NAME GRADING KEY

STUDENT ID

RECITATION INSTRUCTOR

RECITATION TIME

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DIRECTIONS

1. Write your name, 10–digit PUID, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes, calculators or any electronic devices may be used on this exam.

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(6) 1. Find the domain of the function $h(x) = \dfrac{1}{\sqrt[4]{x^2 - 5x}}$.

$x^2 - 5x > 0$ (2)

$x(x-5) > 0$

$x(x-5)$: $+$ at 0 (0) $-$ at 5 (0) $+$, x

$-\infty < x < 0$ (2) , $5 < x < \infty$ (2) ← or → $\boxed{(-\infty, 0) \cup (5, \infty)}$

$\boxed{6}$

(8) 2. If $f(x) = 1 - 3x$ and $g(x) = \cos x$, find the following

2pts each NPC

$(f \circ g)(x) = f(g(x)) = f(\cos x) = 1 - 3\cos x$

$(f \circ g)(x) = 1 - 3\cos x$

$(g \circ f)(x) = g(f(x)) = g(1-3x) = \cos(1-3x)$

$(g \circ f)(x) = \cos(1-3x)$

$(f \circ f)(x) = f(f(x)) = f(1-3x) = 1 - 3(1-3x)$

$(f \circ f)(x) = 1 - 3(1-3x)$

$(g \circ g)(x) = g(g(x)) = g(\cos x) = \cos(\cos x)$

$(g \circ g)(x) = \cos(\cos x)$

$\boxed{8}$

(6) 3. Find all values of x in the interval $[0, 2\pi]$ that satisfy the equation $\cos x + \sin 2x = 0$.

$\cos x + 2\sin x\cos x = 0$ ②

$\cos x(1+2\sin x) = 0 \longrightarrow \cos x = 0$ or $1+2\sin x = 0$

$\cos x = 0 \rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\sin x = -\frac{1}{2} \longrightarrow x = \frac{7\pi}{6}, \frac{11\pi}{6}$

① ① ① ①

$\boxed{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}}$ $\boxed{6}$

(6) 4. If $f(x) = \ln(x+3)$ find a formula for the inverse function $f^{-1}(x)$.

$y = f(x) \Longleftrightarrow x = f^{-1}(y)$

$y = \ln(x+3)$

$e^y = x+3$

$x = e^y - 3$

$\therefore f^{-1}(y) = e^y - 3 \rightarrow f^{-1}(x) = e^x - 3$

-3 pts for $f^{-1}(x) = e^y - 3$

$f^{-1}(x) =$ $e^x - 3$ $\boxed{6}$

(4) 5. Solve each equation for x.

2pts each NPC

(a) $2\ln x = 1$ $\quad \ln x = \frac{1}{2} \rightarrow x = e^{1/2} = \sqrt{e}$

$x =$ $\sqrt{e}$

(b) $e^{-x} = 5$ $\quad -x = \ln 5 \rightarrow x = -\ln 5$

$x =$ $-\ln 5$ $\boxed{4}$

(6) 6. If $f(x) = \begin{cases} \dfrac{x^2 - x}{x^2 - 1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}$ explain why f is discontinuous at $a = 1$.

$\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{x^2 - x}{x^2 - 1} = \lim_{x\to 1} \frac{x(x-1)}{(x+1)(x-1)} = \lim_{x\to 1} \frac{x}{x+1} = \frac{1}{2}$ ③

$f(1) = 1$. f is discontinuous at $a=1$ because $\lim_{x\to 1} f(x) \neq f(1)$ ③ $\boxed{6}$

(6) 7. Find the exact numerical value of the following:

2pts each NPC

(a) $\log_{49} e^{4\ln 7} = \log_{49} e^{\ln 7^4} = \log_{49} (49)^2 = 2$

2

(b) $\ln(\log_2 2008 - \log_2 1004) = \ln \log_2 \frac{2008}{1004}$

$= \ln \log_2 2 = \ln 1 = 0$

0

(c) $\sin(\ln\sqrt{e^\pi}) = \sin \ln (e^\pi)^{1/2} = \sin(\frac{1}{2}\ln e^\pi) = \sin\frac{\pi}{2} = 1$

1 $\boxed{6}$

(10) 8. For each of the following, fill in the boxes below with a finite number or one of the symbols $+\infty$, $-\infty$, or DNE (does not exist). It is not necessary to give reasons for your answers.

2pts each NPC

(a) $\lim_{x\to\pi^-} \cot x = \lim_{x\to\pi^-} \frac{\cos x}{\sin x} = -\infty$

$\boxed{-\infty}$

(b) $\lim_{x\to 0^-} \frac{x}{|\sin x|} = \lim_{x\to 0^-} \frac{x}{-\sin x} = -\lim_{x\to 0^-} \frac{x}{\sin x} = -1$

$\boxed{-1}$

(c) $\lim_{r\to 9} \frac{\sqrt{r}}{(r-9)^2} = \infty$

$\boxed{\infty}$

(d) $\lim_{x\to\infty} \cos x =$ DNE

$\boxed{\text{DNE}}$

(e) $\lim_{x\to 3^+} \frac{2|x-3|}{x-3} = \lim_{x\to 3^+} \frac{2(x-3)}{x-3} = 2$

$\boxed{2}$

$\boxed{10}$

(6) 9. Find the equations of the vertical and horizontal asympotes of the graph of $y = \frac{5x^2-2x+1}{x^2-x-2}$.

$f(x) = \frac{5x^2-2x+1}{(x+1)(x-2)}$

$\lim_{x\to(-1)^+} f(x) = -\infty \quad \therefore x=-1$ is V.A.

$\lim_{x\to 2^+} f(x) = \infty \quad \therefore x=2$ is V.A.

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{5x^2-2x+1}{x^2-x-2} = \lim_{x\to\infty} \frac{5-\frac{2}{x}+\frac{1}{x^2}}{1-\frac{1}{x}-\frac{2}{x^2}} = 5$

$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{5x^2-2x+1}{x^2-x-2} = \lim_{x\to-\infty} \frac{5-\frac{2}{x}+\frac{1}{x^2}}{1-\frac{1}{x}-\frac{2}{x^2}} = 5$

Vertical asymptotes
$x = -1$ (2), $x = 2$ (2)
Horizontal asymptotes
$y = 5$ (2)

$\boxed{6}$

(8) 10. Show that there is a root of the equation $x^2 - x - 1 = \frac{1}{x+1}$ in the interval $(1, 2)$. State the name of the theorem you are using.

$x^2 - x - 1 - \frac{1}{x+1} = 0$

$f(x) = x^2 - x - 1 - \frac{1}{x+1}$ (1) $\quad f$ is continuous on $[1, 2]$

$f(1) = 1 - 1 - 1 - \frac{1}{2} = -\frac{3}{2} < 0$ (2)

$f(2) = 4 - 2 - 1 - \frac{1}{3} = \frac{2}{3} > 0$ (2) $\quad f(1) < 0 < f(2)$

$\therefore f(c) = 0$ for some $c \in (1, 2)$

by the Intermediate Value Theorem (3)

(10) 11. Find the derivative of the function $g(t) = \sqrt{t}$ using the definition of the derivative $g'(t) = \lim_{h\to 0} \frac{g(t+h)-g(t)}{h}$. (0 credit for using a formula for the derivative).

$$g'(t) = \lim_{h\to 0} \frac{g(t+h)-g(t)}{h} = \lim_{h\to 0} \frac{\sqrt{t+h}-\sqrt{t}}{h} \quad 4$$

$$= \lim_{h\to 0} \frac{\sqrt{t+h}-\sqrt{t}}{h} \cdot \frac{\sqrt{t+h}+\sqrt{t}}{\sqrt{t+h}+\sqrt{t}}$$

−1pt for early omission of $\lim_{h\to 0}$

$$= \lim_{h\to 0} \frac{t+h-t}{h(\sqrt{t+h}+\sqrt{t})}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{t+h}+\sqrt{t}} \quad 4 \quad = \frac{1}{2\sqrt{t}} \quad 2$$

$$\boxed{\frac{1}{2\sqrt{t}}}$$

$\boxed{10}$

(8) 12. Find an equation of the tangent line to the curve $y = x\sqrt{x}$ that is parallel to the line $y = 1 + 3x$

The slope of the line $y = 1+3x$ is 3 (2)

curve: $y = x^{3/2}$

(2) $\frac{dy}{dx} = \frac{3}{2}x^{1/2}$

$\frac{3}{2}x^{1/2} = 3 \to \sqrt{x} = 2 \to x = 4$ (1)

$x = 4 \to y = 4\sqrt{4} = 8$ (1)

(2) $\boxed{y-8 = 3(x-4)}$

$\boxed{8}$

(16) 13. Find the derivatives of the following functions. Do not simplify. 4 pts each NPC

(a) $g(t) = 4\sec t + \tan t$.

$\boxed{g'(t) = 4\sec t \tan t + \sec^2 t}$ $\boxed{4}$

(b) $y = e^x(1 + \cot x)$.

$\boxed{\frac{dy}{dx} = e^x(-\csc^2 x) + (1+\cot x)e^x}$ $\boxed{4}$

(c) $f(x) = \dfrac{xe^x}{\sin x}$.

$\boxed{f'(x) = \frac{\sin x(xe^x+e^x) - xe^x \cos x}{\sin^2 x}}$ $\boxed{4}$

(d) $u = \sqrt[5]{t} + 4\sqrt{t^5}$.

$= t^{1/5} + 4t^{5/2}$

$\boxed{\frac{du}{dt} = \frac{1}{5}t^{-\frac{4}{5}} + 10t^{3/2}}$ $\boxed{4}$