NAME Grading Key.

STUDENT ID ____________

RECITATION INSTRUCTOR ____________

RECITATION TIME ____________

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Page 4	/26
TOTAL	/100

DIRECTIONS

1. Write your name, student ID number, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes or calculators may be used on this exam.

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(12) 1. Find the following derivatives. It is not necessary to simplify.

(a) $f(t) = \dfrac{1}{(t^2 - 2t - 5)^4} = (t^2-2t-5)^{-4}$

NPC

4 pts

$f'(t) = \dfrac{-8(t-1)}{(t^2-2t-5)^5}$ or $f'(t) = \boxed{-4(t^2-2t-5)^{-5}(2t-2)}$

(b) $F(x) = \tan^{-1}(e^x)$

NPC

4 pts

$F'(t) = \dfrac{e^x}{1+e^{2x}}$ or $F'(x) = \boxed{\dfrac{e^x}{1+(e^x)^2}}$

(c) $H(x) = \sqrt{1+\cos(2x)} = (1+\cos(2x))^{1/2}$

NPC

4 pts

$H'(x) = \dfrac{-\sin 2x}{\sqrt{1+\cos 2x}}$ or $H'(x) = \boxed{\frac{1}{2}(1+\cos 2x)^{-1/2}(-2\sin 2x)}$

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(8) 2. Find an equation for the tangent line to the graph of $y = \sin x + \cos 2x$ at $(\frac{\pi}{6}, 1)$.

3 pts $y' = \cos x - 2\sin 2x$

3 pts $y'(\frac{\pi}{6}) = \frac{\sqrt{3}}{2} - 2\frac{\sqrt{3}}{2}$

$= -\frac{\sqrt{3}}{2}$

2 pts if consistent

$\boxed{y - 1 = -\frac{\sqrt{3}}{2}(x - \frac{\pi}{6})}$

(9) 3. Find the value of each of the following inverse trigonometric functions.

NPC

(a) $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = y \iff \tan y = -\frac{1}{\sqrt{3}},\ -\frac{\pi}{2} < y < \frac{\pi}{2}$

$\therefore y = -\frac{\pi}{6}$

$\boxed{-\frac{\pi}{6}}$ 3 pts

(b) $\cos^{-1}\left(-\frac{1}{2}\right) = y \iff \cos y = -\frac{1}{2},\ 0 \le y \le \pi$

$y = \frac{2\pi}{3}$

$\boxed{\frac{2\pi}{3}}$ 3 pts

(c) $\sin^{-1}\left(\sin\left(\frac{3\pi}{2}\right)\right) = \sin^{-1}(-1) = y,\ -\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$\iff \sin y = -1$

$y = -\frac{\pi}{2}$

$\boxed{-\frac{\pi}{2}}$ 3 pts

(9) 4. Find $\frac{dy}{dx}$ by implicit differentiation if $\sqrt{xy} = 1 + x^2y$.

3 pts $\frac{1}{2}(xy)^{-1/2}\left(x\frac{dy}{dx} + y\right) = x^2\frac{dy}{dx} + 2xy$ 3 pts

$\frac{dy}{dx}\left(\frac{x}{2}(xy)^{-1/2} - x^2\right) = 2xy - \frac{y}{2}(xy)^{-1/2}$

$\frac{dy}{dx} = \dfrac{2xy - \frac{y}{2}(xy)^{-1/2}}{\frac{x}{2}(xy)^{-1/2} - x^2}$ or $\frac{dy}{dx} = \boxed{\dfrac{4(xy)^{3/2} - y}{x - 2x^2\sqrt{xy}}}$ 3 pts

(6) 5. Find the second derivative of the function $H(t) = \tan 3t$.

2 pts $H'(t) = 3\sec^2 3t$

$H''(t) = 6\sec 3t \cdot \sec 3t \tan 3t \cdot 3$

4 pts

or $H''(t) = \boxed{18\sec^2 3t \tan 3t}$

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(9) 6. The position of a particle at time t is given by $s = 2t^3 - 6t^2 + 4t + 1$. Find the velocity of the particle at the instant when the acceleration is zero.

2 pts $v(t) = 6t^2 - 12t + 4$.

2 pts $a(t) = 12t - 12$, 3 pts $12t - 12 = 0 \Rightarrow t = 1$

$v(1) = -2$

2 pts

velocity = $\boxed{-2}$

(8) 7. If $f(x) = x^2 \ln x$, find $f''(e)$.

$f'(x) = x^2 \cdot \frac{1}{x} + 2x \ln x$

3 pts $= x + 2x \ln x$

$f''(x) = 1 + 2x \cdot \frac{1}{x} + 2 \ln x$

3 pts $= 3 + 2 \ln x$

2 pts

$f''(e) = \boxed{5}$

(5) 8. Find the differential of $y = \sin(e^x)$.

$dy = e^x \cos(e^x)\, dx$

Take off 1 pt if "dx" is omitted

5 pts

$dy = \boxed{e^x \cos(e^x)\, dx}$

(8) 9. Use a linear approximation to estimate the number $\sqrt{36.1}$.

2 pts $f(x) = \sqrt{x}$, $f'(x) = \frac{1}{2\sqrt{x}}$ 2 pts

$\sqrt{x} \approx \sqrt{36} + \frac{1}{2\sqrt{36}}(x - 36)$ 2 pts (correct formula)

$\sqrt{36.1} \approx 6 + \frac{1}{12}(0.1)$

$\approx 6 + \frac{1}{120}$ or

2 pts

$\sqrt{36.1} \approx \boxed{\frac{721}{120}}$

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(8) 10. Find the derivative of $y = x^{\ln x}$. It is not necessary to simplify.

3 pts $y = e^{(\ln x)^2}$

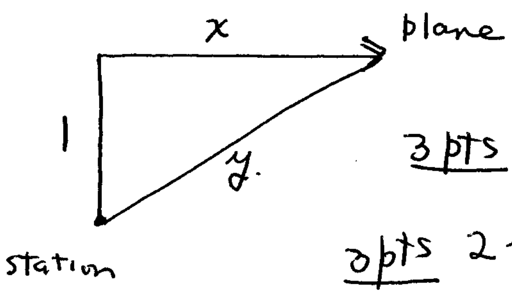
$$\frac{dy}{dx} = e^{(\ln x)^2}\left(2 \ln x \cdot \frac{1}{x}\right).$$

OR

5 pts

$$\frac{dy}{dx} = \boxed{x^{\ln x}\left(\frac{2\ln x}{x}\right)}$$

(9) 11. A plane flying horizontally at an altitude of 1 mi and a speed of 500 mi/h passes directly over a radar station. Find the rate at which the direct distance from the plane to the station is increasing when this distance is 2 mi.



Given $\frac{dx}{dt} = 500$, Find $\frac{dy}{dt}$ when $y = 2$.

3 pts $y^2 = x^2 + 1$ $\therefore$ when $y = 2$, $x = \sqrt{3}$.

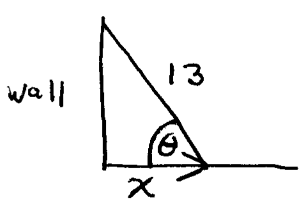
3 pts $2y \frac{dy}{dt} = 2x \frac{dx}{dt}$

$$2 \frac{dy}{dt} = \sqrt{3} \cdot 500$$

3 pts

rate = $\boxed{250\sqrt{3}}$ $\frac{mi}{h}$.

(9) 12. A 13 ft. ladder is leaning against a vertical wall when its base starts to slide away. When the base is 12 ft. from the wall, it is moving at a rate of 5 ft/s. At what rate is the angle θ between the ladder and the ground changing at this time?



Find $\frac{d\theta}{dt}$ when $x = 12$ + $\frac{dx}{dt} = 5$.

$\cos\theta = \frac{12}{13} \Rightarrow \sin\theta = \frac{5}{13}$

3 pts $\cos\theta = \frac{x}{13}$

3 pts $-\sin\theta \frac{d\theta}{dt} = \frac{1}{13}\frac{dx}{dt}$

$$-\frac{5}{13}\frac{d\theta}{dt} = \frac{1}{13} \cdot 5$$

$$\frac{d\theta}{dt} = -1$$

3 pts

$\frac{d\theta}{dt} = \boxed{-1}$ $\frac{rad}{sec}$.