NAME SOLUTIONS

STUDENT ID # ______________________

RECITATION INSTRUCTOR ______________________

RECITATION TIME ______________________

LECTURER ______________________

INSTRUCTIONS

1. There are 8 different test pages (including this cover page). Make sure you have a complete test.
2. Fill in the above items in print. ID # is your 9 digit ID (probably your social security number). Also write your name at the top of pages 2–8.
3. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet.
4. No books, notes or calculators may be used on this exam.
5. Each problem is worth 8 points. The maximum possible score is 200 points.
6. Using a #2 pencil, fill in each of the following items on your answer sheet:
   (a) On the top left side, write your name (last name, first name), and fill in the little circles.
   (b) On the bottom left side, under SECTION, write in your division and section number and fill in the little circles. (For example, for division 9 section 1, write 0901. For example, for division 38 section 2, write 3802).
   (c) On the bottom, under STUDENT IDENTIFICATION NUMBER, write in your 9 digit student ID number, and fill in the little circles.
   (d) Using a #2 pencil, put your answers to questions 1–25 on your answer sheet by filling in the circle of the letter of your response. Double check that you have filled in the circles you intended. If more than one circle is filled in for any question, your response will be considered incorrect. Use a #2 pencil.
7. After you have finished the exam, hand in your answer sheet and your test booklet to your recitation instructor.

1. If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{27 - x^3}$, the domain of $g \circ f$ is

dom. $f = [0, \infty]$ dom. $g = (-\infty, 3]$

$(g \circ f)(x) = g(f(x))$

$x \in \text{dom}(g \circ f)$: $x \in \text{dom} f$, and $f(x) \in \text{dom} g$

$x \in \text{dom}(g \circ f)$: $x \geq 0$, and $\sqrt{x} \leq 3$

$\therefore \quad 0 \leq x \leq 9$

A. $[0, 9]$
B. $(-\infty, 9]$
C. $(-\infty, 0] \cup [9, \infty)$
D. $[0, \infty)$
E. $[0, 3]$

2. If $f(x) = \dfrac{1+x}{1-x}$, $f^{-1}(2) =$

$x = f^{-1}(2) \iff f(x) = 2$

$\dfrac{1+x}{1-x} = 2$

$1 + x = 2 - 2x$

$3x = 1$

$x = \frac{1}{3}$

A. $\frac{1}{3}$
B. $\frac{2}{3}$
C. $\frac{4}{3}$
D. 4
E. $-\frac{1}{3}$

3. $\displaystyle\lim_{x\to\infty} \frac{\sqrt{x^2+4x}}{8x+20} = \lim_{x\to\infty} \frac{x\sqrt{1+\frac{4}{x}}}{8x+20}$

$\displaystyle = \lim_{x\to\infty} \frac{1\sqrt{1+\frac{4}{x}}}{8+\frac{20}{x}}$

$= \dfrac{1}{8}$

A. $\frac{1}{64}$
B. $2\sqrt{2}$
C. $\frac{1}{4}$
D. $\frac{1}{8}$
E. does not exist

4. If $f(x) = \dfrac{4x^2+6}{3x^2-1}$, $f'(1) =$

$f'(x) = \dfrac{(3x^2-1)8x - (4x^2+6)6x}{(3x^2-1)^2}$

$f'(1) = \dfrac{2 \cdot 8 \cdot 1 - 10 \cdot 6 \cdot 1}{2^2} = \dfrac{16 - 60}{4}$

$= -\dfrac{44}{4} = -11$

A. 11
B. -11
C. -7
D. 7
E. 4

5. Find an equation for the line tangent to the curve $y = x^2 \ln(x^2)$ at the point where $x = e$.

$$\frac{dy}{dx} = x^2 \frac{1}{x^2} 2x + 2x \ln(x^2)$$
$$= 2x + 2x \ln(x^2)$$

at $x=e$: $\frac{dy}{dx} = 2e + 2e \ln e^2 = 2e + 4e = 6e$

and $y = e^2 \ln(e^2) = 2e^2$

tangent line: $y - 2e^2 = 6e(x-e)$ $\quad y - 6ex - 2e^2 + 6e^2 = 0$

A. $y - 3ex + e^2 = 0$

B. $y - 6ex + 4e^2 = 0$

C. $y + ex - 3e^2 = 0$

D. $y - 3ex + 2e^2 = 0$

E. $y + 6ex - 7e^2 = 0$

6. If $f(x) = (x^2+1)e^{x^2}$, $f''(x) =$

$$f'(x) = (x^2+1)e^{x^2} 2x + 2x e^{x^2}$$
$$= (2x^3 + 4x) e^{x^2}$$
$$f''(x) = (2x^3+4x)e^{x^2} 2x + (6x^2+4) e^{x^2}$$
$$= (4x^4 + 8x^2 + 6x^2 + 4) e^{x^2}$$
$$= (4x^4 + 14x^2 + 4) e^{x^2}$$

A. $(4x^4 + 4x^2)e^{x^2}$

B. $(4x^2 + 2x + 1)e^{x^2}$

C. $(2x^2 + x + 1)e^{x^2}$

D. $(2x^4 + 12x^2 + 4x)e^{x^2}$

E. $(4x^4 + 14x^2 + 4)e^{x^2}$

7. If $y = \cos^3(2x)$, then $\frac{dy}{dx} =$

$$\frac{dy}{dx} = 3\cos^2(2x)(-\sin(2x)) \cdot 2$$
$$= -6\cos^2(2x)\sin(2x)$$

A. $6\cos^2(2x)\sin(2x)$

B. $-6\sin^2(2x)\sin(2x)$

C. $6\sin^2(2x)\sin(2x)$

D. $-6\cos^2(2x)\sin(2x)$

E. $6\cos^2(2x)$

8. If $x\cos y = y$, find $\frac{dy}{dx}$ at $(x,y) = (0,0)$.

$$x\cos y = y$$
$$-x\sin y \frac{dy}{dx} + \cos y = \frac{dy}{dx}$$

At $(x,y) = 00$: $\quad 0 + 1 = \frac{dy}{dx}$

A. $\frac{1}{2}$

B. 1

C. 0

D. -1

E. 2

9. If $y = \sin^{-1}(\tan x)$, $\dfrac{dy}{dx} =$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-\tan^2 x}} \sec^2 x$$

A. $\dfrac{-2\tan x \sec^2 x}{\sqrt{1+\tan^2 x}}$

B. $\dfrac{-\sec^2 x}{\sqrt{1-\tan^2 x}}$

C. $\dfrac{\sec^2 x}{1+\tan^2 x}$

D. $\dfrac{2\tan x \sec^2 x}{\sqrt{1-\tan^2 x}}$

E. $\dfrac{\sec^2 x}{\sqrt{1-\tan^2 x}}$

10. If $y = x^{\ln x}$, find $\dfrac{dy}{dx}$ at $x = e$.

$$y = x^{\ln x} = \left(e^{\ln x}\right)^{\ln x} = e^{(\ln x)^2}$$

$$\frac{dy}{dx} = e^{(\ln x)^2}(2\ln x)\cdot\frac{1}{x}$$

$$\left.\frac{dy}{dx}\right|_{x=e} = e^{(\ln e)^2}(2\ln e)\cdot\frac{1}{e} = e\cdot 2\cdot\frac{1}{e} = 2$$

A. 1

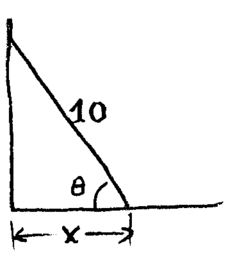
B. 2

C. 3

D. e

E. 0

11. A 10-ft plank of wood is leaning against a wall. The bottom of the plank is being pushed toward the wall at the rate of 1 ft/sec. Find the rate of change of the acute angle that the plank makes with the ground, when this angle is 60°.



$$\frac{dx}{dt} = -1$$

$$\cos\theta = \frac{x}{10}$$

$$-\sin\theta\cdot\frac{d\theta}{dt} = \frac{1}{10}\frac{dx}{dt}$$

When $\theta = 60^\circ$:

$$-\frac{\sqrt{3}}{2}\frac{d\theta}{dt} = \frac{1}{10}(-1)$$

$$\frac{d\theta}{dt} = \frac{1}{5\sqrt{3}} = \frac{\sqrt{3}}{15}$$

A. $\dfrac{\sqrt{2}}{5}$ rad/sec

B. $\sqrt{2}$ rad/sec

C. $\sqrt{3}$ rad/sec

D. $\dfrac{\sqrt{3}}{15}$ rad/sec

E. $2\sqrt{2}$ rad/sec

12. Estimate $\sqrt{36.3}$ using a linear approximation at $a = 36$.

A. 6.010
B. 6.015
C. 6.020
D. 6.025
E. 6.030

Linear approximation of f at a:

$f(x) \approx f(a) + f'(a)(x-a)$, for x near a

$f(x) = \sqrt{x}$ $\quad f'(x) = \frac{1}{2\sqrt{x}}$ $\quad f(36) = 6,\ f'(36) = \frac{1}{12}$

$\sqrt{x} \approx 6 + \frac{1}{12}(x-36)$

$\sqrt{36.3} \approx 6 + \frac{1}{12}(36.3-36) = 6 + \frac{0.3}{12} = 6 + 0.025 = 6.025$

13. The minimum and maximum values of $f(x) = x^2 - 8|x|$ on the interval $[-2, 2]$ are

A. -12 and 0
B. -16 and 0
C. -16 and 16
D. 0 and 16
E. 0 and 12

$$f(x) = \begin{cases} x^2 + 8x & \text{if } -2 \le x \le 0 \\ x^2 - 8x & \text{if } 0 \le x \le 2 \end{cases}$$

$$f'(x) = \begin{cases} 2x + 8 & \text{if } -2 \le x < 0 \\ 2x - 8 & \text{if } 0 < x \le 2 \end{cases}$$

$f'(x) = 0$ when $x = \pm 4$, and $f'(0)$ does not exist.

$\therefore$ $x = 0$ is the only critical number in $[-2, 2]$

$f(-2) = -12$ (min) $\quad f(0) = 0$ (max) $\quad f(2) = -12$ (min)

14. The graph of the function $f(x) = 3x^5 - 5x^4$ has inflection points when $x =$

A. 0 and 1
B. 0 and 2
C. 1
D. 2
E. never

$f'(x) = 15x^4 - 20x^3$

$f''(x) = 60x^3 - 60x^2 = 60x^2(x-1)$

$f''(x)$: $- - - - - 0 - - - 0 + + + + +$ on the x-axis, with 0 and 1 marked

inflection point (at $x = 1$)

15. If m and n are constants, $\lim_{x \to 0} \frac{\cos mx - \cos nx}{x^2} =$

A. $m - n$
B. $n - m$
C. $m^2 - n^2$
D. $\frac{n^2 - m^2}{2}$
E. ∞

$$\lim_{x \to 0} \underbrace{\frac{\cos mx - \cos nx}{x^2}}_{\frac{0}{0}} \overset{L'H}{=} \lim_{x \to 0} \underbrace{\frac{-m \sin mx + n \sin nx}{2x}}_{\frac{0}{0}}$$

$$\overset{L'H}{=} \lim_{x \to 0} \frac{-m^2 \cos mx + n^2 \cos nx}{2} = \frac{-m^2 + n^2}{2}$$

16. On the interval $(0,4)$, the function $f(x) = 2x^3 - 9x^2 + 12x + 10$ is

A. increasing on $(0,1)$ and decreasing on $(1,4)$.

B. decreasing on $(0,1)$ and increasing on $(1,4)$.

C. decreasing on $(0,1)$, increasing on $(1,2)$ and decreasing on $(2,4)$.

D. increasing on $(0,1)$, decreasing on $(1,2)$ and increasing on $(2,4)$.

E. decreasing on $(0,1)$, increasing on $(1,3)$ and decreasing on $(3,4)$.

$$f'(x) = 6x^2 - 18x + 12 = 6(x^2 - 3x + 2) = 6(x-1)(x-2)$$

$f'(x)$: $+++\,0\,---\,0\,++++++$ on the number line $0 \quad 1 \quad 2 \quad 4$

incr. decr incr

17. If $f''(x) = 2 - \sin x$, $f(0) = 1$ and $f'(0) = 1$, find the value of $f(\pi)$.

A. 2π

B. $\pi^2 + 1$

C. π^2

D. $2\pi + \pi^2$

E. $2\pi + 1$

$f'(x) = 2x + \cos x + C_1$

$x=0$: $1 = 0 + 1 + C_1 \rightarrow C_1 = 0$

$f'(x) = 2x + \cos x$

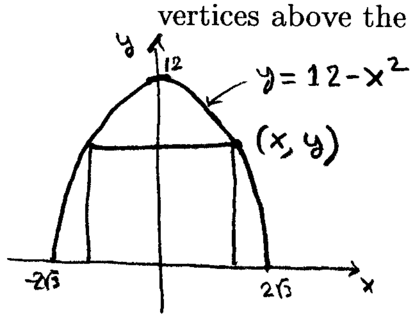
$f(x) = x^2 + \sin x + C_2$

$x=0$: $1 = 0 + 0 + C_2 \rightarrow C_2 = 1$

$f(x) = x^2 + \sin x + 1$

$f(\pi) = \pi^2 + 0 + 1$

18. Find the area of the largest rectangle that has its base on the x–axis and its other two vertices above the x–axis and lying on the parabola $y = 12 - x^2$.



A. 12

B. 18

C. $8\sqrt{2}$

D. $6\sqrt{12}$

E. 32

$A = 2xy$

$A = 2x(12 - x^2)$, $0 \le x \le 2\sqrt{3}$

$\frac{dA}{dx} = 24 - 6x^2$

$\frac{dA}{dx} = 0$: $24 - 6x^2 = 0$

$x = 2$

When $x = 2$: $A = 32$ ← max

When $x = 0$ or $x = 2\sqrt{3}$: $A = 0$ ∴

19. $\dfrac{d}{dx}\displaystyle\int_x^{x^2} \ln t\,dt = \dfrac{d}{dx}\left[\int_x^1 \ln t\,dt + \int_1^{x^2} \ln t\,dt\right]$

$= \dfrac{d}{dx}\left[-\displaystyle\int_1^x \ln t\,dt + \int_1^{x^2} \ln t\,dt\right]$

$= -\ln x + \ln(x^2)\cdot 2x$

A. $2x\ln(x^2) - \ln x$
B. $\ln(x^2)$
C. $\dfrac{1}{x^2} - \dfrac{1}{x}$
D. $\ln(x^2) - \ln x$
E. 2

20. $\displaystyle\int_1^2 \left(\frac{1}{t^2} - t\right) dt = \left[-\frac{1}{t} - \frac{t^2}{2}\right]_1^2$

$= \left(-\frac{1}{2} - 2\right) - \left(-\frac{1}{1} - \frac{1}{2}\right)$

$= -\frac{1}{2} - 2 + 1 + \frac{1}{2} = -1$

A. $-\dfrac{7}{4}$
B. $\dfrac{7}{4}$
C. $-\dfrac{1}{4}$
D. -1
E. 1

21. $\displaystyle\int_0^1 x\sin(x^2+1)\,dx = \frac{1}{2}\int_1^2 \sin u\,du = -\frac{1}{2}\cos u\Big|_1^2$

$u = x^2+1$
$du = 2x\,dx$
$x=0 \rightarrow u=1$
$x=1 \rightarrow u=2$

$= -\frac{1}{2}\cos 2 + \frac{1}{2}\cos 1$

A. $2(\cos 2 - \cos 1)$
B. $2(\cos 1 - \cos 2)$
C. $\frac{1}{2}(\cos 2 - \cos 1)$
D. $\frac{1}{2}(\cos 1 - \cos 2)$
E. 0

22. $\displaystyle\int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos^2 x}\,dx = -\int_1^{\frac{1}{2}} \frac{1}{u^2}\,du = \frac{1}{u}\Big|_1^{\frac{1}{2}}$

$u = \cos x$
$du = -\sin x\,dx$
$x=0 \rightarrow u=1$
$x=\frac{\pi}{3} \rightarrow u=\frac{1}{2}$

$= \frac{1}{\frac{1}{2}} - 1 = 1$

A. $\dfrac{3}{2}$
B. $\dfrac{1}{2}$
C. $\dfrac{2}{\sqrt{3}}$
D. $-\dfrac{1}{2}$
E. 1

23. A certain radioactive substance has a half–life of 50 years. How long will it take for its mass to decay to 75% of its original size?

A. $50\,\dfrac{\ln(0.75)}{\ln(0.5)}$

B. $50\,\dfrac{\ln(0.5)}{\ln(0.75)}$

C. $50\,\dfrac{\ln(0.25)}{\ln(0.5)}$

D. $50\,\dfrac{\ln(0.5)}{\ln(0.25)}$

E. $50\ \ln 3$

$m(t) = m_0 e^{kt}$

half-life is 50 years:

$\frac{1}{2} m_0 = m_0 e^{k(50)}$

$\ln(0.5) = k(50) \rightarrow k = \dfrac{\ln(0.5)}{50}$

$m(t) = m_0 e^{\frac{\ln(0.5)}{50} t}$

t? so that $m(t) = 0.75\, m_0$

$(0.75) m_0 = m_0 e^{\frac{\ln(0.5)}{50} t}$

$\ln(0.75) = \dfrac{\ln(0.5)}{50} t \rightarrow t = 50\, \dfrac{\ln(0.75)}{\ln(0.5)}$

24. Find an equation of the ellipse with center $(-1, 2)$, focus $(2, 2)$ and vertex $(3, 2)$.

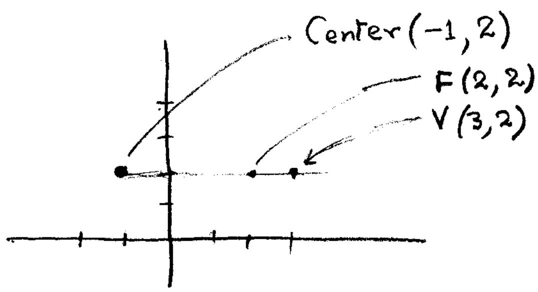
A. $16(x+1)^2+9(y-2)^2 = 144$

B. $9(x+1)^2+16(y-2)^2 = 144$

C. $7(x+1)^2+16(y-2)^2 = 112$

D. $16(x-1)^2+7(y+2)^2 = 112$

E. $9(x-1)^2+16(y+2)^2 = 144$



$a = 3-(-1) = 4$

$c = 2-(-1) = 3$

$a^2 - b^2 = c^2$

$16 - b^2 = 9$

$b^2 = 7$

$$\frac{(x-(-1))^2}{16} + \frac{(y-2)^2}{7} = 1 \qquad 7(x+1)^2 + 16(y-2)^2 = 112$$

25. The parabola $8x + y^2 + 6y + 17 = 0$ has focus at

A. $(2, 3)$

B. $(-3, -3)$

C. $(-1, -3)$

D. $(-2, -3)$

E. $(-2, 3)$

$8x + (y^2 + 6y + 9) = -17 + 9$

$(y+3)^2 = -8x - 8$

$(y+3)^2 = -8(x+1)$

vertex $(-1,-3)$, $4p = -8 \rightarrow p = -2$