MA 162 **Exam 3** **Spring 2008**

Name SOLUTIONS

10-digit PUID

RECITATION Division and Section Numbers

Recitation Instructor

Instructions:

1. Fill in all the information requested above and on the scantron sheet.
2. This booklet contains 22 problems. Problems 1 - 17 are worth 4 points each, problems 18 - 20 are worth 6 points each and problems 21 and 22 are worth 7 points each. The maximum score is 100 points.
3. For each problem mark your answer on the scantron sheet and also circle it in this booklet.
4. Work only on the pages of this booklet.
5. Books, notes, calculators or any electronic devices are not to be used on this test.

1. If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

$\sum_{n=1}^{\infty} \frac{1}{n}$ diverges and $\lim_{n\to\infty} \frac{1}{n} = 0$

A. True
B. False

2. If $\sum_{n=1}^{\infty} |a_n|$ converges then $\sum_{n=1}^{\infty} a_n$ converges.

Absolute convergence implies convergence.

A. True
B. False

3. If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 2$, then $\sum_{n=1}^{\infty} a_n$ converges.

Root Test: $\lim_{n\to\infty} |a_n|^{1/n} = L = \begin{cases} <1 \rightarrow \text{conv.} \\ >1 \rightarrow \text{div.} \\ =1 \rightarrow \text{no conclusion} \end{cases}$

A. True
B. False

4. If $\lim_{n\to\infty} \frac{\frac{1}{n}}{a_n} = 2$, then $\sum_{n=1}^{\infty} a_n$ diverges.

Limit comparison test and $\sum^{\infty} \frac{1}{n}$ diverges

A. True
B. False

5. If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{2}$, then $\sum_{n=1}^{\infty} a_n$ converges.

Ratio Test: $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L = \begin{cases} <1 \rightarrow \text{conv.} \\ >1 \rightarrow \text{div.} \\ =1 \rightarrow \text{inconclusive} \end{cases}$

A. True
B. False

6. If $a_n > b_n \geq 0$ for all n and $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

$0 \leq \frac{1}{n^2} < \frac{1}{n}$ and $\sum^{\infty} \frac{1}{n}$ diverges, but $\sum^{\infty} \frac{1}{n^2}$ converges.

A. True
B. False

7. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+n}$ converges absolutely.

$\sum^{\infty} \frac{1}{n^2+n}$ converges

Comparison Test with $\sum^{\infty} \frac{1}{n^2}$
or
Limit Comparison Test with $\sum^{\infty} \frac{1}{n^2}$

A. True
B. False

8. $\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$ converges conditionally.

$\sum^{\infty} \frac{1}{n^2}$ converges and $\frac{|\sin(n)|}{n^2} < \frac{1}{n^2} \rightarrow \sum^{\infty} \frac{\sin(n)}{n^2}$
converges absolutely.

A. True
B. False

9. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges.

$\sum^{\infty} \frac{1}{\sqrt{n}} = \sum^{\infty} \frac{1}{n^{1/2}}$ divergent p-series, $p = \frac{1}{2} < 1$.

A. True
B. False

10. $\sum_{n=1}^{\infty} \left(\frac{5}{4}\right)^{n-1}$ converges.

$\sum^{\infty} \left(\frac{5}{4}\right)^{n-1}$ divergent geometric series, $r = \frac{5}{4} > 1$.

A. True
B. False

11. $\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n}$ converges absolutely.

$\sum^{\infty} \frac{1}{2^n} = \sum^{\infty} \left(\frac{1}{2}\right)^n$ convergent geom. series,
$r = \frac{1}{2} < 1$.

A. True
B. False

12. $\sum_{n=1}^{\infty} a_n = \lim_{N\to\infty} \left(\sum_{n=1}^{N} a_n\right)$.

An infinite series is the limit of its sequence
of partial sums.

A. True
B. False

13. $\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$ converges.

$\int_2^{\infty} \frac{1}{x\ln(x)}\,dx = \lim_{t\to\infty} \int_2^t \frac{1}{x\ln(x)}\,dx = \lim_{t\to\infty} \left(\ln(\ln x)\Big|_2^t\right)$

$= \lim_{t\to\infty} \left(\ln(\ln(t)) - \ln(\ln(2))\right) = \infty - 2 = \infty$

A. True

B. False

14. $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$ diverges.

$\sum^{\infty} \frac{1}{n}$ diverges and $\lim_{n\to\infty} \frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}} = 1 > 0,$

A. True

B. False

15. If $f(x) = 4 + x - x^2 \boxed{+ x^3} - x^4 + \cdots$, then $f'''(0) = 6$.

$\frac{f'''(0)}{3!} = 1 \longrightarrow f'''(0) = 3! = 6$

A. True

B. False

16. If $f(x) = \sum_{n=0}^{\infty} \frac{n}{(n+1)!} x^n$, then $f^{(5)}(0) = \frac{1}{6}$.

$\frac{f^{(n)}(0)}{n!} = \frac{n}{(n+1)!} \Rightarrow \frac{f^{(5)}(0)}{5!} = \frac{5}{6!} \Rightarrow f^{(5)}(0) = \frac{5(5!)}{6!} = \frac{5}{6}$

A. True

B. False

17. The radius of convergence of the series $\sum_{n=0}^{\infty} (2x)^n$ is 2.

ROOT TEST :

$\lim_{n\to\infty} \left|(2x)^n\right|^{1/n} = \lim_{n\to\infty} 2|x| = 2|x|.$

$2|x| < 1 \rightarrow |x| < \frac{1}{2} \rightarrow$ radius of conv. $= \frac{1}{2}$

A. True

B. False

18. $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^{n-2}}{3^n} =$

$$= \sum_{n=1}^{\infty} \frac{(-2)^{-1}}{3^1} \frac{(-2)^{n-1}}{3^{n-1}} = \sum_{n=1}^{\infty} \left(-\frac{1}{6}\right)\left(\frac{-2}{3}\right)^{n-1}$$

$$= \frac{-\frac{1}{6}}{1-\left(-\frac{2}{3}\right)} = \frac{-\frac{1}{6}}{\frac{5}{3}} = \left(-\frac{1}{6}\right)\left(\frac{3}{5}\right) = -\frac{1}{10}$$

A. $-\frac{3}{5}$

B. $-\frac{1}{5}$

C. $-\frac{2}{5}$

D. $-\frac{6}{5}$

E. $-\frac{1}{10}$

19. The interval of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2} (x+1)^n$ is.

Ratio Test: $\displaystyle\lim_{n\to\infty} \left| \frac{(-1)^n (x+1)^{n+1}}{(n+1)^2} \cdot \frac{n^2}{(-1)^n (x+1)^n} \right|$

$$= \lim_{n\to\infty} \frac{n^2}{(n+1)^2} |x+1| = |x+1|.$$

Series converges for $|x+1| < 1 \to -1 < x+1 < 1 \to -2 < x < 0$

end-points $\begin{cases} x=-2 \to \sum^{\infty} \frac{(-1)^n}{n^2}(-1)^n = \sum^{\infty} \frac{1}{n^2} \text{ conv.} \\ x=0 \to \sum^{\infty} \frac{(-1)^n}{n^2} 1^n \text{ conv.} \end{cases}$

I.O.C. is $[-2, 0]$

A. $[-2, 0]$

B. $(-2, 0]$

C. $[0, 2]$

D. $(0, 2]$

E. $[0, 2)$

20. If $\dfrac{1}{1+2x} = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots$ then $c_3 =$

$f(x) = \dfrac{1}{1+2x}$

$f'(x) = \dfrac{-1}{(1+2x)^2}(2)$

$f''(x) = \dfrac{2}{(1+2x)^3} 2^2$

$f'''(x) = \dfrac{-6}{(1+2x)^4} 2^3$

$$c_3 = \frac{f'''(0)}{3!} = \frac{(-6)2^3}{6} = -8$$

A. $\dfrac{8}{3!}$

B. $-\dfrac{8}{3!}$

C. 8

D. -8

E. 4

21. Using power series, the smallest number of terms needed to approximate $\int_0^{1/10} \frac{1}{1+x^2}\,dx$ to within 10^{-6} is

$$\int_0^{\frac{1}{10}} \frac{1}{1+x^2}\,dx = \int_0^{\frac{1}{10}} \frac{1}{1-(-x^2)}\,dx$$

$$= \int_0^{\frac{1}{10}} (1 - x^2 + x^4 - x^6 + x^8 - \cdots)\,dx$$

$$= x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \cdots \Big|_0^{\frac{1}{10}}$$

$$= \frac{1}{10} - \frac{1}{3(10)^3} + \frac{1}{5(10)^5} - \frac{1}{7(10)^7} + \frac{1}{9(10)^9} - \cdots$$

$$\frac{1}{7(10)^7} < \frac{1}{10^6} \text{ but } \frac{1}{5(10)^5} > \frac{1}{10^6} \Rightarrow \text{smallest \# of terms is } 3$$

A. 1
B. 2
C. 3
D. 4
E. 5

22. The first 4 nonzero terms of the power series representation for $f(x) = (1+x)^{-3}$ are

A. $1 - 3x + 6x^2 - 10x^3$
B. $1 - 3x + 12x^2 - 60x^3$
C. $1 - 3x + 6x^2 - 6x^3$
D. $1 - 3x + 3x^2 - x^3$
E. $1 - 3x + 4x^2 - 8x^3$

$$\frac{1}{1+x} = \frac{1}{1-(-x)} = 1 - x + x^2 - x^3 + x^4 - x^5 + \cdots$$

$$\frac{-1}{(1+x)^2} = \frac{d}{dx}\left(\frac{1}{1+x}\right) = 0 - 1 + 2x - 3x^2 + 4x^3 - 5x^4 + \cdots$$

$$\frac{2}{(1+x)^3} = \frac{d}{dx}\left(\frac{-1}{(1+x)^2}\right) = 0 - 0 + 2 - 6x + 12x^2 - 20x^3 + \cdots$$

$$\rightarrow \frac{1}{(1+x)^3} = 1 - 3x + 6x^2 - 10x^3 + \cdots$$