NAME Solutions

10-DIGIT PUID

REC. INSTR. REC. TIME

LECTURER

INSTRUCTIONS:

1. There are 14 different test pages (including this cover page). Make sure you have a complete test.
2. Fill in the above items in print. Also write your name at the top of pages 2–14.
3. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet. No partial credit will be given, but if you show your work on the test booklet, it may be used in borderline cases.
4. No books, notes, calculators or any electronic devices may be used on this exam.
5. Each problem is worth 8 points. The maximum possible score is 200 points.
6. Using a #2 pencil, fill in each of the following items on your answer sheet:
   (a) On the top left side, write your name (last name, first name), and fill in the little circles.
   (b) On the bottom left side, under SECTION, write in your division and section number and fill in the little circles. (For example, for division 9 section 1, write 0901. For example, for division 38 section 1, write 3801).
   (c) On the bottom, under TEST/QUIZ NUMBER, write 01 and fill in the little circles.
   (d) On the bottom, under STUDENT IDENTIFICATION NUMBER, write in your 10–digit PUID, and fill in the little circles.
   (e) Using a #2 pencil, put your answers to questions 1–25 on your answer sheet by filling in the circle of the letter of your response. Double check that you have filled in the circles you intended. If more than one circle is filled in for any question, your response will be considered incorrect. Use a #2 pencil.
7. After you have finished the exam, hand in your answer sheet and your test booklet to your recitation instructor.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!},\ |x| < \infty \qquad \sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}\, x^{2n+1},\ |x| < \infty$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}\, x^{2n},\ |x| < \infty \qquad \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n,\ |x| < 1$$

1. Find a vector that has the direction opposite of $2\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$ but has length $\sqrt{5}$.

A. $-\frac{2}{\sqrt{5}}\mathbf{i} + \frac{4}{\sqrt{5}}\mathbf{j} - \frac{5}{\sqrt{5}}\mathbf{k}$

B. $\frac{2}{\sqrt{5}}\mathbf{i} - \frac{4}{\sqrt{5}}\mathbf{j} + \frac{5}{\sqrt{5}}\mathbf{k}$

C. $\frac{2\sqrt{5}}{\sqrt{11}}\mathbf{i} - \frac{4\sqrt{5}}{\sqrt{11}}\mathbf{j} + \frac{5\sqrt{5}}{\sqrt{11}}\mathbf{k}$

D. $-\frac{2\sqrt{5}}{\sqrt{11}}\mathbf{i} + \frac{4\sqrt{5}}{\sqrt{11}}\mathbf{j} - \frac{5\sqrt{5}}{\sqrt{11}}\mathbf{k}$

E. $-\frac{2}{3}\mathbf{i} + \frac{4}{3}\mathbf{j} - \frac{5}{3}\mathbf{k}$

$$|2\vec{i} - 4\vec{j} + 5\vec{k}| = \sqrt{(2)^2 + (-4)^2 + (5)^2} = \sqrt{45}$$

$$\frac{-\sqrt{5}}{\sqrt{45}}(2\vec{i} - 4\vec{j} + 5\vec{k})$$

$$= -\frac{2\sqrt{5}}{\sqrt{45}}\vec{i} + \frac{4\sqrt{5}}{\sqrt{45}}\vec{j} - \frac{5\sqrt{5}}{\sqrt{45}}\vec{k}$$

$$= \frac{-2}{3}\vec{i} + \frac{4}{3}\vec{j} - \frac{5}{3}\vec{k}$$

note: $\sqrt{45} = \sqrt{9 \cdot 5} = 3\sqrt{5}$

2 For what values of b are the vectors $< -2, b, 1 >$ and $< 3, b, -b >$ orthogonal?

A. $b = 1, 3$

B. $b = -3, 2$

C. $b = -2, 3$

D. $b = 1, 2$

E. $b = 2, 3$

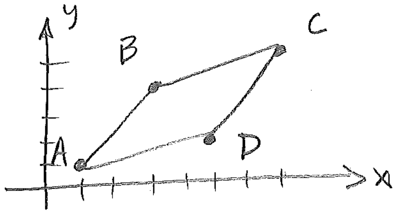
$$\langle -2, b, 1 \rangle \cdot \langle 3, b, -b \rangle = 0$$

$$\rightarrow -6 + b^2 - b = 0$$

$$\rightarrow (b-3)(b+2) = 0$$

$$\rightarrow b = -2, 3$$

3. Find the area of the parallelogram with vertices $A(1,1)$, $B(3,4)$, $C(7,5)$ and $D(5,2)$.



$\overrightarrow{AB} = \langle 2, 3, 0 \rangle$

$\overrightarrow{AD} = \langle 4, 1, 0 \rangle$

A. 11

B. 13

C. 10

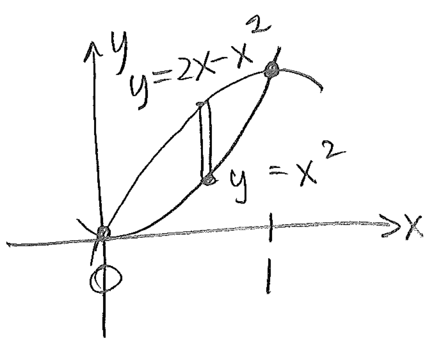
D. 12

E. 9

$$\overrightarrow{AB} \times \overrightarrow{AD} = \begin{vmatrix} i & j & k \\ 2 & 3 & 0 \\ 4 & 1 & 0 \end{vmatrix} = \langle 0-0, -(0-0), 2-12 \rangle = \langle 0, 0, -10 \rangle$$

$$|\langle 0, 0, -10 \rangle| = 10$$

4. Find the area of the region bounded by $y = x^2$ and $y = 2x - x^2$.



A. $\frac{11}{3}$

B. $\frac{10}{3}$

C. $\frac{16}{3}$

D. $\frac{1}{3}$

E. $\frac{4}{3}$

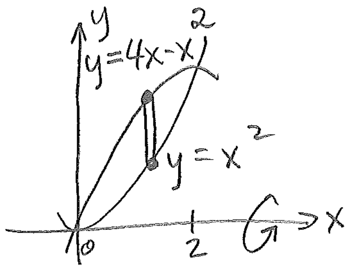
$$\text{Area} = \int_0^1 \left((2x - x^2) - (x^2)\right) dx$$

$$= \int_0^1 (2x - 2x^2)\, dx$$

$$= \left(x^2 - \tfrac{2}{3}x^3\right)\Big|_0^1$$

$$= \left(1 - \tfrac{2}{3}\right) - (0 - 0) = \frac{1}{3}$$

5. Find the volume of the solid obtained by rotating the region bounded by $y = x^2$ and $y = 4x - x^2$ about the x–axis.



A. $\frac{11}{3}\pi$

B. $\frac{10}{3}\pi$

C. $\frac{16}{3}\pi$

D. $\frac{8}{3}\pi$

E. $\frac{32}{3}\pi$

$$V = \int_0^2 \pi \left((4x - x^2)^2 - (x^2)^2 \right) dx$$

$$= \int_0^2 \pi \left(16x^2 - 8x^3 + x^4 - x^4 \right) dx$$

$$= \pi \left(\frac{16}{3} x^3 - 2x^4 \right) \Big|_0^2$$

$$= \pi \left(\frac{128}{3} - 32 \right) = \pi \left(\frac{128 - 96}{3} \right) = \frac{32}{3}\pi$$

6. It took 2700 J of work to stretch a spring from its natural length of $2m$ to a length of $5m$. Find the spring's force constant.

A. 150

B. $\frac{800}{3}$

C. 400

D. $\frac{1400}{3}$

E. 600

$$\text{work} = \int_a^b kx \, dx$$

$$2700 = \int_0^3 kx \, dx$$

$$= \frac{k}{2} x^2 \Big|_0^3 = \frac{9}{2} k$$

$$\Rightarrow k = (2700)\left(\frac{2}{9}\right) = 600$$

let u be first function in following list.

7. Evaluate $\int_1^e \frac{\ln x}{x^2}\, dx$.

$u =$ L. I. A. T. E.

let $u = \ln x$ $\quad dv = x^{-2}\,dx$

$\rightarrow du = \frac{1}{x}\,dx \quad v = -\frac{1}{x}$

A. $\frac{1-e^3}{2e^3}$

B. $\frac{3-3e^4}{e^4}$

C. $\frac{e-2}{e}$

D. $\frac{e^2-2}{e^2}$

E. $\frac{4-3e^3}{e^3}$

$\int_1^e (\ln x)(x^{-2})\,dx = (\ln x)\left(-\frac{1}{x}\right)\Big|_1^e - \int_1^e -\frac{1}{x^2}\,dx$

$= \left(-\frac{1}{x}\ln x - \frac{1}{x}\right)\Big|_1^e$

$= \left(-\frac{1}{e}\cdot 1 - \frac{1}{e}\right) - \left(-\frac{1}{e}\cdot 0 - 1\right)$

$= -\frac{2}{e} + 1 = 1 - \frac{2}{e} = \frac{e-2}{e}$

8. Evaluate $\int_{\pi/2}^{3\pi/4} \frac{\cos^3\theta}{\sin\theta}\, d\theta$.

$= \int_{\pi/2}^{3\pi/4} \frac{\cos^2\theta}{\sin\theta} \cdot \cos\theta\, d\theta$

$= \int_{\pi/2}^{3\pi/4} \frac{1-\sin^2\theta}{\sin\theta} \cos\theta\, d\theta$

$= \int_{\pi/2}^{3\pi/4} (\cot\theta - \sin\theta\cos\theta)\,d\theta$

$= \left(\ln|\sin\theta| - \frac{1}{2}\sin^2\theta\right)\Big|_{\pi/2}^{3\pi/4}$

$= \left(\ln\left(\sqrt{2}\right) - \frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)^2\right) - \left(\ln(1) - \frac{1}{2}(1)^2\right)$

$= \left(-\frac{1}{2}\ln 2 - \frac{1}{4}\right) - \left(0 - \frac{1}{2}\right)$

$= \frac{1}{4} - \frac{1}{2}\ln 2$

A. $\frac{3}{4} - \frac{1}{2}\ln 2$

B. $\frac{1}{2}\ln 2 - \frac{3}{4}$

C. -1

D. $\frac{1}{4} - \frac{1}{2}\ln 2$

E. $\frac{1}{2}\ln 2 - \frac{1}{4}$

9. A trigonometric substitution can be used to convert the definite integral $\int_2^5 \frac{dt}{\sqrt{t^2-4t+13}}$ into which of the following definite integrals?

A. $\int_0^{\pi/4} \sec\theta \, d\theta$

B. $\int_0^{\pi/4} \cos\theta \, d\theta$

C. $\int_0^{\pi/3} \sin\theta \, d\theta$

D. $\int_0^{\pi/3} \cos\theta \, d\theta$

E. $\int_0^{\pi/3} \sec\theta \, d\theta$

$t^2-4t+13 = t^2-4t+4+9$
$= (t-2)^2+9$

let $t-2 = 3\tan\theta$

Then $dt = 3\sec^2\theta \, d\theta$

and $\sqrt{(t-2)^2+9} = 3\sec\theta$

Also $\theta = \tan^{-1}\left(\frac{t-2}{3}\right)$

$\theta(2) = \tan^{-1}(0) = 0$

$\theta(5) = \tan^{-1}\left(\frac{3}{3}\right) = \pi/4$

$$\therefore \int_2^5 \frac{dt}{\sqrt{t^2-4t+13}} = \int_0^{\pi/4} \frac{3\sec^2\theta \, d\theta}{3\sec\theta} = \int_0^{\pi/4} \sec\theta \, d\theta$$

10. Compute $\int_0^2 \frac{8x-4}{x^2-2x-3} \, dx$.

A. $-8\ln 3$

B. $-5\ln 3$

C. $-3\ln 3$

D. $-2\ln 3$

E. $-\ln 3$

$$\frac{8x-4}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{x+1}$$

$\rightarrow 8x-4 = A(x+1) + B(x-3)$

let $x=3 \rightarrow 20 = 4A + 0B \rightarrow A=5$

let $x=-1 \rightarrow -12 = 0A - 4B \rightarrow B=3$

$$\int_0^2 \left(\frac{5}{x-3} + \frac{3}{x+1}\right) dx = \left(5\ln|x-3| + 3\ln|x+1|\right)\Big|_0^2$$
$$= (5\ln(1) + 3\ln(3)) - (5\ln(3) + 3\ln(1))$$
$$= 0 + 3\ln(3) - (5\ln(3) + 0)$$
$$= -2\ln 3$$

11. Find whether the series $\sum_{n=1}^{\infty} 3\left(-\frac{1}{2}\right)^n$ converges or diverges, and find its sum if it converges.

$\sum_{n=1}^{\infty} 3\left(-\frac{1}{2}\right)^n$ is a convergent geometric series with $r = -\frac{1}{2}$ and $-1 < r < 1$.

$$\sum_{n=1}^{\infty} 3\left(-\frac{1}{2}\right)^n = \sum_{n=0}^{\infty} -\frac{3}{2}\left(-\frac{1}{2}\right)^{n-1}$$

$$= \frac{-\frac{3}{2}}{1-\left(-\frac{1}{2}\right)} = \frac{-\frac{3}{2}}{\frac{3}{2}} = -1$$

A. Diverges.

B. Converges and sum = 0

C. Converges and sum = 2

D. Converges and sum = −1

E. Converges and sum = −3

12. Which of these improper integrals converge?

DIV. I. $\int_0^3 \frac{1}{x-2}\,dx$. $= \int_0^2 \frac{1}{x-2}\,dx + \int_2^3 \frac{1}{x-2}\,dx$.

CONV. II. $\int_0^3 \frac{1}{\sqrt{3-x}}\,dx$. $= \lim_{t\to 3^-} \int_0^t (3-x)^{-1/2}\,dx$

DIV. III. $\int_3^\infty \frac{1}{\sqrt{x}}\,dx$.

$$= \lim_{t\to 3^-}\left(-2(3-x)^{1/2}\Big|_0^t\right)$$

$$= \lim_{t\to 3^-}\left(-2(3-t)^{1/2} + 2\sqrt{3}\right)$$

$$= 0 + 2\sqrt{3}$$

→ integral converges

$$\int_0^2 \frac{1}{x-2}\,dx = \lim_{t\to 2^-} \int_0^t \frac{1}{x-2}\,dx$$

$$= \lim_{t\to 2^-}\left(\ln|x-2|\Big|_0^t\right)$$

$$= \lim_{t\to 2^-}\left(\ln|t-2| - \ln 2\right)$$

$$= -\infty - \ln 2$$

→ integral diverges.

A. All

B. Only (I)

C. Only (II)

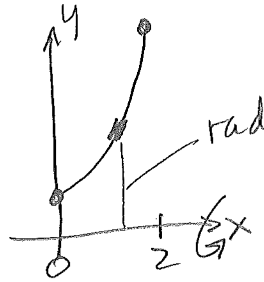
D. Only (III)

E. (I) and (II)

$$= \lim_{t\to\infty} \int_3^t x^{-1/2}\,dx = \lim_{t\to\infty} 2x^{1/2}\Big|_3^t$$

$$= \lim_{t\to\infty}\left(2\sqrt{t} - 2\sqrt{3}\right) = \infty - 2\sqrt{3}$$ → integral diverges.

13. The curve $y = x^2 + 1,\ 0 \le x \le 2$, is rotated about the x–axis. The area of the surface is given by



A. $\int_0^2 2\pi(x^2+1)\sqrt{1+4x^2}\, dx$

B. $\int_0^2 2\pi x\sqrt{1+4x^2}\, dx$

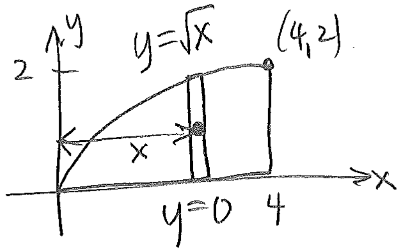
C. $\int_0^2 \pi(x^2+1)^2\, dx$

D. $\int_0^2 2\pi x(x^2+1)\, dx$

E. $\int_0^2 (x^2+1)\, dx$

Surface Area $= \int_0^2 2\pi\,(x^2+1)\sqrt{1+(2x)^2}\, dx$

14. Consider the lamina bounded by the graph of $y = \sqrt{x}$, the x–axis and the line $x = 4$, with density $\rho = 1$. The x–coordinate $\overline{x}$ of the center of mass of the lamina is



$\overline{x} = \dfrac{M_y}{m}$

A. 1

B. 2

C. 3

D. $\frac{5}{2}$

E. $\frac{12}{5}$

$m = \int_0^4 1\left(\sqrt{x}-0\right)dx = \frac{2}{3}x^{3/2}\Big|_0^4 = \frac{16}{3}$

$M_y = \int_0^4 (x)(1)(\sqrt{x}-0)\, dx = \int_0^4 x^{3/2}\,dx = \frac{2}{5}x^{5/2}\Big|_0^4 = \frac{64}{5}$

$\overline{x} = \dfrac{M_y}{m} = \dfrac{\frac{64}{5}}{\frac{16}{3}} = \left(\frac{64}{5}\right)\left(\frac{3}{16}\right) = \frac{12}{5}$

15. Let $a = \lim_{n\to\infty} ne^{-n}$ and $b = \lim_{n\to\infty} \frac{n!}{(2n)!}$. Then

$$a = \lim_{n\to\infty} \frac{n}{e^n} \overset{\frac{\infty}{\infty}}{=} \lim_{n\to\infty} \frac{1}{e^n} = 0$$

A. $a = 1$ and $b = \frac{1}{2}$

B. $a = 0$ and $b = \frac{1}{2}$

C. $a = 1$ and $b = 0$

D. $a = 0$ and $b = 0$

E. $a = e^{-1}$ and $b = 0$

$$b = \lim_{n\to\infty} \frac{n!}{(2n)!}$$

$$= \lim_{n\to\infty} \frac{\cancel{(n)(n-1)(n-2)\cdots(2)(1)}\ 1}{(2n)(2n-1)(2n-2)\cdots(n+2)(n+1)\cancel{(n)(n-1)(n-2)\cdots(2)(1)}}$$

$$= 0$$

16. The series $\sum_{n=0}^{\infty} (-1)^n \frac{\tan^{-1} n}{1+n^2}$ is

A. absolutely convergent

B. conditionally convergent

C. divergent since $\lim_{n\to\infty} (-1)^n \frac{\tan^{-1} n}{1+n^2} \neq 0$

D. divergent even though $\lim_{n\to\infty} (-1)^n \frac{\tan^{-1} n}{1+n^2} = 0$

E. divergent by the ratio test

$0 \le \left|(-1)^n \frac{\tan^{-1} n}{1+n^2}\right| \le \frac{\pi/2}{1+n^2}$ and $\sum^{\infty} \frac{\pi/2}{1+n^2}$ converges by the Limit Comparison Test with convergent $\sum^{\infty} \frac{1}{n^2}$.

Therefore $\sum^{\infty} \left|(-1)^n \frac{\tan^{-1} n}{1+n^2}\right|$ converges by the comparison test with $\sum^{\infty} \frac{\pi/2}{1+n^2}$, and therefore $\sum (-1)^n \frac{\tan^{-1} n}{1+n^2}$ is absolutely convergent.

17. Which of the following series converge?

CONV. (I) $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{n^2} = \sum^{\infty} \frac{1}{n^{3/2}}$ convergent p-series, $p = \frac{3}{2} > 1$.

DIV. (II) $\sum_{n=1}^{\infty} \frac{n^n}{n!}$ Ratio Test: $\lim_{n\to\infty} (a_{n+1})\left(\frac{1}{a_n}\right) = \lim_{n\to\infty}\left(\frac{(n+1)^{n+1}}{(n+1)!}\,\frac{n!}{n^n}\right)$

$= \lim_{n\to\infty}\left(\frac{\cancel{(n+1)}(n+1)^n\,\cancel{n!}}{\cancel{(n+1)!}\;n^n}\right) = \lim_{n\to\infty}\left(\frac{n+1}{n}\right)^n$

$= \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e > 1$

CONV. (III) $\sum_{n=1}^{\infty} \frac{1}{5^n - 2}$

CONV. (IV) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[4]{n}}$

(III) $\lim_{n\to\infty}\left(\frac{\frac{1}{5^n}}{\frac{1}{5^n-2}}\right) = \lim_{n\to\infty} \frac{5^n-2}{5^n}$

$= \lim_{n\to\infty}\left(1 - \frac{2}{5^n}\right) = 1 > 0$, and

$\sum^{\infty} \frac{1}{5^n}$ conv. geom. series.

A. (III) only

B. (III) and (IV) only

C. All

D. (I), (III) and (IV) only

E. (II), (III) and (IV) only

(IV) Converges by Alt. Series Test. $\lim_{n\to\infty} \frac{1}{\sqrt[4]{n}} = 0$ and $\frac{1}{\sqrt[4]{n+1}} < \frac{1}{\sqrt[4]{n}}$.

18. Use a Maclaurin series and the Estimation Theorem for alternating series to approximate $\sin\left(\frac{1}{2}\right)$ using the fewest number of terms necessary so that the error is less than 0.001.

$0.001 = \frac{1}{1000}$

$\sin\left(\frac{1}{2}\right) = \frac{1}{2} - \frac{\left(\frac{1}{2}\right)^3}{3!} + \frac{\left(\frac{1}{2}\right)^5}{5!} - \cdots$

$= \frac{1}{2} - \frac{1}{48} + \frac{1}{(32)(120)} - \cdots$

$\frac{1}{(32)(120)} < \frac{1}{1000}$

$\Rightarrow$ use 2 terms

$\frac{1}{2} - \frac{1}{48}$

$= \frac{24-1}{48} = \frac{23}{48}$

A. $\frac{1}{2}$

B. $\frac{23}{48}$

C. $\frac{3}{4}$

D. $\frac{33}{40}$

E. $\frac{41}{60}$

19. Consider the power series $\sum_{n=1}^{\infty} \frac{2^n}{n} x^n$. The radius of convergence R and the interval of convergence of this series are

Root Test: $\lim_{n\to\infty} \left|\frac{2^n x^n}{n}\right|^{1/n}$

$= \lim_{n\to\infty} \frac{2|x|}{n^{1/n}} = 2|x|.$

Series Conv. Abs for $2|x| < 1$

$\longrightarrow -\frac{1}{2} < x < \frac{1}{2}$

A. $R = \frac{1}{2},\ \left(-\frac{1}{2}, \frac{1}{2}\right)$

B. $R = \frac{1}{2},\ \left[-\frac{1}{2}, \frac{1}{2}\right)$

C. $R = \frac{1}{2},\ \left[-\frac{1}{2}, \frac{1}{2}\right]$

D. $R = 1,\ (-1, 1)$

E. $R = 1,\ [-1, 1)$

Endpoints: $x = -\frac{1}{2} \to \sum^{\infty} \frac{2^n\left(-\frac{1}{2}\right)^n}{n} = \sum^{\infty} \frac{(-1)^n}{n}$ converges.

$x = \frac{1}{2} \longrightarrow \sum^{\infty} \frac{2^n\left(\frac{1}{2}\right)^n}{n} = \sum^{\infty} \frac{1}{n}$ diverges.

20. The interval of convergence for the series $\sum_{n=1}^{\infty} \frac{10^n}{n!}(x-1)^n$ is

Ratio Test: $\lim_{n\to\infty} \left| a_{n+1} \cdot \frac{1}{a_n} \right|$

$= \lim_{n\to\infty} \left| \frac{10^{n+1}(x-1)^{n+1}}{(n+1)!} \frac{n!}{10^n (x-1)^n} \right|$

$= \lim_{n\to\infty} \left(\frac{10}{n+1}\right)|x-1| = 0$ for all x.

Therefore series converges for all x,

A. $(0, 2)$

B. $[0, 2)$

C. $(9, 11)$

D. $[9, 11)$

E. $(-\infty, \infty)$

21. In the Taylor series expansion for $f(x) = \dfrac{x-1}{x-2}$ about $a = 1$, the coefficient of $(x-1)^{10}$ is

A. -2
B. -1
C. 0
D. 1
E. 2

$$f(x) = (x-1)\left(\frac{1}{(x-1)-1}\right) = -(x-1)\left(\frac{1}{1-(x-1)}\right)$$

$$= -(x-1)\sum_{n=0}^{\infty}(x-1)^n$$

$$= \sum_{n=0}^{\infty} -(x-1)^{n+1}$$

$\Rightarrow$ coefficients of all terms are -1.

22. Evaluate $\displaystyle\lim_{x\to 0} \frac{\cos(x^2) - 1 + \frac{x^4}{2}}{x^8}$.

A. $\frac{1}{2}$
B. $\frac{1}{8}$
C. $\frac{1}{6}$
D. $\frac{1}{120}$
E. $\frac{1}{24}$

$$= \lim_{x\to 0}\left(\frac{\left(\cancel{1}-\cancel{\frac{(x^2)^2}{2!}}+\frac{(x^2)^4}{4!}-\frac{(x^2)^6}{6!}+\cdots\right)\cancel{-1}+\cancel{\frac{x^4}{2}}}{x^8}\right)$$

$$= \lim_{x\to 0}\left(\frac{+\frac{x^8}{4!}-\frac{x^{12}}{6!}+\frac{x^{16}}{8!}+\cdots}{x^8}\right)$$

$$= \lim_{x\to 0}\frac{\cancel{x^8}\left(\frac{1}{4!}-\frac{x^4}{6!}+\frac{x^8}{8!}-\cdots\right)}{\cancel{x^8}\left(1\right)}$$

$$= \frac{\frac{1}{4!}}{1} = \frac{1}{24}$$

$$\frac{1}{4!}+\frac{1}{2} = \frac{1}{24}+\frac{1}{2} = \frac{13}{24}$$

23. Find the slope of the tangent line to the curve described by $x = \ln\ t,\ y = 1 + t^2$ at $t = 1$.

$t=1 \rightarrow (x,y) = (\ln 1, 1+1^2) = (0,2)$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t}{\frac{1}{t}}$. $\left.\frac{dy}{dx}\right|_{t=1} = 2$

tangent line : $y - 2 = 2(x-0)$

A. 0

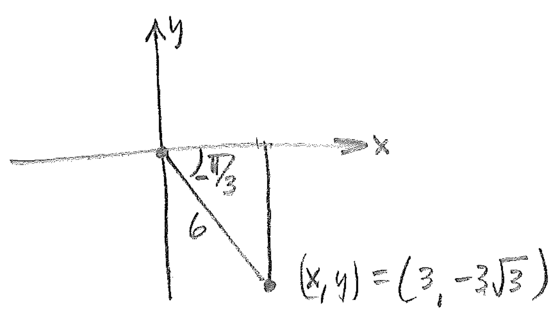
B. 3

C. -1

D. 2

E. $\frac{1}{3}$

24. A point P has Cartesian coordinates $(x, y) = (3, -3\sqrt{3})$. Which of the following gives polar coordinates of P?



$r = \pm\sqrt{x^2+y^2} = \pm\sqrt{9+27} = \pm 6$

$\tan\theta = \frac{y}{x} = \frac{-3\sqrt{3}}{3} = -\sqrt{3} \rightarrow \theta = -\pi/3, \ldots$

A. $\left(-6, \frac{\pi}{3}\right)$

B. $\left(6, -\frac{\pi}{3}\right)$

C. $\left(6, \frac{\pi}{6}\right)$

D. $\left(-6, \frac{\pi}{6}\right)$

E. $\left(6, -\frac{\pi}{6}\right)$

25. Identify the curve $r^2 = r\tan(\theta)\sec(\theta)$ by finding the Cartesian equation for it.

A. ellipse

B. line

C. circle

D. parabola

E. hyperbola

$r^2 = r \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta}$

$\rightarrow (r\cos\theta)^2 = r\sin\theta$

$\rightarrow x^2 = y$

$\rightarrow$ parabola