NAME GRADING KEY

10-digit PUID ____________

RECITATION INSTRUCTOR ____________

RECITATION TIME ____________

Page 1	/18
Page 2	/36
Page 3	/16
Page 4	/30
TOTAL	/100

DIRECTIONS

1. Write your name, 10–digit PUID, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes, calculators or any electronic devices may be used on this exam.

---

(9) 1. Find the absolute maximum and absolute minimum values of the function $f(x) = x^3 - 3x^2 - 9x$ on the interval $[0, 4]$.

$f'(x) = 3x^2 - 6x - 9$ ②

$f'(x) = 0$; $x^2 - 2x - 3 = 0 \rightarrow (x-3)(x+1) = 0$

$x = 3$ ③ , $\cancel{x = -1}$, -1 is not in $(0, 4)$

$f(0) = 0$

$f(3) = 27 - 3 \cdot 9 - 9 \cdot 3 = -27$

$f(4) = 64 - 3 \cdot 16 - 9 \cdot 4 = -20$

abs. max. $f(0) = 0$ ②

abs. min. $f(3) = -27$ ②

9

(9) 2. (a) The Mean Value Theorem asserts that for the function $f(x) = \sin x$ on the interval $[0, 2\pi]$

there is a number c in $(0, 2\pi)$ ②
such that $f'(c) = \frac{\sin 2\pi - \sin 0}{2\pi}$

or ③
$f'(c) = 0$

(b) Find all numbers c that satisfy the conclusion in (a)

$f'(c) = 0$

$\cos c = 0 \longrightarrow c = \frac{\pi}{2}, \frac{3\pi}{2}$

② ②

$\frac{\pi}{2}$, $\frac{3\pi}{2}$

9

(30) 3. Find each of the following as a real number, $+\infty, -\infty$, or write DNE (does not exist).

5pts each NPC

(a) $\lim_{x\to 0} \frac{\sin 4x}{\tan 5x} \overset{L'H}{=} \lim_{x\to 0} \frac{4\cos 4x}{5\sec^2 5x} = \frac{4}{5}$

$\frac{0}{0}$

$\boxed{\frac{4}{5}}$

(b) $\lim_{x\to 0} \frac{\cos x - 1}{x^2} \overset{L'H}{=} \lim_{x\to 0} \frac{-\sin x}{2x}$

$\frac{0}{0}$

$= -\frac{1}{2} \lim_{x\to 0} \frac{\sin x}{x} = -\frac{1}{2}$

$\boxed{-\frac{1}{2}}$

(c) $\lim_{x\to \frac{\pi}{4}} (1-\tan x)\sec x = \left(1-\tan\frac{\pi}{4}\right)\sec\frac{\pi}{4}$

$= (1-1)\sqrt{2} = 0$

$\boxed{0}$

(d) $\lim_{x\to 0^+} \sin x \ln x = \lim_{x\to 0^+} \frac{\ln x}{\csc x} \overset{L'H}{=} \lim_{x\to 0} \frac{\frac{1}{x}}{-\csc x \cot x}$

$0\cdot\infty$ $\quad \frac{\infty}{\infty}$

$= -\lim_{x\to 0^+} \frac{\sin x}{x} \tan x = -1\cdot 0 = 0$

$\boxed{0}$

(e) $\lim_{x\to(\frac{\pi}{2})^-} \frac{\cos x}{1-\sin x} \overset{L'H}{=} \lim_{x\to(\frac{\pi}{2})^-} \frac{-\sin x}{-\cos x}$

$\frac{0}{0}$

$= \lim_{x\to(\frac{\pi}{2})^-} \tan x = \infty$

$\boxed{\infty}$

(f) $\lim_{x\to\infty} (1+\frac{4}{x})^{7x} = \lim_{x\to\infty} e^{\ln(1+\frac{4}{x})^{7x}} = e^{\lim_{x\to\infty}(7x)\ln(1+\frac{4}{x})} = e^{28}$

$\lim_{x\to\infty} (7x)\ln(1+\frac{4}{x}) = 7\lim_{x\to\infty} \frac{\ln(1+\frac{4}{x})}{\frac{1}{x}} \overset{L'H}{=}$

$\infty\cdot 0$

$\overset{L'H}{=} 7\lim_{x\to\infty} \frac{\frac{1}{1+\frac{4}{x}}\left(-\frac{4}{x^2}\right)}{-\frac{1}{x^2}} = 7\lim_{x\to\infty} \frac{4}{1+\frac{4}{x}} = 7\cdot 4 = 28$

$\boxed{e^{28}}$

$\boxed{30}$

(6) 5. 0 is a critical number of the function $f(x) = e^{-|x|}$. Complete the following

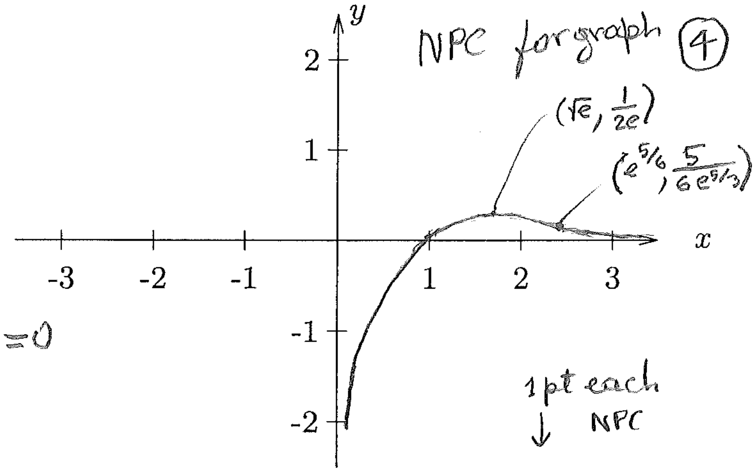
(a) If $x > 0$, $f'(x) = \boxed{-e^{-x}}$; $f'(x)$ is positive, [✓ negative] (circle one). ②

(b) If $x < 0$, $f'(x) = \boxed{e^{x}}$; $f'(x)$ is [✓ positive], negative (circle one). ②

(c) At $x = 0$, f has a
[✓ local maximum,] local minimum, neither (circle one) ②

$\boxed{6}$

(16) 5. Let $f(x) = \frac{\ln x}{x^2}$. Give all the requested information and sketch the graph of the function on the axes below. Give both coordinates of the intercepts, local extrema and points of inflection, and give an equation for each asymptote. Write NONE where appropriate.



NPC for graph ④

1pt each ↓ NPC

Domain $(0, \infty)$

Intercepts:

y-int. NONE

x-int. $\frac{\ln x}{x^2} = 0 \rightarrow x = 1$

Symmetry NONE

H. A. $\lim_{x\to\infty} \frac{\ln x}{x^2} \overset{L'H}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{2x} = 0$ $\quad \frac{\infty}{\infty}$ $\quad \boxed{y=0}$

V. A. $\lim_{x\to 0^+} \frac{\ln x}{x^2} = -\infty$ $\quad \boxed{x=0}$

$$f'(x) = \frac{x^2 \frac{1}{x} - (\ln x) 2x}{x^4}$$

$$= \frac{1 - 2\ln x}{x^3}$$

$f'(x) = 0$: $\ln x = \frac{1}{2} \rightarrow x = \sqrt{e}$

$y = \frac{\ln\sqrt{e}}{e} = \frac{1}{2e}$

$f'(x)$ 0 + + + + 0 − − − − → x

0 $\quad \sqrt{e}$

↑loc. max

$$f''(x) = \frac{x^3(-\frac{2}{x}) - (1 - 2\ln x)3x^2}{x^6}$$

$$= \frac{-2 - 3 + 6\ln x}{x^4}$$

$$= \frac{-5 + 6\ln x}{x^4}$$

$f''(x) = 0$: $\ln x = \frac{5}{6} \rightarrow x = e^{5/6}$

$f''(x)$: 0 − − − 0 + + + + → x

$e^{5/6}$

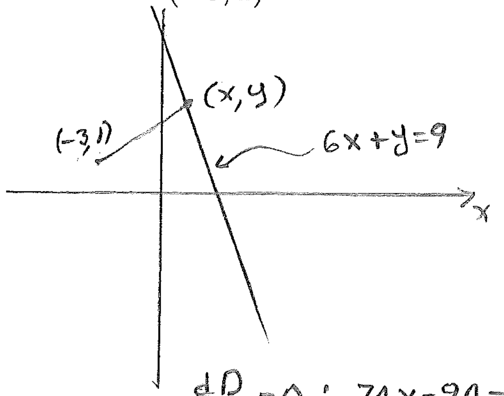
Points of inflection:

$x = e^{5/6}$

$y = \frac{\frac{5}{6}}{e^{5/3}} = \frac{5}{6e^{5/3}}$

domain	$(0, \infty)$
intercepts	$(1, 0)$
symmetry	NONE
horizontal asymptotes	$y = 0$
vertical asymptotes	$x = 0$
intervals of increase	$(0, \sqrt{e})$
intervals of decrease	$(\sqrt{e}, \infty)$
local maxima	$(\sqrt{e}, \frac{1}{2e})$
local minima	NONE
intervals of concave down	$(0, e^{5/6})$
intervals of concave up	$(e^{5/6}, \infty)$
points of inflection	$(e^{5/6}, \frac{5}{6e^{5/3}})$

(12) 6. Find the x–coordinate of the point on the line $6x + y = 9$ that is closest to the point $(-3, 1)$.



Let D = distance of a point (x, y) on the line from (-3, 1)

$$D = \sqrt{(x+3)^2 + (y-1)^2}$$

$$D = \sqrt{(x+3)^2 + (8-6x)^2}, \quad -\infty < x < \infty \quad \leftarrow ⑤$$

$$\frac{dD}{dx} = \frac{1}{2\sqrt{(x+3)^2+(8-6x)^2}}\left[2(x+3)+2(8-6x)(-6)\right]$$

$$= \frac{74x - 90}{2\sqrt{(x+3)^2+(8-6x)^2}} \quad ⑤$$

$\frac{dD}{dx} = 0$: $74x - 90 = 0 \rightarrow x = \frac{90}{74}$

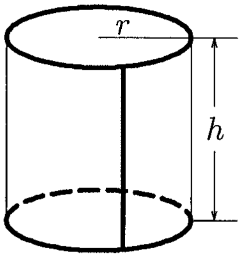
$$\frac{dD}{dx} \quad - - - \underset{90/74}{0} + + +$$

or $\frac{45}{37}$

$$\boxed{x = \frac{90}{74} \quad ②}$$

$\boxed{12}$

(12) 7. A closed cylindrical tank is made from three metal sheets (top, bottom, side), which are welded together along three seams, as indicated. The volume of the tank is 1000 cm^3. Find the radius of the tank so that the total length L of the seams is as small as possible.



$$\pi r^2 h = 1000 \quad ③$$

$$L = 4\pi r + h \quad ③$$

$$h = \frac{1000}{\pi r^2}$$

$$L = 4\pi r + \frac{1000}{\pi r^2} \quad ③, \qquad 0 < r < \infty$$

$$\frac{dL}{dr} = 4\pi - \frac{2000}{\pi r^3}$$

$\frac{dL}{dr} = 0$: $4\pi - \frac{2000}{\pi r^3} = 0 \rightarrow r^3 = \frac{1000}{2\pi^2} \rightarrow r = \frac{10}{\sqrt[3]{2\pi^2}}$ ③

$$\frac{dL}{dr} \quad - - - \underset{\underset{\overset{\uparrow}{\text{min}}}{\frac{10}{\sqrt[3]{2\pi^2}}}}{0} + + + + \rightarrow r$$

$$\boxed{r = \frac{10}{\sqrt[3]{2\pi^2}}}$$

$\boxed{12}$

(6) 8. Find f if $f'(x) = 2\cos x + \sec^2 x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$, and $f(\frac{\pi}{3}) = 4$.

$$f(x) = 2\sin x + \tan x + C$$

$x = \frac{\pi}{3}$: $f(\frac{\pi}{3}) = 2\sin\frac{\pi}{3} + \tan\frac{\pi}{3} + C$

$$4 = 2 \cdot \frac{\sqrt{3}}{2} + \sqrt{3} + C$$

$$C = 4 - 2\sqrt{3}$$

$$\boxed{f(x) = \overset{②}{2\sin x} + \overset{②}{\tan x} + \overset{②}{4 - 2\sqrt{3}}}$$

$\boxed{6}$