NAME GRADING KEY

10-DIGIT PUID ______

RECITATION INSTRUCTOR ______

RECITATION TIME ______

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Page 2	/34
Page 3	/26
Page 4	/24
TOTAL	/100

DIRECTIONS

1. Write your name, 10-digit PUID, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes, calculators or any electronic devices may be used on this exam.

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(16) 1. Find the derivative of the following functions. (It is not necessary to simplify).

NPC

(a) $y = (1 + \cos^2 x)^6$

$\frac{dy}{dx} = 6(1+\cos^2 x)^5 2\cos x(-\sin x)$ or →

$\boxed{-12(1+\cos^2 x)^5 \cos x \sin x}$ 4

(b) $g(t) = \frac{1}{(t^4+1)^3} = (t^4+1)^{-3}$

$g'(t) = -3(t^4+1)^{-4} 4t^3$ or →

$\boxed{-12(t^4+1)^{-4} t^3}$ 4

(c) $y = \sin^2(3x)$

$\frac{dy}{dx} = 2\sin(3x)\cos(3x)\cdot 3$ or →

$\boxed{6\sin(3x)\cos(3x)}$ 4

(d) $y = \ln(x^4 \tan x)$

$\frac{dy}{dx} = \frac{1}{x^4\tan x}(x^4\sec^2 x + 4x^3\tan x)$ or →

$\boxed{\dfrac{x^4\sec^2 x + 4x^3\tan x}{x^4\tan x}}$ 4

Or $y = 4\ln x + \ln(\tan x)$

$\therefore \frac{dy}{dx} = \frac{4}{x} + \frac{1}{\tan x}\sec^2 x$ ← or

(8) 2. Find $\frac{dy}{dx}$ by implicit differentiation, if $x^2y + xy^2 = 3x$.

$\underbrace{x^2\frac{dy}{dx} + 2xy}_{2} + \underbrace{x2y\frac{dy}{dx} + y^2}_{2} = \underbrace{3}_{1}$

3 $\boxed{\frac{dy}{dx} = \frac{3-2xy-y^2}{x^2+2xy}}$ $\boxed{8}$

(8) 3. Find the second derivative of $y = xe^{2x}$.

$\frac{dy}{dx} = xe^{2x}2 + e^{2x}$ 4

$\frac{d^2y}{dx^2} = xe^{2x}\cdot 2\cdot 2 + e^{2x}2 + 2e^{2x}$

4 $\boxed{4xe^{2x} + 4e^{2x}}$ $\boxed{8}$

(12) 4. Find the exact value of each expression.

NPC

(a) $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = y \iff \sin y = -\frac{1}{\sqrt{2}}, -\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$y = -\frac{\pi}{4}$

$\boxed{-\frac{\pi}{4}}$ 3

(b) $\sec(\tan^{-1} 2) = \sec y$

Let $y = \tan^{-1} 2 \iff \tan y = 2, -\frac{\pi}{2} < y < \frac{\pi}{2}$

$\tan^2 y + 1 = \sec^2 y \to \sec y = \pm\sqrt{2^2+1} = \sqrt{5}$

+ because

$\boxed{\sqrt{5}}$ 3

(c) $\tan^{-1}\left(\tan\frac{5\pi}{6}\right) = y \iff \tan y = \tan\frac{5\pi}{6}, -\frac{\pi}{2} < y < \frac{\pi}{2}$

$\therefore y = -\frac{\pi}{6}$

$\boxed{-\frac{\pi}{6}}$ 3

(d) $\cos^{-1} 0 = y \iff \cos y = 0, 0 \le y \le \pi$

$y = \frac{\pi}{2}$

$\boxed{\frac{\pi}{2}}$ 3

(6) 5. Find the slope of the tangent line to the curve $y = \sinh x$ at the point $\left(\ln 2, \frac{3}{4}\right)$ and express your answer as the ratio of two integers.

$\frac{dy}{dx} = \cosh x = \frac{e^x + e^{-x}}{2}$ 2

$\left.\frac{dy}{dx}\right|_{x=\ln 2} = \frac{e^{\ln 2} + e^{-\ln 2}}{2} = \frac{2 + \frac{1}{2}}{2} = \frac{5}{4}$ 4

$\boxed{\frac{5}{4}}$ $\boxed{6}$

(12) 6. Find the derivatives of the following functions. (It is not necessary to simplify).

(a) $y = \tan^{-1}(x^2)$

NPC

$$\frac{dy}{dx} = \frac{1}{(x^2)^2+1} 2x$$

or

$$\boxed{\frac{2x}{x^4+1}}$$

4

(b) $f(x) = \sin^{-1}(2x+1)$

$$f'(x) = \frac{1}{\sqrt{1-(2x+1)^2}} 2$$

$$\boxed{\frac{2}{\sqrt{1-(2x+1)^2}}}$$

4

(c) $y = x^{\sqrt{x}} = e^{\ln x^{\sqrt{x}}} = e^{\sqrt{x}\ln x}$

$$\frac{dy}{dx} = e^{\sqrt{x}\ln x}\left(\sqrt{x}\frac{1}{x} + \frac{1}{2\sqrt{x}}\ln x\right)$$

$$= e^{\sqrt{x}\ln x}\left(\frac{1}{\sqrt{x}} + \frac{\ln x}{2\sqrt{x}}\right)$$

or

$$\boxed{x^{\sqrt{x}}\ \frac{2+\ln x}{2\sqrt{x}}}$$

4

(6) 7. Find the linearization $L(x)$ of the function $f(x) = x^5$ at $a = 1$.

$$L(x) = f(a) + f'(a)(x-a)$$

$$= f(1) + f'(1)(x-1)$$

or 2

$f(1) = 1$

$f'(x) = 5x^4$

$f'(1) = 5$

$$\boxed{L(x) = \underbrace{1}_{1} + \underbrace{5(x-1)}_{3}}$$

6

(8) 8. Find the differential dy if

(a) $y = \sqrt{1+x^2}$

$$dy = \frac{1}{2\sqrt{1+x^2}} 2x\, dx$$

−1 pt for missing dx

$$\boxed{dy = \frac{x}{\sqrt{1+x^2}}\, dx}$$

4

(b) $y = \sec(3x)$

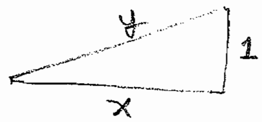
$$dy = \sec(3x)\tan(3x)\cdot 3\, dx$$

$$\boxed{dy = 3\sec(3x)\tan(3x)\, dx}$$

4

(12) 9. A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 1 m higher than the bow of the boat. If the rope is pulled in at a rate of 1 m/sec, how fast is the boat approaching the dock when it is 8 m from the dock.

Let x be the distance of the boat from the dock and y be the length of the rope



Given $\frac{dy}{dt} = -1$ m/sec (2)

Find $\frac{dx}{dt}$ when $x = 8$m

$x^2 + 1 = y^2$ (3)

$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$ (3)

When $x=8$: $y = \sqrt{65}$

$2\cdot 8\frac{dx}{dt} = 2\sqrt{65}(-1) \rightarrow \frac{dx}{dt} = -\frac{\sqrt{65}}{8}$ m/sec (4)

The boat is approaching the dock at the rate of $\frac{\sqrt{65}}{8}$ m/sec

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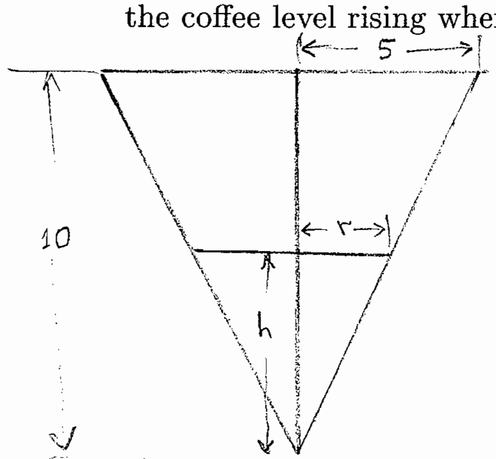
(12) 10. A coffee cup has the shape of an inverted circular cone with height 10 cm and radius at the top 5 cm. If coffee is poured into the cup at the rate of 2 cm^3/sec, how fast is the coffee level rising when the coffee is 5 cm deep?



Let V be the volume of the coffee in the cup and h be the depth of the coffee

Given $\frac{dV}{dt} = 2$ cm^3/sec (2)

Find $\frac{dh}{dt}$ when $h = 5$cm

$V = \frac{1}{3}\pi r^2 h$ (2), $\frac{r}{h} = \frac{5}{10} \rightarrow r = \frac{1}{2}h$ (2)

$V = \frac{\pi}{12}h^3$ (2)

$\frac{dV}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt}$

When $h=5$: $2 = \frac{\pi}{4}5^2\frac{dh}{dt} \longrightarrow \frac{dh}{dt} = \frac{8}{25\pi}$ cm/sec (4)

The coffee level is rising at the rate of $\frac{8}{25\pi}$ cm/sec

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