NAME GRADING KEY

STUDENT ID ______

RECITATION INSTRUCTOR ______

RECITATION TIME ______

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DIRECTIONS

1. Write your name, 10–digit PUID, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes, calculators or any electronic devices may be used on this exam.

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(6) 1. Find all values of x in the interval $[0, 2\pi]$ that satisfy the equation $\sec x = 2\sin x$.

$\sec x = 2\sin x \longrightarrow \frac{1}{\cos x} = 2\sin x \qquad 1 = 2\sin x\cos x$ (2)

$\sin 2x = 1$ (2)

$2x = \frac{\pi}{2}, \frac{5\pi}{2}$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

(1) (1)

$\boxed{\frac{\pi}{4}, \frac{5\pi}{4}}$

6

(6) 2. If $f(x) = \sqrt{x}$ and $g(x) = \dfrac{1}{x-1}$, find the functions $f \circ g$ and $g \circ f$ and their domains.

$(f\circ g)(x) = f(g(x)) = \sqrt{\frac{1}{x-1}} = \frac{1}{\sqrt{x-1}}$ $\qquad x-1>0$

or

$(g\circ f)(x) = g(f(x)) = \frac{1}{\sqrt{x}-1}$

$x \geq 0, \; x \neq 1$

$(f \circ g)(x) = \frac{1}{\sqrt{x-1}}$ (2)

domain : $(1, \infty)$ (1)

$(g \circ f)(x) = \frac{1}{\sqrt{x}-1}$ (2)

domain : $[0,1) \cup (1,\infty)$ (1)

6

(6) 3. Find a formula for the inverse of $f(x) = 2x^3 + 3$.

-3pts for $f^{-1}(x) = \sqrt[3]{\frac{y-3}{2}}$

$x = f^{-1}(y) \iff y = f(x)$

$y = 2x^3 + 3$

$x = \sqrt[3]{\frac{y-3}{2}} \quad \therefore f^{-1}(y) = \sqrt[3]{\frac{y-3}{2}}$

$f^{-1}(x) = \sqrt[3]{\frac{x-3}{2}}$ 6

(4) 4. Solve the equation $e^{2x+3} - 7 = 0$ for x.

$e^{2x+3} - 7 = 0$

$e^{2x+3} = 7 \longrightarrow \ln e^{2x+3} = \ln 7$

$2x + 3 = \ln 7$

$x = \frac{\ln 7 - 3}{2}$ 4

(6) 5. If $f(x) = \begin{cases} cx^2 & \text{if } x \le 2 \\ c - x & \text{if } x > 2 \end{cases}$, find the value of the constant c for which $\lim_{x\to 2} f(x)$ exists.

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} cx^2 = c4$

$4c = c - 2$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (c - x) = c - 2$

$c = -\frac{2}{3}$ 6

(6) 6. Find the equations of the vertical and horizontal asymptotes of the graph of $y = \frac{x^2+4}{x^2-1}$.

$\lim_{x\to 1^+} \frac{x^2+4}{x^2-1} = +\infty \quad \therefore x = 1$ is V.A.

$\lim_{x\to (-1)^+} \frac{x^2+4}{x^2-1} = -\infty \quad \therefore x = -1$ is V.A.

$\lim_{x\to\infty} \frac{x^2+4}{x^2-1} = 1 \quad \lim_{x\to -\infty} \frac{x^2+4}{x^2-1} = 1$

$\therefore y = 1$ is H.A.

Vertical asymptotes	Horizontal asymptotes
$x = 1$, $x = -1$ (2) (2)	$y = 1$ (2)

6

(8) 7. Find the exact numerical value of the following:

2pts each

(a) $e^{2\ln 3} = e^{\ln 3^2} = 3^2$

9

(b) $\log_{10} 25 + \log_{10} 4 = \log_{10}[(25)\cdot 4] =$

$= \log_{10} 100 = 2$

2

(c) $\tan(\pi e^{-\ln 4}) = \tan\left(\pi \frac{1}{e^{\ln 4}}\right) = \tan\frac{\pi}{4} = 1$

1

(d) $\cos(\ln 1) = \cos 0 = 1$

1

8

(15) 8. For each of the following, fill in the boxes below with a finite number, or one of the symbols $+\infty$, $-\infty$, or DNE (does not exist). It is not necessary to give reasons for your answers.

3pts each
NPC

(a) $\lim\limits_{h\to 0} \dfrac{(2+h)^3-8}{h} = \lim\limits_{h\to 0} \dfrac{2^3+3\cdot 2^2 h+3\cdot 2h^2+h^3-8}{h}$

$= \lim\limits_{h\to 0} (12+6h+h^2) = 12$

$\boxed{12}$ 3

(b) $\lim\limits_{t\to 0} \left(\dfrac{1}{t} - \dfrac{1}{t^2+t}\right) = \lim\limits_{t\to 0} \dfrac{t^2+t-t}{t(t^2+t)}$

$= \lim\limits_{t\to 0} \dfrac{1}{t+1} = 1$

$\boxed{1}$ 3

(c) $\lim\limits_{x\to(-4)^-} \dfrac{|x+4|}{x+4} =$

$x<-4 \to x+4<0 \to |x+4| = -(x+4)$

$\therefore \lim\limits_{x\to(-4)^-} \dfrac{|x+4|}{x+4} = \lim\limits_{x\to(-4)^-} \dfrac{-(x+4)}{x+4} = -1$

$\boxed{-1}$ 3

(d) $\lim\limits_{x\to 0} x^2 \sin\dfrac{\pi}{x} =$ $\quad -1 \le \sin\frac{\pi}{x} \le 1$

$-x^2 \le x^2 \sin\frac{\pi}{x} \le x^2$

as $x\to 0$: $-x^2 \to 0$, $x^2 \to 0$ $\quad \therefore$ $x^2\sin\frac{\pi}{x} \to 0$ (Squeeze Theorem)

$\boxed{0}$ 3

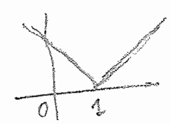
(e) $\lim\limits_{x\to 0} \dfrac{x-1}{x^2(x+3)} = -\infty$

$\boxed{-\infty}$ 3

$\boxed{15}$

(4) 9. True or False. (Circle T or F)

(a) The function $f(x) = |x-1|$ is continuous at $x=1$. **T** F



1pt each

(b) The function $f(x) = |x|$ is differentiable at $x=0$. T **F**

(c) The function $f(x) = |x|$ is differentiable at $x=-1$. **T** F

(d) The function $g(x) = \ln(x-1)$ is continuous at $x=0$. T **F**

g not defined at $x=0$

$\boxed{4}$

(6) 10. Find an equation of the tangent line to the curve $y = \dfrac{2x}{x+1}$ at the point $(1,1)$.

$\dfrac{dy}{dx} = \dfrac{(x+1)2-2x\cdot 1}{(x+1)^2} = \dfrac{2}{(x+1)^2}$ 2

$\left.\dfrac{dy}{dx}\right|_{x=1} = \dfrac{2}{2^2} = \dfrac{1}{2}$ 2

$\boxed{y-1 = \frac{1}{2}(x-1) \text{ or } y=\frac{1}{2}x+\frac{1}{2}}$ 2

$\boxed{6}$

(11) 11. Find the derivative of the function $f(x) = \dfrac{1}{x^2}$ using the definition of the derivative $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$. (0 credit for using a formula for the derivative).

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} \quad 4$$

$$= \lim_{h\to 0} \frac{\frac{x^2-(x^2+2xh+h^2)}{(x+h)^2x^2}}{h} = \lim_{h\to 0} \frac{-2xh-h^2}{h(x+h)^2x^2}$$

$$= \lim_{h\to 0} \frac{-2x-h}{(x+h)^2x^2} \quad 4 \quad = -\frac{2x}{x^4} \quad 3 \quad = -\frac{2}{x^3}$$

$\boxed{11}$

(6) 12. For what values of x is the tangent line to the curve $y = 3x^2 - 1$ parallel to the line $x - 2y = -2$.

slope of line $x-2y=-2$ is $\frac{1}{2}$ 2

$\frac{dy}{dx} = 6x \qquad 6x = \frac{1}{2}$ 3

$x = \frac{1}{12}$ 1

$\boxed{x = \frac{1}{12}}$

$\boxed{6}$

(16) 13. Find the derivatives of the following functions. (It is not necessary to simplify).

4pts each. NPC

(a) $v = t^2 - \dfrac{1}{4\sqrt{t^3}}$.

$= t^2 - \frac{1}{4}t^{-3/2}$

$\boxed{\frac{dv}{dt} = 2t + \frac{3}{8}t^{-5/2}}$

$\boxed{4}$

(b) $y = (1 - e^x)\tan x$.

$\boxed{\frac{dy}{dx} = (1-e^x)\sec^2 x - e^x \tan x}$

$\boxed{4}$

(c) $f(x) = x^\pi + e^2$.

$\boxed{f'(x) = \pi x^{\pi - 1}}$

$\boxed{4}$

(d) $y = \dfrac{1 + \sin x}{x + \cos x}$.

$\boxed{\frac{dy}{dx} = \frac{(x+\cos x)\cos x - (1+\sin x)(1-\sin x)}{(x+\cos x)^2}}$

$\boxed{4}$