\right) t^{\frac{3}{2}}\ \vec{k}$, for $0 \leq t \leq 2$ is

A) 1

B) 4

C) $\frac{2}{3}\sqrt{2}$

D) $\frac{4}{3}$

E) 2

3 The intersection of the surface $z = x^2 + y^2 + 1$ with the plane $z = 2$ is

A) A parabola

B) A hyperbola

C) An ellipse

D) A circle

E) A line

4 Suppose that the graph of a function $f(x, y)$ intersects the plane $y = 0$ along the curve $z = x^2 + 2x + 1$. What is the value of $f_x(1, 0)$?

A) 0

B) 3

C) 1

D) 2

E) 4

5 Assuming that the equation $xyz + xz^3 = 8$ defines z implicitly as a function of x and y, find $\frac{\partial z}{\partial x}$ at the point $(2, 3, 1)$.

A) $-\frac{1}{3}$

B) -1

C) $-\frac{1}{6}$

D) 0

E) $-\frac{5}{6}$

symmetric

6) Which of the following are the ~~parametric~~ symmetric equations for the line perpendicular to the surface $x^2 + y^2 + z^2 = 4$ at $(\sqrt{2}, 1, 1)$?

A) $\frac{x-1}{\sqrt{2}} = y - 1 = z - 1.$

B) $x - \sqrt{2} = y - 1 = z - 1$

C) $\frac{x-\sqrt{2}}{\sqrt{2}} = y - 1 = z - 1.$

D) $\frac{x-\sqrt{2}}{2} = \frac{y-1}{2} = \frac{y-1}{2}$

E) $x - \sqrt{2} = y - 1 = \frac{z-1}{3}.$

Remark: Questions 7 to 9 require detailed solutions. No points will be given to answers without explanations. It is important to justify your steps. Even if you arrive at the correct answer, points will be deducted if your explanation is incorrect.

7) Find an equation of the plane tangent to the graph of the function $f(x, y) = \cos(\pi xy)$ at the point $(1, \frac{1}{3}, \frac{1}{2})$.

$z = \cos(\pi x y)$

eq. of the plane tangent to the graph of $f(x,y)$ at (x_0, y_0, z_0):

$$z = z_0 + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) \quad ②$$

$(x_0, y_0, z_0) = (1, \frac{1}{3}, \frac{1}{2})$

$f_x(x,y) = -\sin(\pi x y)\cdot \pi y$ ② $\quad f_x(1, \frac{1}{3}) = -\sin(\frac{\pi}{3})\cdot \pi \frac{1}{3} = -\frac{\pi}{3}\left(\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{2\sqrt{3}}$ ①

$f_y(x,y) = -\sin(\pi x y)\cdot \pi x$ ② $\quad f_y(1, \frac{1}{3}) = -\sin(\frac{\pi}{3})\pi\cdot 1 = -\frac{\pi\sqrt{3}}{2}$ ①

$$\boxed{z = \frac{1}{2} - \frac{\pi}{2\sqrt{3}}(x - 1) - \frac{\pi\sqrt{3}}{2}\left(y - \frac{1}{3}\right)} \quad ③$$

Or

$\underbrace{z - \cos(\pi x y)}_{F(x,y,z)} = 0$ ② $\quad$ grad $F(x,y,z) = \pi y \sin(\pi x y)\vec{i} + \pi x \sin(\pi x y)\vec{j} + \vec{k}$ ② ②

grad $F(1, \frac{1}{3}, \frac{1}{2}) = \frac{\pi}{3}\sin(\frac{\pi}{3})\vec{i} + \pi\cdot 1 \sin(\frac{\pi}{3})\vec{j} + \vec{k}$

$= \frac{\pi}{3}\frac{\sqrt{3}}{2}\vec{i}$ ① $+ \frac{\sqrt{3}}{2}\vec{j}$ ① $+ \vec{k}$

$$\boxed{\frac{\pi}{2\sqrt{3}}(x - 1) + \frac{\pi\sqrt{3}}{2}\left(y - \frac{1}{3}\right) + \left(z - \frac{1}{2}\right) = 0} \quad ③$$

8) Does

$$\lim_{(x,y)\to(0,0)} \frac{5x^2+3y^2}{x^2+y^2}$$

exist? If so find its value.

As $(x,y) \longrightarrow (0,0)$ along the x-axis:

$$\lim_{(x,0)\to(0,0)} \frac{5x^2+3y^2}{x^2+y^2} = \lim_{(x,0)\to(0,0)} \frac{5x^2}{x^2} = 5 \quad 4$$

As $(x,y) \longrightarrow (0,0)$ along the y-axis:

$$\lim_{(0,y)\to(0,0)} \frac{5x^2+3y^2}{x^2+y^2} = \lim_{(0,y)\to(0,0)} \frac{3y^2}{y^2} = 3. \quad 4$$

$\therefore$ the limit does not exist. 3

9) Find the position $\vec{r}(t)$ of an object with acceleration $\vec{a}(t) = 3\vec{k}$, initial velocity $\vec{v}_0 = 2\vec{i}$ and initial position $\vec{r}_0 = 3\vec{j}$.

$\vec{a}(t) = 3\vec{k}$

$\vec{v}(t) = 3t\vec{k} + \vec{C}_1$ 1

At $t=0$: $2\vec{i} = \vec{0} + \vec{C}_1$

$\vec{v}(t) = 2\vec{i} + 3t\vec{k}$ 4

$\vec{r}(t) = 2t\vec{i} + \frac{3t^2}{2}\vec{k} + \vec{C}_2$ 1

At $t=0$: $3\vec{j} = \vec{0} + \vec{0} + \vec{C}_2$

$\vec{r}(t) = 2t\vec{i} + 3\vec{j} + \frac{3}{2}t^2\vec{k}$ 5