NAME GRADING KEY

STUDENT ID ____________

RECITATION INSTRUCTOR ____________

RECITATION TIME ____________

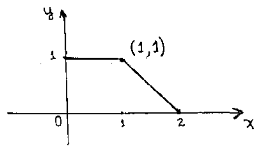
Page 1	/8
Page 2	/30
Page 3	/34
Page 4	/28
TOTAL	/100

DIRECTIONS

1. Write your name, student ID number, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3 and 4.
2. The test has four (4) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes or calculators may be used on this exam.

---

(8) 1. The graph of the function f is shown below:



(a) Find an expression for $f(x)$.

For $0 \le x \le 1$, $f(x) = 1$

For $1 \le x \le 2$: $y - 1 = (-1)(x-1)$

$y = 2 - x$

$f(x) = 2 - x$

$$f(x) = \begin{cases} 1 & \text{if } 0 \le x \le 1 \\ 2-x & \text{if } 1 \le x \le 2 \end{cases}$$ 2 2

(b) The domain of f is: $[0,2]$ or $0 \le x \le 2$ 1

The range of f is: $[0,1]$ or $0 \le y \le 1$ 1

(c) True or False? (i) f is continuous at $x = 1$. T F 1

(ii) f is differentiable at $x = 1$. T F 1

8

Name: ______________________________

(4) 2. If $f(x) = \sin x$ and $g(x) = 1 - \sqrt{x}$, find the functions $f \circ g$ ~~ans~~ and $g \circ f$.

$(f \circ g)(x) = f(g(x)) = \sin(1 - \sqrt{x})$

$(g \circ f)(x) = g(f(x)) = 1 - \sqrt{\sin x}$

$(f \circ g)(x) = \sin(1 - \sqrt{x})$ 2
$(g \circ f)(x) = 1 - \sqrt{\sin x}$ 2

$\boxed{4}$

(6) 3. Find all the values of x in the interval $[0, 2\pi]$ that satisfy the equation $2\sin^2 x = 1$.

$\sin x = \pm \frac{1}{\sqrt{2}}$ 2

-1pt for each extra answer

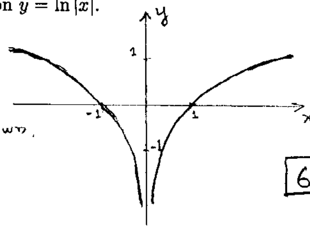
1	1	1	1
$\frac{\pi}{4}$	$\frac{3\pi}{4}$	$\frac{5\pi}{4}$	$\frac{7\pi}{4}$

$\boxed{6}$

(6) 4. Make a rough sketch of the graph of the function $y = \ln|x|$.

Each branch must go through (-1,0) or (1,0) and must have the y-axis as a vertical asymptote as shown.

2 pts for only one branch



$\boxed{6}$

(8) 5. Find a formula for the inverse f^{-1} of the function

$$f(x) = \frac{1 + e^x}{1 - e^x}$$

$y = \frac{1 + e^x}{1 - e^x}$ 2

$y - ye^x = 1 + e^x$

$y - 1 = e^x(y+1)$

$e^x = \frac{y-1}{y+1}$

$x = \ln \frac{y-1}{y+1}$ 4 $\quad f^{-1}(y) = \ln \frac{y-1}{y+1}$

or

$x = \frac{1 + e^y}{1 - e^y}$ 2

$x - xe^y = 1 + e^y$

$x - 1 = e^y(x+1)$

$e^y = \frac{x-1}{x+1}$

$y = \ln \frac{x-1}{x+1}$ 4 $\quad f^{-1}(x) = \ln \frac{x-1}{x+1}$

$f^{-1}(x) = \ln \frac{x-1}{x+1}$ 2

$\boxed{8}$

(6) 6. Evaluate the following:

(a) $e^{|\ln \frac{1}{2}|} = e^{|-\ln 2|} = e^{\ln 2}$ NPC $\boxed{2}$ $\boxed{2}$

(b) $\cos(\pi e^{-\ln 6}) = \cos\left(\pi \frac{1}{e^{\ln 6}}\right) = \cos\left(\frac{\pi}{6}\right)$ $\boxed{\frac{\sqrt{3}}{2}}$ $\boxed{2}$

(c) $\ln e^{-5}$ $\boxed{-5}$ $\boxed{2}$

Name: ______________________________

(12) 7. Find each of the following. Fill in the boxes below with a finite number, or one of the symbols: ∞, $-\infty$, or DNE (does not exist). It is not necessary to give reasons for your answers.

2 pts for each correct answer

NPC

$\displaystyle\lim_{x\to(-4)^-} \frac{|x+4|}{x+4} = \boxed{-1}$

$\displaystyle\lim_{x\to -4} \frac{|x+4|}{x+4} = \boxed{\text{DNE}}$

$\displaystyle\lim_{x\to(-\frac{\pi}{2})^-} \sec x = \boxed{-\infty}$

$\displaystyle\lim_{x\to\infty} \sqrt{\frac{1+3x^3}{5x^3-x^2+2x}} = \boxed{\sqrt{\tfrac{3}{5}}}$

$\displaystyle\lim_{t\to 0} \frac{\sin 5t}{t} = \boxed{5}$

$\displaystyle\lim_{x\to 2^+} \ln(x-2) = \boxed{-\infty}$

12

(8) 8. Find $\displaystyle\lim_{t\to 0}\frac{\sqrt{2-t}-\sqrt{2}}{t}$.

$\boxed{-1\text{pt if } \lim_{t\to 0} \text{ is missing one or more times}}$

3

$$\lim_{t\to 0}\frac{\sqrt{2-t}-\sqrt{2}}{t} = \lim_{t\to 0}\frac{\sqrt{2-t}-\sqrt{2}}{t}\cdot\frac{\sqrt{2-t}+\sqrt{2}}{\sqrt{2-t}+\sqrt{2}} = \lim_{t\to 0}\frac{2-t-2}{t\left(\sqrt{2-t}+\sqrt{2}\right)} =$$

$$= \lim_{t\to 0}\left(-\frac{1}{\sqrt{2-t}+\sqrt{2}}\right) = -\frac{1}{2\sqrt{2}} \quad 2$$

3

$\boxed{-\frac{1}{2\sqrt{2}}}$

8

(6) ~~(5)~~ 9. Find the constant c that makes g continuous on $(-\infty,\infty)$.

$$g(x) = \begin{cases} x^2 - c^2, & \text{if } x < 4 \\ cx + 20, & \text{if } x \geq 4 \end{cases}$$

f is continuous on $(-\infty,4)\cup(4,\infty)$ because it is a polynomial

f will be continuous at $x=4$ if $\lim_{x\to 4} f(x) = f(4)$

$\lim_{x\to 4^-} f(x) = 16 - c^2$ 1 $\quad \lim_{x\to 4^+} = 4c + 20$ 1

$16 - c^2 = 4c + 20$ 2

$c^2 + 4c + 4 = 0$

$(c+2)^2 = 0 \rightarrow c = -2$ 2

With $c = -2$

$\lim_{x\to 4} f(x) = 12$

$f(4) = 12$

$\boxed{c = -2}$

6

(8) 10. Find an interval of the form $(n, n+1)$, where n is an integer, such that the equation $x^4 + 2x - 25 = 0$ has a solution in the interval. State the name of the theorem you are using.

$f(x) = x^4 + 2x - 25$ 1

$f(2) = 16 + 4 - 25 = -5 < 0$ 2

$f(3) = 81 + 6 - 25 = 62 > 0$ 2

f is continuous in $[2,3]$ and $f(2) < 0 < f(3)$

Intermediate value theorem 2

1 $\boxed{(2,3) \text{ or } (-3,-2)}$

8

Name: ____________________

(10) 11. Find the derivative of $g(x) = \frac{1}{x^2}$, using the definition of the derivative:

$$g'(x) = \lim_{h\to 0} \frac{g(x+h) - g(x)}{h}.$$

$\boxed{-1\text{pt if } \lim_{h\to 0} \text{ is missing one or more times}}$

$$g'(x) = \lim_{h\to 0} \frac{g(x+h) - g(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} \quad 4$$

$$= \lim_{h\to 0} \frac{x^2 - (x+h)^2}{h(x+h)^2x^2} = \lim_{h\to 0} \frac{-2xh - h^2}{h(x+h)^2x^2}$$

$$= \underbrace{\lim_{h\to 0} \frac{-2x - h}{(x+h)^2x^2}}_{4} = \frac{-2x}{x^4} = -\frac{2}{x^3} \quad 2 \qquad \boxed{-\frac{2}{x^3}}$$

(6) 12. Find the equation of the tangent line to the graph of $f(x) = 3x^2 - 5x$ at the point $(2, 2)$.

$f'(x) = 6x - 5$ 2

$f'(2) = 6\cdot 2 - 5 = 7$ 1

$y - 2 = 7(x - 2)$ 3

or → $\boxed{y = 7x - 12}$ $\boxed{6}$

(12) 13. Find the derivatives of the following functions. (It is not necessary to simplify.)

(a) $y = \sqrt{x}e^x$

NPC

$\boxed{\sqrt{x}e^x + \frac{1}{2\sqrt{x}}e^x}$ $\boxed{3}$

(b) $f(x) = \frac{x}{1 - x^2}$.

$f'(x) = \frac{(1-x^2)1 - x(-2x)}{(1-x^2)^2}$ or → $\boxed{\frac{1+x^2}{(1-x^2)^2}}$ $\boxed{3}$

(c) $g(t) = 4\sec t + \tan t$.

$\boxed{4\sec t \tan t + \sec^2 t}$ $\boxed{3}$

(d) $y = \frac{\sin x}{1 + \cos x}$.

$\frac{dy}{dx} = \frac{(1+\cos x)\cos x - \sin x(-\sin x)}{(1+\cos x)^2}$ or → $\boxed{\frac{1+\cos x}{(1+\cos x)^2} = \frac{1}{1+\cos x}}$ $\boxed{3}$