NAME GRADING KEY

STUDENT ID ______

RECITATION INSTRUCTOR ______

RECITATION TIME ______

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DIRECTIONS

1. Write your name, student ID number, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3, 4 and 5.
2. The test has five (5) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes or calculators may be used on this test.

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(12) 1. Circle the letter of the correct response. (You need not show work for this problem).

(a) Which of the following statements are true?

✓ (I) If $\sum_{n=1}^{\infty} a_n$ converges absolutely, then $\sum_{n=1}^{\infty} a_n$ converges. NPC

X (II) If $0 < a_n$ for all $n \geq 1$ and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 2$, then $\sum_{n=1}^{\infty} a_n$ converges.

X (III) If $0 < a_n \leq b_n$ for all $n \geq 1$ and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ diverges.

A. I only B. I and II only C. II and III only D. III only E. None $\boxed{6}$

(b) Which of the following series converge?

(I) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2-1}}$ conv. Compare with $\sum_{n=2}^{\infty} \frac{1}{n^2}$, limit comp. test

(II) $\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} - \frac{1}{\sqrt{5}} + \ldots$ conv. Alt. ser. test NPC

(III) $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{5}} + \ldots$ div. p-series $p = \frac{1}{2} < 1$.

A. II only B. I only C. I and II only D. II and III only E. all $\boxed{6}$

(9) 2. Find the limit if it exists, or write DNE if the limit does not exist. (You need not show work for this problem).

NPC

(a) $\lim_{n\to\infty} (-1)^n \frac{n^2}{n+1}$

$\boxed{\text{DNE}}$ $\boxed{3}$

(b) $\lim_{n\to\infty} \frac{-n^2+2n-1}{3-4n+3n^2}$

$\boxed{-\frac{1}{3}}$ $\boxed{3}$

(c) $\lim_{n\to\infty} \sqrt[n]{5n^2} = \lim_{n\to\infty} \sqrt[n]{5}\,\sqrt[n]{n}\,\sqrt[n]{n}$

$= 1 \cdot 1 \cdot 1 = 1$

$\boxed{1}$ $\boxed{3}$

(8) 3. Find the sum of the series, if it converges.

$$\sum_{n=1}^{\infty} 5\left(\frac{4}{7}\right)^n = 5 \cdot \frac{4}{7} \cdot \frac{1}{1-\frac{4}{7}}$$

$$= \frac{20}{3}$$

If answer is wrong, give 3 points for attempting to sum using geometric series.

$\boxed{\frac{20}{3}}$ $\boxed{8}$

(10) 4. Use the integral test to determine whether the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ is convergent or divergent. You must show all work.

$$\int_2^\infty \frac{1}{x(\ln x)^2}\,dx = \lim_{b\to\infty} \int_2^b \frac{1}{x(\ln x)^2}\,dx =$$

4 2

$$= \lim_{b\to\infty} \left[-\frac{1}{\ln x}\right]_2^b = \lim_{b\to\infty}\left[\frac{1}{\ln 2} - \frac{1}{\ln b}\right]$$

$$= \frac{1}{\ln 2}$$ 2

2

The series is convergent

10

(10) 5. Determine whether the series $\sum_{n=1}^{\infty} \frac{\sin n}{n^2+1}$ diverges, converges conditionally, or converges absolutely. You must show all work.

Converges absolutely?

$\sum_{n=1}^{\infty} \left|\frac{\sin n}{n^2+1}\right|$ 2 conv? Compare with $\sum_{n=1}^{\infty} \frac{1}{n^2}$. 2

which conv. 1

(p-series, $p=2>1$) 1

$$\frac{|\sin n|}{n^2+1} \le \frac{1}{n^2+1} \le \frac{1}{n^2} \text{ for } n=1,2,\ldots$$ 2

By comparison test series conv. abs.

2

The series converges absolutely

10

(6) 6. Determine whether the series $\sum_{n=1}^{\infty} \frac{n+1}{n+2}$ converges or diverges. You must justify your answer.

$$\lim_{n\to\infty} \frac{n+1}{n+2} = 1 \neq 0$$ 4

"converges" → 0 points for problem

2

The series diverges

6

(20) 7. Determine whether each series is convergent or divergent. You must justify your conclusion with a mathematically correct reason. Write your conclusion in the small box.

(a) $\sum_{n=0}^{\infty} \frac{e^n}{n!}$

Show all necessary work here:

$$\lim_{n\to\infty} \frac{\frac{e^{n+1}}{(n+1)!}}{\frac{e^n}{n!}} = \lim_{n\to\infty} \frac{e^{n+1}\, n!}{e^n\,(n+1)!} =$$

4

$$= \lim_{n\to\infty} \frac{e}{n+1} = 0 < 1$$

3 1

1 1

By the ratio test, the series is convergent

(b) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}}$

Show all necessary work here:

3 1

Compare with $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ which converges (p-series $p=\frac{3}{2}>1$) 1

3

$$\frac{1}{\sqrt{n^3+1}} \le \frac{1}{n^{3/2}} \quad \text{for } n=1,2,\ldots$$

[or $\lim_{n\to\infty} \frac{\frac{1}{\sqrt{n^3+1}}}{\frac{1}{n^{3/2}}} = \lim_{n\to\infty} \sqrt{\frac{n^3}{n^3+1}} = 1 \neq 0$]

1 1 1

1 1

By the comparison test, the series is convergent

or [limit comparison]

(15) 8. Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{2}{3^{n+1}n^2}x^n$. Don't forget to test for convergence at the end points of the interval. You must show all work.

Generalized ratio test

4 $\lim_{n\to\infty}\left|\dfrac{\frac{2}{3^{n+2}(n+1)^2}x^{n+1}}{\frac{2}{3^{n+1}n^2}x^n}\right| = \lim_{n\to\infty}\frac{3^{n+1}n^2}{3^{n+2}(n+1)^2}|x|$

$= \lim \frac{1}{3}\left(\frac{n}{n+1}\right)^2|x| = \frac{|x|}{3}$ 3

$\therefore$ series converges (absolutely) for $\frac{|x|}{3}<1$, or $|x|<3$, or $-3<x<3$ 2

At $\underline{x=3}$ $\sum_{n=1}^{\infty}\frac{2}{3^{n+1}n^2}3^n = \frac{2}{3}\sum_{n=1}^{\infty}\frac{1}{n^2}$ 1 conv. 1 (p-series, $p=2>1$)

At $\underline{x=-3}$ $\sum_{n=1}^{\infty}\frac{2}{3^{n+1}n^2}(-3)^n = \frac{2}{3}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$ 1 conv 1 (alt. ser. test) or (conv, abs.)

$\boxed{-3 \leq x \leq 3}$ 1 1

15

(10) 9. Given that $e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \ldots$, approximate the value of the integral $\int_0^{\frac{1}{10}} e^{-x^2}\,dx$ with error less than 10^{-5}, using the <u>smallest possible</u> number of terms of the series. (You may leave your answer as a sum of fractions).

$$\int_0^{\frac{1}{10}} e^{-x^2}dx = \int_0^{\frac{1}{10}}\left(1-x^2+\frac{x^4}{2!}-\frac{x^6}{3!}+\cdots\right)dx$$

$$= \Big[\underbrace{x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \cdots}_{4}\Big]_0^{\frac{1}{10}}$$

4 for plugging in and integrating term by term

$$= \underbrace{\frac{1}{10} - \frac{1}{3000}} + 10^{-6} - \cdots$$

3pts for additional terms

$\int_0^{\frac{1}{10}} e^{-x^2}\,dx \approx \frac{1}{10} - \frac{1}{3000}$ 3 3

10