NAME GRADING KEY

STUDENT ID ____________

RECITATION INSTRUCTOR ____________

RECITATION TIME ____________

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DIRECTIONS

1. Write your name, student ID number, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes or calculators may be used on this exam.

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(12) 1. Find the absolute maximum and absolute minimum values of $f(x) = xe^{-x}$ on the interval $[0, 2]$.

$f'(x) = -xe^{-x} + e^{-x} = e^{-x}(1-x)$ (2)

$f'(x) = 0$: $e^{-x}(1-x) = 0 \longrightarrow x = 1$. (2)

$f(0) = 0$ abs. min

$f(1) = \frac{1}{e}$ abs. max

$f(2) = \frac{2}{e^2}$

$\frac{1}{e} > \frac{2}{e^2}$?

$e > 2$ yes

(3) (1)

abs. max. $\boxed{f(1) = \frac{1}{e}}$

abs. min. $\boxed{f(0) = 0}$

(3) (1)

12

(8) 2. If $f(1) = 10$ and $f'(x) \geq 2$ for $1 \leq x \leq 4$, find the largest number A for which you are sure that $f(4) \geq A$. (Hint: Mean Value Theorem).

(4) $\frac{f(4) - f(1)}{4-1} = f'(c)$, $1 < c < 4$

$f'(c) \geq 2 \rightarrow \frac{f(4) - f(1)}{4-1} \geq 2$

$f(4) \geq 6 + 10 = 16$ (4)

$\boxed{A = 16}$

8

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(18) 3. Find each of the following as a real number, $+\infty$, $-\infty$ or DNE (does not exist).

(a) $\lim_{x\to 0} \frac{\cos x - 1}{\sin x}$ $\frac{0}{0}$ $\overset{L'H}{=} \underbrace{\lim_{x\to 0} \frac{-\sin x}{\cos x}}_{4} = \frac{0}{1} = 0$ 1

−1 pt for inconsistent use of "=" and "lim" (one time only)

$\boxed{0}$ 5

(b) $\lim_{x\to\infty} \frac{e^x}{x^2}$ $\frac{\infty}{\infty}$ $\overset{L'H}{=} \overbrace{\lim_{x\to\infty} \frac{e^x}{2x}}^{3}$ $\frac{\infty}{\infty}$ $= \overbrace{\lim_{x\to\infty} \frac{e^x}{2}}^{1} = \overset{1}{\infty}$

$\boxed{\infty}$ 5

(c) $\lim_{x\to\infty} \left(1+\frac{2}{x}\right)^x = \lim_{x\to\infty} e^{x\ln(1+\frac{2}{x})} = e^{\underbrace{\lim_{x\to\infty} x\ln(1+\frac{2}{x})}_{3}}$

$\lim_{x\to\infty} x\ln(1+\frac{2}{x})$ $\infty\cdot 0$ $= \lim_{x\to\infty} \frac{\ln(1+\frac{2}{x})}{\frac{1}{x}}$ 1 $\frac{0}{0}$ $\overset{L'H}{=} \lim_{x\to\infty} \frac{\frac{1}{1+\frac{2}{x}}\left(-\frac{2}{x^2}\right)}{-\frac{1}{x^2}}$ 2 $=$

$= \lim_{x\to\infty} \frac{2}{1+\frac{2}{x}} = 2$ 1

$\therefore \lim_{x\to\infty}\left(1+\frac{2}{x}\right)^x = e^2$ 1

$\boxed{e^2}$ 8

(12) 4. The number 0 is the only critical number of $f(x) = \ln(1+x^2)$. Decide whether $f(0) = 0$ is a local maximum or a local minimum using

(a) the first derivative test.

$f'(x) = \frac{2x}{1+x^2}$ 2

$- - - - - \overset{0}{|} + + + + +$ x

$f'(x) < 0$ for $-\infty < x < 0$ and $f'(x) > 0$ for $0 < x < \infty$ 4 $\therefore f(0)=0$ is a local min. 6

(b) the second derivative test.

or 2

$f''(x) = \frac{(1+x^2)2 - 2x\cdot 2x}{(1+x^2)^2} = \frac{2-2x^2}{(1+x^2)^2}$

$f''(0) = 2 > 0$ $\therefore f(0) = 0$ is a local minimum 4 6

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(20) 5. Let $f(x) = \dfrac{x}{x-1}$. Give all the requested information and sketch the graph of the function on the axes below. Give both coordinates of the intercepts, local extrema and points of inflection, and give an equation for each asymptote. Write NONE where appropriate.

$y = \dfrac{x}{x-1}$

Domain all $x \neq 1$

y-intercept: $x=0 \to y=0$ $(0,0)$

x-intercept: $y=0 \to x=0$ $(0,0)$

$f(-x) = \dfrac{-x}{-x-1} \neq \dfrac{x}{x+1}$

no symmetry

$\lim\limits_{x\to\infty} \dfrac{x}{x-1} = 1$, $\lim\limits_{x\to-\infty} \dfrac{x}{x-1} = 1$

$\therefore$ h.a. $y=1$

$\lim\limits_{x\to 1^+} \dfrac{x}{x-1} = +\infty$

$\therefore$ v.a. $x=1$

$\lim\limits_{x\to 1^-} \dfrac{x}{x-1} = -\infty$

$f'(x) = \dfrac{(x-1)1 - x\cdot 1}{(x-1)^2} = -\dfrac{1}{(x-1)^2}$

f decr. on $(-\infty, 0)$ and on $(0,\infty)$

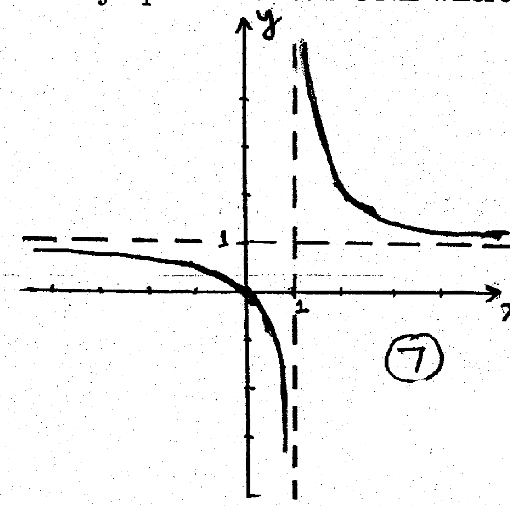
$f'(x) \neq 0$ for all x in domain

$f''(x) = -\dfrac{(x-1)^2\cdot 0 - 1\cdot 2(x-1)}{(x-1)^4}$

$= \dfrac{2}{(x-1)^3}$

$f''(x) < 0$ on $(-\infty, 1)$ C.D.

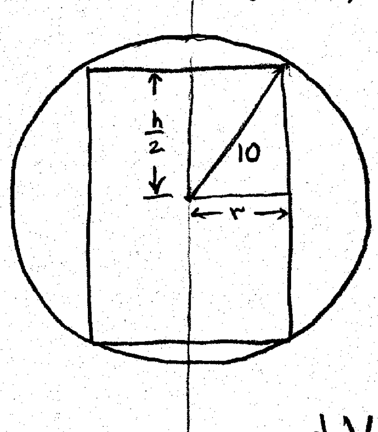
$f''(x) > 0$ on $(1,\infty)$ C.U.



(7)

domain	all $x \neq 1$ or $(-\infty,1)\cup(1,\infty)$	(1)
intercepts	$(0,0)$	(1)
symmetry	NONE	(1)
horizontal asymptotes	$y=1$	(1)
vertical asymptotes	$x=1$	(1)
intervals of increase	NONE	(1)
intervals of decrease	$(-\infty, 1)$ and $(1,\infty)$	(2) NPC
local maxima	NONE	(1)
local minima	NONE	(1)
intervals of concave down	$(-\infty, 1)$	(1)
intervals of concave up	$(1,\infty)$	(1)
points of inflection	NONE	(1)

(15) 6. A right circular cylinder is inscribed in a sphere of radius 10. Find the height of the cylinder with largest possible volume. (Let h denote the height and r denote the radius of the cylinder).



$V = \pi r^2 h$. (2)

$r^2 + \left(\frac{h}{2}\right)^2 = 100$ (5)

$V = \pi\left(100 - \frac{h^2}{4}\right)h$ or (2)

$V = 100\pi h - \frac{\pi}{4}h^3$ $\quad 0 < h < 20$

$\frac{dV}{dh} = 100\pi - \frac{3\pi}{4}h^2$ (2)

$\frac{dV}{dh} = 0$: $\quad 100\pi - \frac{3\pi}{4}h^2 = 0 \rightarrow h^2 = \frac{400}{3} \rightarrow h = \frac{20}{\sqrt{3}}$ (4)

$\left(\frac{dV}{dh} > 0 \text{ for } h < \frac{20}{\sqrt{3}} \text{ and } \frac{dV}{dh} < 0 \text{ for } h > \frac{20}{\sqrt{3}}\right)$

$\therefore$ abs. max

$\boxed{\frac{20}{\sqrt{3}}}$ 15

(5) 7. Find the most general antiderivative of $f(x) = \sec^2 x + 5\cos x$.

-1 pt for missing $+C$

$\boxed{\tan x + 5\sin x + C}$ 5

(10) 8. A particle is moving with acceleration $a(t) = \cos t + \sin t$. Its initial position is $s(0) = -5$ and its initial velocity is $v(0) = 3$. Find the position function $s(t)$.

$a(t) = \cos t + \sin t$

$v(t) = \sin t - \cos t + C_1$ (2)

$v(0) = 3$: $\quad 3 = 0 - 1 + C_1 \longrightarrow C_1 = 4$

$v(t) = \sin t - \cos t + 4$ (3)

$s(t) = -\cos t - \sin t + 4t + C_2$ (2)

$s(0) = -5$: $\quad -5 = -1 - 0 + 0 + C_2 \longrightarrow C_2 = -4$

$s(t) = -\cos t - \sin t + 4t - 4$ (3)

$\boxed{s(t) = -\cos t - \sin t + 4t - 4}$ 10