NAME GRADING KEY

10–digit PUID ______

RECITATION INSTRUCTOR ______

RECITATION TIME ______

Page 1	/14
Page 2	/27
Page 3	/27
Page 4	/32
TOTAL	/100

DIRECTIONS

1. Write your name, 10–digit PUID, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes or calculators may be used on this exam.

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(6) 1. Circle the correct choice. $\tan^2 x - \tan^2 x \sin^2 x =$

$$\tan^2 x - \tan^2 x \sin^2 x$$
$$= \tan^2 x\,(1 - \sin^2 x)$$
$$= \left(\frac{\sin^2 x}{\cos^2 x}\right)(\cos^2 x)$$
$$= \sin^2 x$$

A. $\tan^2 x$

6 B. $\sin^2 x$

C. $\cos^2 x$

D. 1

E. $\sec^2 x$

$\boxed{6}$

(8) 2. If $f(x) = 1 - x^3$ and $g(x) = \dfrac{1}{x}$ find the composite functions $f \circ g$ and $g \circ f$ and give their domains.

$$(f \circ g)(x) = f(g(x)) = f\left(\tfrac{1}{x}\right) = 1 - \left(\tfrac{1}{x}\right)^3 = \frac{x^3 - 1}{x^3}$$

3 $(f \circ g)(x) = 1 - \dfrac{1}{x^3}$

1 Domain of $(f \circ g)$ $x \neq 0$

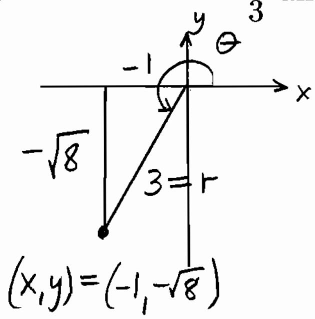
$$(g \circ f)(x) = g(f(x)) = g(1 - x^3) = \frac{1}{1 - x^3}$$

3 $(g \circ f)(x) = \dfrac{1}{1 - x^3}$

1 Domain of $(g \circ f)$ $x \neq 1$

$\boxed{8}$

(5) 3. If $\cos\theta = -\frac{1}{3}$ and $\pi < \theta < \frac{3\pi}{2}$, find the following:



$\sin\theta = \frac{y}{r} = \frac{-\sqrt{8}}{3}$

$\tan\theta = \frac{y}{x} = \frac{-\sqrt{8}}{-1}$

$\sec\theta = \frac{1}{\cos\theta} = -3$

$\csc\theta = \frac{1}{\sin\theta} = -\frac{3}{\sqrt{8}}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{-1}{-\sqrt{8}}$

$\sin\theta = -\frac{\sqrt{8}}{3}$	1 pt each
$\tan\theta = \sqrt{8}$	NPC
$\sec\theta = -3$	
$\csc\theta = -\frac{3}{\sqrt{8}}$	
$\cot\theta = \frac{1}{\sqrt{8}}$	

5

(8) 4. Find a formula for the inverse of $f(x) = e^{x^3}$.

$x = f^{-1}(y) \Longleftrightarrow y = f(x)$ 2

$y = f(x) = e^{x^3} \longrightarrow \ln y = x^3$

$\longrightarrow x = \sqrt[3]{\ln y} \rightarrow f^{-1}(y) = \sqrt[3]{\ln y} \rightarrow f^{-1}(x) = \sqrt[3]{\ln x}$ 4

$\boxed{f^{-1}(x) = \sqrt[3]{\ln x}}$ 2

8

(9) 5. Find the equations of the vertical and horizontal asymptotes of the function $y = \frac{x+2}{x+5}$.

$\lim_{x \to \pm\infty} \frac{x+2}{x+5} = 1$, $\therefore$ $y = 1$ is hor. asymptote

$\lim_{x \to (-5)^-} \frac{x+2}{x+5} = \infty$

or $\lim_{x \to (-5)^+} \frac{x+2}{x+5} = -\infty$

$\therefore x = -5$ is a vertical asymptote.

Vertical asymptotes	Horizontal asymptotes
$x = -5$	$y = 1$

5 for one correct. 9 for both correct.

9

(5) 6. (a) Complete the definition: The function f is continuous at a if $\boxed{\lim_{x \to a} f(x) = f(a)}$ 2

(b) Let $f(x) = \frac{\sqrt{x+9} - 3}{x}$. Use part (a) to find the value of $f(0)$ so that f is continuous at $x = 0$. $\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{\sqrt{x+9} - 3}{x}$

$= \lim_{x \to 0} \left(\frac{\sqrt{x+9} - 3}{x}\right)\left(\frac{\sqrt{x+9} + 3}{\sqrt{x+9} + 3}\right)$

$\boxed{f(0) = \frac{1}{6}}$ 3

$= \lim_{x \to 0} \frac{\cancel{(x+9)} - \cancel{9}}{\cancel{x}(\sqrt{x+9} + 3)} = \lim_{x \to 0} \frac{1}{\sqrt{x+9} + 3} = \frac{1}{3+3} = \frac{1}{6}$

5

(18) 7. For each of the following, fill in the boxes below with a finite number, or one of the symbols $+\infty$, $-\infty$, or DNE (does not exist). It is not necessary to give reasons for your answers.

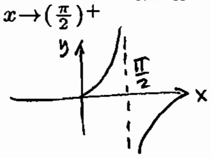
3 pts each. NPC

(a) $\lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{t^2+t}\right) = \lim_{t\to 0}\frac{t^2+t-t}{t(t^2+t)}$

$= \lim_{t\to 0}\frac{1}{t+1} = \frac{1}{0+1} = 1$

$\boxed{1}$

(b) $\lim_{x\to(\frac{\pi}{2})^+}\tan x = -\infty$



$\boxed{-\infty}$

(c) $\lim_{x\to(-4)^-}\frac{|x+4|}{x+4} = \lim_{x\to(-4)^-}\frac{-(x+4)}{x+4}$

$= \lim_{x\to(-4)^-} -1 = -1$

$\boxed{-1}$

(d) $\lim_{x\to\infty}\frac{3x^2-x+4}{2x^2+5x-8} = \lim_{x\to\infty}\frac{3-\frac{1}{x}+\frac{4}{x^2}}{2+\frac{5}{x}-\frac{8}{x^2}}$

$= \frac{3-0+0}{2+0-0} = \frac{3}{2}$

$\boxed{\frac{3}{2}}$

(e) $\lim_{x\to 0}\frac{\sin 3x}{x} = \lim_{x\to 0} 3\cdot\frac{\sin 3x}{3x} = 3\cdot 1 = 3$

$\boxed{3}$

(f) $\lim_{x\to 0}\sin\left(\frac{1}{x}\right) =$ does not exist

$\boxed{\text{DNE}}$

(9) 8. Let $f(x) = x^3 - x^2 + x - 2$.

(a) The number c, such that $f(c) = 3$, is in the interval (circle one)

$f(-1) = -5$ $f(0) = -2$ $f(1) = -1$ $f(2) = 4$ $f(3) = 19$

$\underbrace{f(1) < 3 < f(2)}_{1}$ and $\underbrace{f \text{ continuous}}_{1}$ $\rightarrow \begin{cases} 1 < c < 2 \\ \text{and } f(c) = 3 \end{cases}$

A. $(-1,0)$ B. $(0,1)$ 4 C. $(1,2)$ D. $(2,3)$

(b) State the name of the theorem you are using in part (a).

Intermediate Value theorem. 3

or I.V.T.

(10) 9. Find the derivative of the function $f(x) = \dfrac{1}{x^2}$ using the definition of the derivative: $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$. (0 credit for using a formula for the derivative).

$$f'(x) = \lim_{h\to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} \quad (4) \quad = \lim_{h\to 0} \frac{x^2 - (x+h)^2}{h(x+h)^2 x^2}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2} - (\cancel{x^2} + 2xh + h^2)}{h(x+h)^2 x^2} = \lim_{h\to 0} \frac{\cancel{h}(-2x - h)}{\cancel{h}(x+h)^2 x^2} \quad (3)$$

$$= \frac{-2x - 0}{(x+0)^2 x^2} = \frac{-2}{x^3} \quad (3)$$

$\boxed{10}$

(6) 10. Prove that $\lim_{x\to 0} x^4 \cos\left(\dfrac{2}{x}\right) = 0$. State the name of the theorem that you use in the proof.

$-1 \le \cos\left(\frac{2}{x}\right) \le 1, \; x \ne 0$ (1)

$-x^4 \le x^4 \cos\left(\frac{2}{x}\right) \le x^4, \; x \ne 0$ (1)

$\lim_{x\to 0} -x^4 = \lim_{x\to 0} x^4 = 0$ (2)

Therefore $\lim_{x\to 0} x^4 \cos\left(\frac{1}{x}\right) = 0$ by the Squeeze Thm. (2)

$\boxed{6}$

(4) 11. For what value(s) of x does the graph of $f(x) = 2x^3 + 3x^2 - 36x + 5$ have a horizontal tangent?

$f'(x) = 6x^2 + 6x - 36$ (1)

$f'(x) = 6x^2 + 6x - 36 = 6(x+3)(x-2) = 0$ (2)

$\longrightarrow x = -3, 2$ (1)

$\boxed{x = -3, 2}$

$\boxed{4}$

(8) 12. Find the derivatives of the following functions. (It is not necessary to simplify).

(a) $y = (\tan x)(x^2 + e^x)$.

NPC (4)

$\boxed{(\tan x)(2x + e^x) + (x^2 + e^x)(\sec^2 x)}$

(b) $f(x) = \dfrac{e^x - \cos x}{x^3 + \sin x}$.

NPC (4)

$$\boxed{\frac{(x^3 + \sin x)(e^x + \sin x) - (e^x - \cos x)(3x^2 + \cos x)}{(x^3 + \sin x)^2}}$$

$\boxed{8}$

(4) 13. Find an equation of the tangent line to the curve $y = x^3 + 2\sqrt{x}$ at the point $(1, 3)$.

$\dfrac{dy}{dx} = 3x^2 + \dfrac{1}{\sqrt{x}}$ (1), $\left.\dfrac{dy}{dx}\right|_{x=1} = 4$ (1)

tangent line: $y - 3 = 4(x-1)$ (2)

$\boxed{y - 3 = 4(x-1) \text{ or } y = 4x - 1}$

$\boxed{4}$