NAME GRADING KEY

STUDENT ID __________

RECITATION INSTRUCTOR __________

RECITATION TIME __________

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DIRECTIONS

1. Write your name, student ID number, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3, 4 and 5.
2. The test has five (5) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes or calculators may be used on this test.

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(4) 1. Express $\dfrac{x^2+1}{x^2(x^2+9)}$ as a sum of partial fractions. Do not evaluate the constants.

NPC

$$\boxed{\frac{x^2+1}{x^2(x^2+9)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+9}}$$ 4

(13) 2. Evaluate $\displaystyle\int \frac{x+1}{x(x^2+1)}\,dx$

$$\frac{x+1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

(2) (2)

No additional credit beyond this point if anything is wrong here

$$x+1 = Ax^2 + A + Bx^2 + Cx$$

$$0 = A+B, \quad 1 = C, \quad 1 = A$$

$$\frac{x+1}{x(x^2+1)} = \frac{1}{x} + \frac{-x+1}{x^2+1}$$

$$\int \frac{x+1}{x(x^2+1)}\,dx = \int \frac{1}{x}\,dx - \int \frac{x}{x^2+1}\,dx + \int \frac{1}{x^2+1}\,dx =$$

(3) (3) (3)

$$\boxed{\ln|x| - \frac{1}{2}\ln(x^2+1) + \tan^{-1}x + C}$$ 13

−2 pts for each wrong coefficient
−1 pt for missing + C

(18) 3. Determine whether the improper integral converges or diverges. If it converges find its value. Important: Show clearly how limits are involved.

(a) $\int_2^\infty \frac{1}{x(\ln x)^3}\,dx$ $= \lim_{b\to\infty} \int_2^b \frac{1}{x(\ln x)^3}dx$ ④

$u = \ln x \quad du = \frac{1}{x}dx$

$x = 2 \to u = \ln 2$

$x = b \to u = \ln b$

$= \lim_{b\to\infty} \int_{\ln 2}^{\ln b} u^{-3}\,du =$

$= \lim_{b\to\infty} \left[\frac{u^{-2}}{-2}\right]_{\ln 2}^{\ln b} =$

$= \lim_{b\to\infty} \left[-\frac{1}{2}\frac{1}{(\ln b)^2} + \frac{1}{2}\frac{1}{(\ln 2)^2}\right]$ ④

$= \frac{1}{2}\frac{1}{(\ln 2)^2}$ ①

or

$= \lim_{b\to\infty}\left[-\frac{1}{2(\ln x)^2}\right]_2^b$

$= \underbrace{\lim_{b\to\infty}\left[-\frac{1}{2(\ln b)^2} + \frac{1}{2(\ln 2)^2}\right]}_{④}$

$= \frac{1}{2(\ln 2)^2}$ ①

$\boxed{\frac{1}{2(\ln 2)^2}}$ $\boxed{9}$

(b) $\int_0^3 \frac{1}{(x-1)^{4/3}}\,dx$ $\overset{?}{=} \int_0^1 \frac{1}{(x-1)^{4/3}}dx + \int_1^3 \frac{1}{(x-1)^{4/3}}dx$

$\int_0^1 \frac{1}{(x-1)^{4/3}}dx = \lim_{c\to 1^-} \int_0^c \frac{1}{(x-1)^{4/3}}dx$ ④

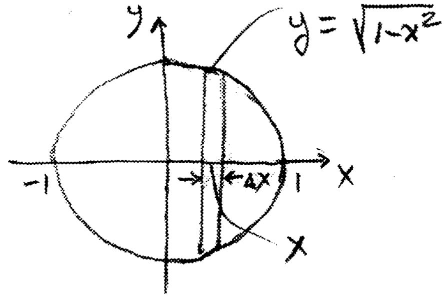
$= \lim_{c\to 1^-} \left[\frac{(x-1)^{-\frac{1}{3}}}{-\frac{1}{3}}\right]_0^c$

$= \lim_{c\to 1^-} \left[-\frac{3}{(x-1)^{1/3}}\right]_0^c$

$= \underbrace{\lim_{c\to 1^-}\left[-3 - \frac{3}{(c-1)^{1/3}}\right]}_{③} = \infty$ ① diverges ①

$\boxed{\text{diverges}}$ $\boxed{9}$

(10) 4. The base of a solid is a circle of radius 1. The cross sections perpendicular to a given diameter are squares. Find the volume of the solid.



$$\Delta V = \left(2\sqrt{1-x^2}\right)^2 \Delta x$$

$$V = \int_{-1}^{1} 4(1-x^2)\,dx$$

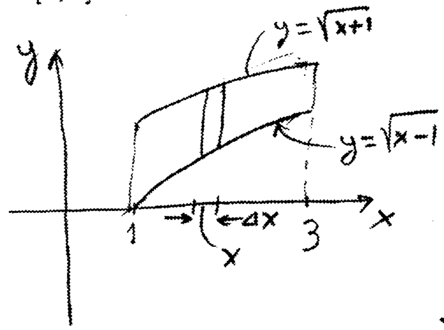
$$= 4\left(x - \frac{x^3}{3}\right)\Big|_{-1}^{1}$$

$$= 4\left(1-\frac{1}{3}\right) - 4\left(-1+\frac{1}{3}\right)$$

$$= \frac{16}{3} \quad 2$$

$$\boxed{\frac{16}{3}}$$ 10

(10) 5. Let R be the region between the graphs of $y=\sqrt{x+1}$ and $y=\sqrt{x-1}$ on the interval $[1,3]$. Find the volume of the solid obtained by revolving R about the x axis.



$$\Delta V = \left[\pi\left(\sqrt{x+1}\right)^2 - \pi\left(\sqrt{x-1}\right)^2\right]\Delta x$$

$$V = \int_{1}^{3} \underbrace{\left[\pi(x+1) - \pi(x-1)\right]}_{5} dx$$

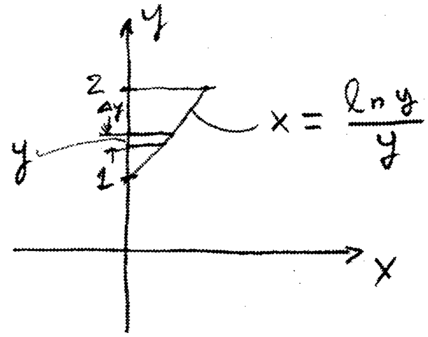
$$= \int_{1}^{3} 2\pi\,dx$$

$$= 2\pi x\Big|_{1}^{3}$$

$$= 4\pi \quad 2$$

$$\boxed{4\pi}$$ 10

(10) 6. Let R be the region between the graph of $x = \dfrac{\ln y}{y}$ and the y axis on the interval $1 \le y \le 2$. Set up an integral (in y) for the volume V of the solid generated by revolving R about the x-axis. Do not evaluate the integral.



$$\Delta V = 2\pi y \frac{\ln y}{y} \Delta y$$

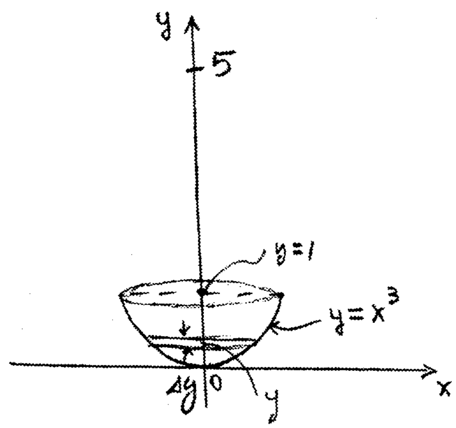
$$V = \int_1^2 2\pi \ln y \, dy$$

0 credit if more than
1 item is wrong.

2 2 5 1

$$V = \int_1^2 2\pi \ln y \, dy$$

(13) 7. A tank has the shape of the surface generated by revolving the curve $y = x^3$, $0 \le x \le 1$ about the y-axis. The tank is full of water. Set up an integral (in y) for the work W required to pump all the water to a level 4 feet above the top of the tank. Do not evaluate the integral. (Water weighs 62.5 lbs/ft^3).



$$\Delta W = (5-y)(62.5)\pi \left(y^{1/3}\right)^2 \Delta y$$

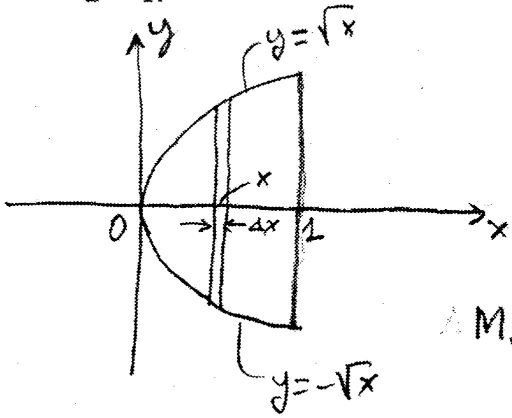
0 credit if more than
2 items are wrong

3 3 1 5 1

$$W = \int_0^1 (5-y)(62.5)\pi\, y^{2/3}\, dy$$

13

(10) 8. Find the center of gravity $(\bar{x}, \bar{y})$ of the region bounded by the graphs of $x = y^2$ and $x = 1$.



From symmetry → $\bar{y} = 0$ (2)

$A\bar{x} = M_y$.

$M_y = \int_0^1 x \, 2\sqrt{x} \, dx$ (1) (2) (1)

$= 2 \frac{x^{5/2}}{\frac{5}{2}} \Big|_0^1 = \frac{4}{5}$ (1)

$A = \int_0^1 2\sqrt{x} \, dx = 2\frac{x^{3/2}}{\frac{3}{2}} \Big|_0^1 = \frac{4}{3}$ (1)

$\bar{x} = \frac{M_y}{A} = \frac{\frac{4}{5}}{\frac{4}{3}} = \frac{3}{5}$ (1) (1)

$(\bar{x}, \bar{y}) = \left(\frac{3}{5}, 0\right)$ 10

(12) 9. Find the third Taylor polynomial $p_3(x)$ of $f(x) = \ln(1+x)$.

$P_3(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3$

$f(x) = \ln(1+x)$ $\quad f(0) = 0$

$f'(x) = \frac{1}{1+x}$ $\quad f'(0) = 1$

$f''(x) = -\frac{1}{(1+x)^2}$ $\quad f''(0) = -1$

$f'''(x) = \frac{2}{(1+x)^3}$ $\quad f'''(0) = 2$

$P_3(x) = 0 + \frac{1}{1}x + \frac{-1}{2!}x^2 + \frac{2}{3!}x^3$

−2pts if more than 3 terms or for $P_n(x)$

(3) (3) (3) (3)

$p_3(x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3$ 12