NAME GRADING KEY

STUDENT ID

RECITATION INSTRUCTOR

RECITATION TIME

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TOTAL	/100

DIRECTIONS

1. Write your name, student ID number, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3, and 4 .
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes or calculators may be used on this test.

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(12) 1.(a) Find the center and radius of the sphere with equation $x^2+y^2+z^2-2x-4y+8z+17=0$.

$$(x^2-2x+1)+(y^2-4y+4)+(z^2+8z+16)=-17+1+4+16$$

$$(x-1)^2+(y-2)^2+(z+4)^2=4$$

center: $(1,2,-4)$ 4

radius $=2$ 4

-2 pts if radius $=4$

center: $(1, 2, -4)$ $\boxed{4}$

radius: 2 $\boxed{4}$

(b) Is the origin inside, outside, or on the sphere in part (a)?

Distance of the origin from center

$=\sqrt{1+4+16}=\sqrt{21}>2$ 2 2 →

ok if consistent with wrong (a)

outside $\boxed{4}$

(6) 2. Find a unit vector $\vec{u}$ in the direction opposite that of $\vec{a}=2\vec{\imath}-\vec{\jmath}-2\vec{k}$.

$|\vec{a}|=\sqrt{4+1+4}=3$ 2

unit vector in the direction of $\vec{a}$: $\frac{2}{3}\vec{\imath}-\frac{1}{3}\vec{\jmath}-\frac{2}{3}\vec{k}$

$\vec{u}=-\frac{2}{3}\vec{\imath}+\frac{1}{3}\vec{\jmath}+\frac{2}{3}\vec{k}$ 4

$\vec{u}=-\frac{2}{3}\vec{\imath}+\frac{1}{3}\vec{\jmath}+\frac{2}{3}\vec{k}$ $\boxed{6}$

(10) 3. A constant force with vector representation $\vec{F} = 10\vec{\imath} + 18\vec{\jmath} - 6\vec{k}$ moves an object along a straight line from the point $P(2,3,0)$ to the point $Q(4,9,15)$. Find the work done if the distance is measured in meters and the magnitude of the force is measured in newtons.

$$\overrightarrow{PQ} = 2\vec{i} + 6\vec{j} + 15\vec{k} \quad 2$$

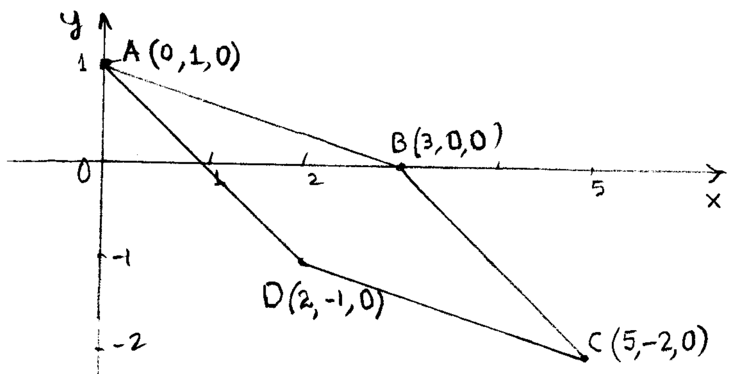
$$\text{Work} = \vec{F} \cdot \overrightarrow{PQ} \quad 4$$

$$= (10\vec{i} + 18\vec{j} - 6\vec{k}) \cdot (2\vec{i} + 6\vec{j} + 15\vec{k})$$

$$= 20 + 108 - 90 = 38 \quad 4$$

$\boxed{\text{38 joules}}$ $\boxed{10}$

(10) 4. Use the cross product to find the area of the parallelogram with vertices $A(0,1)$, $B(3,0)$, $C(5,-2)$, and $D(2,-1)$.



$$\text{Area} = |\overrightarrow{AB} \times \overrightarrow{AD}| \quad 4$$

$$\overrightarrow{AB} = 3\vec{i} - \vec{j}$$

$$\overrightarrow{AD} = 2\vec{i} - 2\vec{j}$$

$$\overrightarrow{AB} \times \overrightarrow{AD} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -1 & 0 \\ 2 & -2 & 0 \end{vmatrix} = -4\vec{k} \quad 4$$

$$|\overrightarrow{AB} \times \overrightarrow{AD}| = 4 \quad 2$$

$\boxed{4}$ $\boxed{10}$

(10) 5. If $\vec{a} = \vec{\jmath} + 2\vec{k}$ and $\vec{b} = 3\vec{\imath} + \vec{\jmath}$, find a vector $\vec{c}$ that is perpendicular to both $\vec{a}$ and $\vec{b}$, has length 2, and has positive z-component.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 2 \\ 3 & 1 & 0 \end{vmatrix} = -2\vec{i} + 6\vec{j} - 3\vec{k} \quad 4$$

-2pts for only one wrong coefficient

$$|\vec{a} \times \vec{b}| = \sqrt{4 + 36 + 9} = 7 \quad 2$$

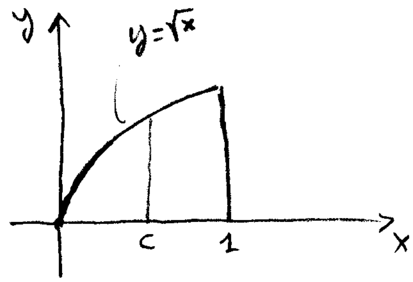
$$\frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = -\frac{2}{7}\vec{i} + \frac{6}{7}\vec{j} - \frac{3}{7}\vec{k}$$

ok if consistent with wrong $\vec{a} \times \vec{b}$

$$\vec{c} = \frac{4}{7}\vec{i} - \frac{12}{7}\vec{j} + \frac{6}{7}\vec{k} \quad 4$$

$\boxed{\vec{c} = \frac{4}{7}\vec{i} - \frac{12}{7}\vec{j} + \frac{6}{7}\vec{k}}$

(8) 6. Find the number c such that the vertical line $x = c$ divides the region bounded the curves $y = \sqrt{x}$, $x = 1$ and the x–axis into two regions with equal area.



$$\int_0^c \sqrt{x}\,dx = \int_c^1 \sqrt{x}\,dx \quad 4$$

$$\frac{2}{3}x^{3/2}\Big|_0^c = \frac{2}{3}x^{3/2}\Big|_c^1$$

$$\frac{2}{3}c^{3/2} = \frac{2}{3} - \frac{2}{3}c^{3/2}$$

$$\frac{4}{3}c^{3/2} = \frac{2}{3}$$

$$c^{3/2} = \frac{1}{2} \to c = \left(\frac{1}{2}\right)^{2/3}$$

3 1

or

$$\int_0^c \sqrt{x}\,dx = \frac{1}{2}\int_0^1 \sqrt{x}\,dx \quad 4$$

$$\frac{2}{3}x^{3/2}\Big|_0^c = \frac{1}{2}\cdot\frac{2}{3}x^{3/2}\Big|_0^1$$

$$\frac{2}{3}c^{3/2} = \frac{1}{3}$$

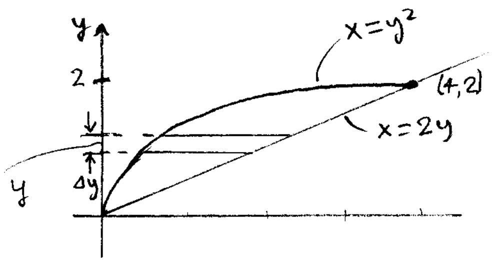
$$c^{3/2} = \frac{1}{2} \to c = \left(\frac{1}{2}\right)^{2/3}$$

3 1

$c = \left(\frac{1}{2}\right)^{2/3}$

8

(10) 7. Use washers to find the volume of the solid obtained by rotating about the y–axis the region bounded by the curves $y^2 = x$ and $x = 2y$.



Curves intersect when $y^2 = 2y \to y = 0, 2$

Volume of typical approximating washer:

$$\Delta V = \pi\left[(2y)^2 - (y^2)^2\right]\Delta y$$

$$V = \int_0^2 \underbrace{\pi\,(4y^2 - y^4)}_{5}\,dy \quad *$$

1 1 1

* 0 credit for problem if more than 1 item is wrong (limits count as 1 item in this rule)

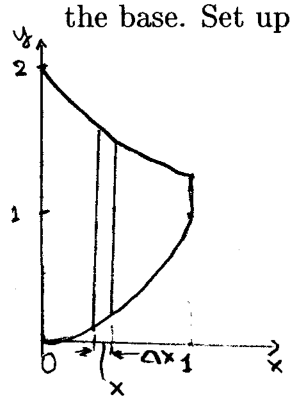
$$= \pi\left(\frac{4}{3}y^3 - \frac{1}{5}y^5\right)\Big|_0^2$$

$$= \pi\left(\frac{32}{3} - \frac{32}{5}\right) = \frac{64\pi}{15} \quad 2$$

$\frac{64\pi}{15}$

10

(10) 8. The base of a solid is the region bounded by the curves $y = 1+e^{-x}$, $y = x^2$, $x = 0$ and $x = 1$. Cross–sections perpendicular to the x–axis are semicircles with diameter on the base. Set up an integral for the volume of the solid. Do not evaluate the integral.



Area of cross-section at x

$$A(x) = \frac{1}{2}\pi\left(\frac{1+e^{-x}-x^2}{2}\right)^2$$

Volume of typical approximating slice

$$\Delta V = \frac{1}{2}\pi\left(\frac{1+e^{-x}-x^2}{2}\right)^2\Delta x$$

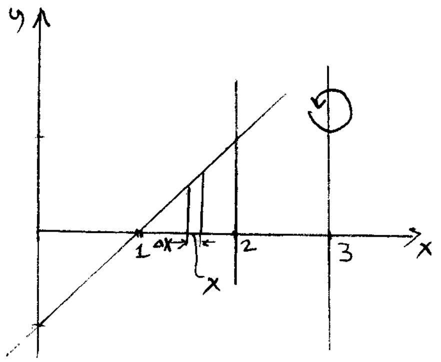
Rule *

$$V = \int_0^1 \underbrace{\frac{1}{2}\pi\left(\frac{1+e^{-x}-x^2}{2}\right)^2}_{5}\,dx$$

2 2 1

10

(8) 9. Using the method of cylindrical shells, set up an integral for the volume of the solid obtained by rotating the region bounded by the curves $y = x - 1$, $x = 2$, and $y = 0$ about the line $x = 3$. Do not evaluate the integral.



Volume of typical approximating shell

$\Delta V = 2\pi(3-x)(x-1)\Delta x$

Rule *

$$\boxed{V = \int_{1}^{2} \underbrace{2\pi(3-x)(x-1)}_{5}\, dx}$$

1 1 1

8

(8) 10. A force of 10 lbs is required to hold a spring stretched 4 in beyond its natural length of 1 ft. How much work is done in stretching the spring 6 in beyond its natural length?

$F = kx$

$10 = k\left(\frac{4}{12}\right) \longrightarrow k = 30$ lbs/ft 3

$F(x) = 30x$

$W = \int_0^{1/2} 30x\,dx = 30 \frac{x^2}{2}\Big|_0^{\frac{1}{2}} = 30 \cdot \frac{1}{8} = \frac{30}{8}$ 3 2

-2pts if in. are not changed to ft.
(45 in-lbs)

$\boxed{\frac{30}{8} \quad ft-lbs}$

8

(8) 11. Find $\int x^{\frac{3}{2}} \ln x dx$.

$u = \ln x \quad dv = x^{3/2}dx$

$du = \frac{1}{x}dx \quad v = \frac{2}{5}x^{5/2}$

$\int u\,dv = uv - \int v\,du$

$\int x^{3/2}\ln x\,dx = \frac{2}{5}x^{5/2}\ln x - \int \frac{2}{5}x^{5/2}\frac{1}{x}dx$

$= \frac{2}{5}x^{5/2}\ln x - \frac{2}{5}\cdot\frac{2}{5}x^{5/2} + C$

-1pt if missing

$$\boxed{\underbrace{\frac{2}{5}x^{5/2}\ln x}_{4} \underbrace{- \left(\frac{2}{5}\right)^2 x^{5/2}}_{4} + C}$$

8