NAME Grading Key

STUDENT ID ______________________

RECITATION INSTRUCTOR ______________________

RECITATION TIME ______________________

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TOTAL	/100

DIRECTIONS

1. Write your name, student ID number, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes or calculators may be used on this exam.

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(8) 1. If $f(x) = \frac{1}{x}$ and $g(x) = x^3 - 2x$, find each of the following functions. (It is not necessary to simplify.)

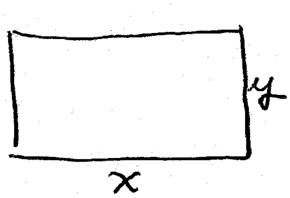
2 pts $(f \circ g)(x) = \boxed{\frac{1}{x^3 - 2x}}$

2 pts $(g \circ f)(x) = \boxed{\left(\frac{1}{x}\right)^3 - 2 \cdot \frac{1}{x}}$ or $= \frac{1}{x^3} - \frac{2}{x}$

2 pts $(f \circ f)(x) = \boxed{\frac{1}{1/x} \overset{\text{or}}{=} x,\ x \neq 0}$

2 pts $(g \circ g)(x) = \boxed{(x^3 - 2x)^3 - 2(x^3 - 2x)}$

(5) 2. A rectangle has a perimeter of 48 m. Express its area, A, as a function of the length of one of its sides.



$$\left.\begin{aligned} A &= xy \\ 48 &= 2x + 2y \end{aligned}\right\} 3 \text{ pts}$$

$$y = 24 - x$$

$A = \boxed{x(24 - x) \overset{\text{or}}{=} 24x - x^2}$

2 pts

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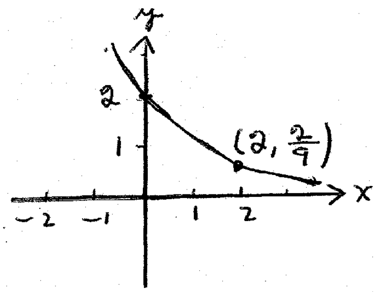
(8) 3. Find the exponential function $f(x) = Ca^x$ whose graph is given below.



$f(0) = 2 \rightarrow 2 = Ca^0 = C$ 3 pts

$f(2) = \frac{2}{9} \rightarrow \frac{2}{9} = 2a^2 \rightarrow a = \frac{1}{3}$ 3 pts

$f(x) = \boxed{2\left(\frac{1}{3}\right)^x}$ 2 pts

(10) 4. Find the inverse of the following function and state its domain.

$g(x) = \ln(7x - 3)$, $g^{-1}(x) = \boxed{\frac{1}{7}(e^x + 3)}$ 3 pts   domain $= \boxed{(-\infty, \infty)}$ or $\mathbb{R}$. 4 pts

$y = \ln(7x - 3)$

$e^y = 7x - 3$. } 3 pts

$\therefore x = \frac{1}{7}(e^y + 3)$

(6) 5. Find the value(s) of a so that the function f is continuous on $(-\infty, \infty)$ if

$$f(x) = \begin{cases} x^2 + a, & x \leq 1 \\ a^2 - x, & x > 1 \end{cases}$$

$1 + a = a^2 - 1$ 4 pts

$a^2 - a - 2 = 0$

$(a-2)(a+1) = 0$

$a = 2, a = -1$

2 pts $\boxed{a = -1, a = 2}$

(6) 6. Find the equations of the horizontal and vertical asymptotes of the function

$$y = \frac{x^2 + 5x - 9}{x^2 - 3x + 2}.$$

$\lim_{x \to \pm\infty} y = \lim_{x \to \pm\infty} \frac{1 + 5/x - 9/x^2}{1 - 3/x + 2/x^2} = 1$

horizontal asymptote(s) $\boxed{y = 1}$ 2 pts

$= \frac{x^2 + 5x - 9}{(x-1)(x-2)}$.

$\lim_{x \to 1^+} y = \infty$, $\lim_{x \to 2^+} y = \infty$

vertical asymptote(s) $\boxed{x = 1, x = 2.}$ 2 pts for each

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(18) 7. For each of the following fill in the boxes below with a finite number, or one of the symbols: ∞, $-\infty$, or DNE (does not exist). It is not necessary to give reasons for your answers.

(a) $\lim_{x \to (-\frac{\pi}{2})^-} \sec x =$ 3 pts $\boxed{-\infty}$

(b) $\lim_{x \to \infty} \frac{x^2 - x + 5}{3x^2 + x - 1} =$ 3 pts $\boxed{\frac{1}{3}}$

(c) $\lim_{x \to 0^-} \frac{2}{x} =$ 3 pts $\boxed{-\infty}$

(d) $\lim_{x \to 5} \frac{x - 5}{|x - 5|} =$ 3 pts $\boxed{\text{DNE}}$

(e) $\lim_{x \to 3} \left(\frac{1}{x - 3} - \frac{6}{x^2 - 9} \right) =$ 3 pts $\boxed{\frac{1}{6}}$

(f) $\lim_{x \to \infty} \cos x =$ 3 pts $\boxed{\text{DNE}}$

(8) 8. Find the derivative of $f(x) = \sqrt{x}$ using the definition of the derivative:
$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

4 pts for a correct start 2 pts.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h \to 0} \frac{x + h - x}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

No credit for using the formula for the derivative.

2 pts

$\boxed{\frac{1}{2\sqrt{x}}}$

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(6) 9. For what values of x does the graph $f(x) = 2x^3 - 3x^2 - 6x + 87$ have a horizontal tangent.

$f'(x) = 6x^2 - 6x - 6 = 0$. 3 pts for correct start

$x^2 - x - 1 = 0$

$x = \frac{1 \pm \sqrt{5}}{2}$

Take off 1 pt if they do not have both ans.

3 pts

$x = \boxed{\frac{1 \pm \sqrt{5}}{2}}$

(15) 10. Find the derivatives of the following functions. (It is not necessary to simplify.)

(a) $y = \frac{e^x}{1+x}$

$\frac{dy}{dx} = \frac{(1+x)e^x - e^x}{(1+x)^2}$ or

5 pts

$\frac{dy}{dx} = \boxed{\frac{xe^x}{(1+x)^2}}$

(b) $f(x) = x^2 \sin x$

5 pts

$f'(x) = \boxed{x^2 \cos x + 2x \sin x}$

(c) $g(t) = \sec t \tan t$

5 pts

$g'(t) = \sec t \sec^2 t + \sec t \tan t \tan t$ or $g'(t) = \boxed{\sec^3 t + \sec t \tan^2 t}$

(10) 11. Find the following limits. (Use the fact that $\lim_{x \to 0} \frac{\sin x}{x} = 1$.) Show your work.

(a) $\lim_{\theta \to 0} \frac{\sin^2 \theta}{\theta} = \lim_{\theta \to 0} \frac{\sin \theta}{\theta} \cdot \sin \theta$ ← 3 pts

$= 1 \cdot 0 = 0$

$\boxed{0}$ 2 pts

(b) $\lim_{x \to 0} \frac{\tan x}{4x} = \lim_{x \to 0} \frac{\sin x}{x} \cdot \frac{1}{4 \cos x}$ ← 3 pts

$= 1 \cdot \frac{1}{4} = \frac{1}{4}$

$\boxed{\frac{1}{4}}$ 2 pts