NAME GRADING KEY

STUDENT ID

RECITATION INSTRUCTOR

RECITATION TIME

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TOTAL	/100

DIRECTIONS

1. Write your name, 10–digit PUID, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes, calculators or any electronic devices may be used on this exam.

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(16) ~~(6)~~ 1. Find the derivative of the following functions. (It is not necessary to simplify).

(a) $y = \sin\sqrt{1+4x}$ NPC

$$\frac{dy}{dx} = \cos\sqrt{1+4x}\cdot\frac{1}{2\sqrt{1+4x}}\cdot 4$$

or

$$\boxed{\frac{dy}{dx} = 2\,\frac{\cos\sqrt{1+4x}}{\sqrt{1+4x}}}$$ 4

(b) $y = e^{e^{2x}}$

$$\frac{dy}{dx} = e^{e^{2x}}\cdot e^{2x}\cdot 2$$

or

$$\boxed{\frac{dy}{dx} = 2e^{2x}e^{e^{2x}}}$$ 4

(c) $y = \tan^2(3\theta)$

$$\frac{dy}{d\theta} = 2\tan(3\theta)\cdot\sec^2(3\theta)\cdot 3$$

or

$$\boxed{\frac{dy}{d\theta} = 6\tan(3\theta)\sec^2(3\theta)}$$ 4

(d) $F(y) = y\ln(1+e^y)$

$$F'(y) = y\frac{1}{1+e^y}e^y + \ln(1+e^y)$$

or

$$\boxed{F'(y) = \frac{ye^y}{1+e^y} + \ln(1+e^y)}$$ 4

(7) 2. If $F(x) = f(g(x))$, where $f(-2) = 8$, $f'(-2) = 4$, $f'(5) = 3$, $g(5) = -2$, and $g'(5) = 6$, find $F'(5)$.

$F'(x) = f'(g(x))\,g'(x)$ 3

$F'(5) = \underbrace{f'(g(5))\,g'(5)}_{2} = f'(-2)\cdot 6 = 4\cdot 6 = 24$ 2

$\boxed{24}$

$\boxed{7}$

(8) 3. Find the slope of the tangent line to the curve $\sin x = \cos y$ at the point $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$.

$\sin x = \cos y$

$\cos x = -\sin y \cdot \frac{dy}{dx}$ 3

$\frac{dy}{dx} = -\frac{\cos x}{\sin y}$

At $(x,y) = \left(\frac{\pi}{6}, \frac{\pi}{3}\right)$: $\frac{dy}{dx} = -\frac{\cos(\frac{\pi}{6})}{\sin(\frac{\pi}{3})}$ 3

$= -\frac{\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}} = -1$ 2

$\boxed{-1}$

$\boxed{8}$

(9) 4. Find the exact value of each expression:

NPC

(a) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$y = \sin^{-1}(\frac{\sqrt{3}}{2}) \iff \sin y = \frac{\sqrt{3}}{2},\ -\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$y = \frac{\pi}{3}$

$\boxed{\frac{\pi}{3}}$ 3

(b) $\cos^{-1}\left(-\frac{1}{2}\right)$

$y = \cos^{-1}(-\frac{1}{2}) \iff \cos y = -\frac{1}{2},\ 0 \le y \le \pi$

$y = \frac{2\pi}{3}$

$\boxed{\frac{2\pi}{3}}$ 3

(c) $\tan^{-1}\left(\tan \frac{7\pi}{6}\right)$

$y = \tan^{-1}(\tan \frac{7\pi}{6}) \iff \tan y = \tan \frac{7\pi}{6},\ -\frac{\pi}{2} < y < \frac{\pi}{2}$

$y = \frac{\pi}{6}$

$\boxed{\frac{\pi}{6}}$ 3

(6) 5. Find the second derivative of $y = x^3 \ln(4x)$.

$y = x^3 \ln(4x)$

$y' = x^3 \frac{1}{4x}\cdot 4 + 3x^2 \ln(4x)$

$= x^2 + 3x^2 \ln(4x)$ 3

$y'' = 2x + 3x^2 \frac{1}{4x}\cdot 4 + 6x\ln(4x)$

$= 5x + 6x\ln(4x)$ 3

Or

$y = x^3(\ln 4 + \ln x)$

$= x^3(\ln 4) + x^3 \ln x$

$y' = 3x^2(\ln 4) + x^3\frac{1}{x} + 3x^2 \ln x$

$= 3x^2(\ln 4) + x^2 + 3x^2 \ln x$ 3

$y'' = 6x\ln 4 + 2x + 3x^2\frac{1}{x} + 6x\ln x$ 3

or

$\boxed{y'' = 5x + 6x\ln(4x)}$

$\boxed{6}$

(12) 6. Find the derivatives of the following functions. (It is not necessary to simplify).

(a) $y = \sin^{-1}\sqrt{x}$

NPC

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(\sqrt{x})^2}} \cdot \frac{1}{2\sqrt{x}}$$

or

$$\boxed{\frac{dy}{dx} = \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}}}$$ 4

(b) $y = \tan^{-1}(\cos^2\theta)$

$$\frac{dy}{d\theta} = \frac{1}{1+(\cos^2\theta)^2} \cdot 2\cos\theta(-\sin\theta)$$

or

$$\boxed{\frac{dy}{d\theta} = -\frac{2\sin\theta\cos\theta}{1+\cos^4\theta}}$$ 4

(c) $y = (\tan x)^{\ln x}$, $0 < x < \frac{\pi}{2}$

$$y = (\tan x)^{\ln x} = \left(e^{\ln(\tan x)}\right)^{\ln x} = e^{(\ln x)\ln(\tan x)}$$

$$\frac{dy}{dx} = e^{(\ln x)\ln(\tan x)} \cdot \left[\ln x \frac{1}{\tan x}\sec^2 x + \frac{1}{x}\ln(\tan x)\right]$$

or

$$\boxed{\frac{dy}{dx} = (\tan x)^{\ln x}\left[\frac{(\ln x)\sec^2 x}{\tan x} + \frac{1}{x}\ln(\tan x)\right]}$$ 4

(8) 7. Use a linear approximation to estimate $(8.06)^{2/3}$

2 $f(x) \approx f(a) + f'(a)(x-a)$, for x near a

$f(x) = x^{2/3}$, $a = 8$, $f(8) = 4$, $f'(x) = \frac{2}{3}x^{-\frac{1}{3}}$, $f'(8) = \frac{2}{3}\cdot\frac{1}{2} = \frac{1}{3}$

3 $x^{2/3} \approx 4 + \frac{1}{3}(x-8)$, for x near 8

$(8.06)^{2/3} \approx 4 + \frac{1}{3}(8.06-8) = 4 + \frac{1}{3}(0.6) = 4 + 0.2 = 4.02$ 3

$\boxed{4.02}$ 8

(6) 8. Find the differential dy of each of the functions:

(a) $y = x\sin x$

$$dy = (x\cos x + \sin x)dx$$

−1 pt for missing dx

$\boxed{dy = (x\cos x + \sin x)dx}$ 3

(b) $y = \ln\sqrt{1+t^2}$

$$dy = \frac{1}{\sqrt{1+t^2}}\,\frac{1}{2\sqrt{1+t^2}}\,2t\,dt$$

or $y = \frac{1}{2}\ln(1+t^2)$

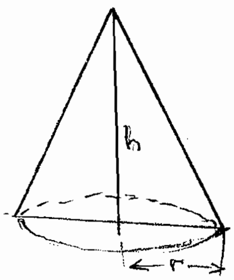
$$dy = \frac{1}{2}\,\frac{1}{1+t^2}\,2t\,dt$$

or

−1 pt for missing dt

$\boxed{dy = \frac{t}{1+t^2}\,dt}$ 3

(14) 9. Gravel is being dumped from a conveyor belt at a rate of 30 ft^3/min, and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 ft high?



Let V be the volume of the conical pile
h be its height and r be the radius of the base.
Given: $h = 2r$, $\frac{dV}{dt} = 30 \ ft^3/min$ (2)
Find $\frac{dh}{dt}$ when $h = 10 \ ft$

$V = \frac{1}{3}\pi r^2 h$ (2), $r = \frac{h}{2} \rightarrow V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$V = \frac{\pi}{12} h^3$ (3)

$\frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$ (3)

When $h = 10$: $30 = \frac{\pi}{4} 100 \frac{dh}{dt} \longrightarrow \frac{dh}{dt} = \frac{30}{25\pi} = \frac{6}{5\pi} \ ft/min$

(4)

The height of the pile is increasing at the rate of $\frac{6}{5\pi}$ ft/min

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(14) 10. A block of ice in the shape of a cube with initial volume 1000 cm^3 is melting in such a way that the length of each edge is decreasing at the rate of 1 cm/hr. Assuming that the block of ice maintains its cubical shape, at what rate is its surface area decreasing when the volume is 27 cm^3?

Let V be the volume of the ice cube,
S be its surface area and x be the length of each of its edges.
Given $\frac{dx}{dt} = -1 \ cm/hr$ (2)
Find $\frac{dS}{dt}$ when $V = 27 cm^3$

$V = x^3$

$S = 6x^2$ (3)

$\frac{dS}{dt} = 12x \frac{dx}{dt}$ (3)

When $V = 27$: $x = 3$ (2)

$\frac{dS}{dt} = (12) 3 (-1) = -36 \ cm^2/hr$

(4)

The surface area is decreasing at the rate of 36 cm^2/hr

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