1. $\int \frac{x+13}{x^2+5x-6}\,dx = \int \frac{2}{x-1} - \frac{1}{x+6}\,dx = 2\ln|x-1| - \ln|x+6| + C$

A. $\ln\left|\frac{x-1}{(x+6)^2}\right| + C$

B. $\ln\left|\frac{x+3}{x+2}\right| + C$

C. $\ln\left|\frac{2(x-1)}{x+6}\right| + C$

D. $\boxed{\ln\left|\frac{(x-1)^2}{x+6}\right| + C}$

E. $\ln\left|\frac{x+2}{x+3}\right| + C$

$\frac{x+13}{x^2+5x-6} = \frac{A}{x-1} + \frac{B}{x+6}$

$x+13 = (A+B)x + 6A - B$

$A = 2$

$B = -1$

2. $\int \frac{2x^3+5x^2+8x+4}{(x^2+2x+2)^2}\,dx = \int \frac{Ax+B}{x^2+2x+2} + \frac{Cx+D}{(x^2+2x+2)^2}\,dx$

A. $\int \frac{2x+2}{x^2+2x+2}\,dx + \int \frac{2x+1}{(x^2+2x+2)^2}\,dx$

B. $\boxed{\int \frac{2x+1}{x^2+2x+2}\,dx + \int \frac{2x+2}{(x^2+2x+2)^2}\,dx}$

C. $\int \frac{2x}{x^2+2x+2}\,dx + \int \frac{2x+1}{(x^2+2x+2)^2}\,dx$

D. $\int \frac{2x}{x^2+2x+2}\,dx + \int \frac{2x+2}{(x^2+2x+2)^2}\,dx$

E. $\int \frac{2x+1}{x^2+2x+2}\,dx + \int \frac{2x}{(x^2+2x+2)^2}\,dx$

$2x^3+5x^2+8x+4 = (Ax+B)(x^2+2x+2) + Cx+D$

$= Ax^3 + (B+2A)x^2 + (2A+2B+C)x + (2B+D)$

$A = 2$

$B = 1$

$C = 2$

$D = 2$

3. The curve $y = \sin x$, $0 \le x \le \frac{\pi}{2}$, is rotated around the x-axis to generate a surface S. Which of the following formulas represents the surface area of S?

A. $\boxed{\int_0^{\frac{\pi}{2}} 2\pi \sin x \sqrt{1+\cos^2 x}\, dx}$

B. $\int_0^1 2\pi y \sqrt{1+(\cos^{-1} y)^2}\, dy$

C. $\int_0^1 2\pi \sin^{-1} y \sqrt{1+\frac{1}{1-y^2}}\, dy$

D. $\int_0^{\frac{\pi}{2}} 2\pi x \sqrt{1+\cos^2 x}\, dx$

E. $\int_0^{\frac{\pi}{2}} 2\pi \sin x \sqrt{1+\frac{1}{1-x^2}}\, dx$

$S.A. = \int 2\pi r\, ds$

$r = y = \sin x$

$ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx = \sqrt{1+\cos^2 x}\, dx$

4. $\int_0^{2\sqrt{2}} \frac{x^2\, dx}{\sqrt{16-x^2}} = \int_0^{\pi/4} \frac{(4\sin\theta)^2\, 4\cos\theta\, d\theta}{\sqrt{16-16\sin^2\theta}} = \int_0^{\pi/4} \frac{(16\sin^2\theta)(4\cos\theta)}{4\cos\theta}\, d\theta$

A. 2π

B. $2\pi - 2$

C. $2\pi + 2$

D. $\boxed{2\pi - 4}$

E. $2\pi + 4$

$x = 4\sin\theta$

$dx = 4\cos\theta\, d\theta$

$x = 2\sqrt{2} = 4\sin\theta \rightarrow \frac{\sqrt{2}}{2} = \sin\theta \rightarrow \theta = \pi/4$

$x = 0 = 4\sin\theta \rightarrow 0 = \sin\theta \rightarrow \theta = 0$

$= \int_0^{\pi/4} 16\sin^2\theta\, d\theta = \int_0^{\pi/4} 16\left(\frac{1-\cos 2\theta}{2}\right) d\theta$

$= \int_0^{\pi/4} 8 - 8\cos 2\theta\, d\theta = 8\theta - 4\sin 2\theta \Big|_0^{\pi/4}$

$= 8\left(\frac{\pi}{4}\right) - 4\sin\left(\frac{\pi}{2}\right) - 8(0) + 4\sin(0)$

$u = 2\sqrt{t}$

$du = \frac{1}{\sqrt{t}}\,dt$

$t = \frac{1}{4} \to u = 1$

$t = \frac{1}{12} \to u = \frac{1}{\sqrt{3}}$

5. $\displaystyle\int_{\frac{1}{12}}^{\frac{1}{4}} \frac{12\,dt}{\sqrt{t} + 4t\sqrt{t}} = \int_{1/\sqrt{3}}^{1} \frac{12}{1+u^2}\,du = 12\tan^{-1}u\Big|_{1/\sqrt{3}}^{1} = 12\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$= 3\pi - 2\pi$

A. 6π

B. 4π

C. $\boxed{\pi}$

D. 3π

E. 2π

6. Find the length of the curve given by

$$x^{2/3} + y^{2/3} = 1$$

for $0 \le x \le 1$ and $0 \le y \le 1$.

$y = \left[1 - x^{2/3}\right]^{3/2}$

$\frac{dy}{dx} = \frac{3}{2}\left[1 - x^{2/3}\right]^{1/2}\left(-\frac{2}{3}x^{-1/3}\right)$

$= -x^{-1/3}\left(1 - x^{2/3}\right)^{1/2}$

A. $\frac{4}{3}$

B. $\frac{8}{5}$

C. $\frac{13}{9}$

D. $\boxed{\frac{3}{2}}$

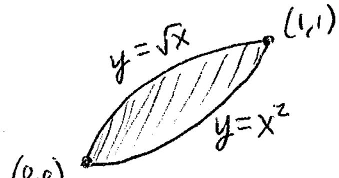
E. $\frac{5}{3}$

$L = \int ds = \int_0^1 \sqrt{1 + \left[\left(-x^{-1/3}\right)\left(1 - x^{2/3}\right)^{1/2}\right]^2}\,dx$

$= \int_0^1 \sqrt{1 + x^{-2/3}\left(1 - x^{2/3}\right)}\,dx$

$= \int_0^1 \sqrt{1 + x^{-2/3} - 1}\,dx = \int_0^1 \sqrt{x^{-2/3}}\,dx$

$= \int_0^1 x^{-1/3}\,dx = \frac{3}{2}x^{2/3}\Big|_0^1$



7. Which expression represents the y-coordinate of the centroid of the region of the plane bounded by $y = x^2$ and $x = y^2$?

A. $\dfrac{\int_0^1 (x - x^{3/2})\,dx}{\int_0^1 (\sqrt{x} - x^2)\,dx}$

B. $\dfrac{\int_0^1 (x - x^{3/2})\,dx}{2\int_0^1 (\sqrt{x} - x^2)\,dx}$

C. $\dfrac{\int_0^1 (x^3 - x^{3/2})\,dx}{\int_0^1 (\sqrt{x} - x^2)\,dx}$

D. $\dfrac{\int_0^1 (x^{3/2} - x^3)\,dx}{2\int_0^1 (\sqrt{x} - x^2)\,dx}$

E. $\boxed{\dfrac{\int_0^1 (x - x^4)\,dx}{2\int_0^1 (\sqrt{x} - x^2)\,dx}}$

$$M_x = \int_0^1 \frac{1}{2}\left[(\sqrt{x})^2 - (x^2)^2\right] dx$$

$$= \frac{1}{2}\int_0^1 (x - x^4)\,dx$$

$$A = \int_0^1 \sqrt{x} - x^2 \,dx$$

$$\bar{y} = \frac{M_x}{A}$$

8. $\displaystyle\int_0^1 \frac{1}{\sqrt{x}}\,dx = \lim_{a\to 0^+} \int_a^1 x^{-1/2}\,dx = \lim_{a\to 0^+} 2x^{1/2}\Big|_a^1$

$$= \lim_{a\to 0^+} 2 - 2\sqrt{a}$$

$$= 2 - 0 = 2$$

A. 1

B. $\frac{2}{3}$

C. $\boxed{2}$

D. $\frac{1}{2}$

E. Divergent

9. Which statement is true for the following sequence?

$$a_n = \sqrt{n+1} - \sqrt{n}$$

where $n = 1, 2, \ldots$

A. The sequence is divergent and decreasing.

B. The sequence is divergent and not monotonic.

C. The sequence is convergent and not monotonic.

D. The sequence is divergent and increasing.

E. $\boxed{\text{The sequence is convergent and decreasing.}}$

$$\lim_{n\to\infty} \left(\sqrt{n+1} - \sqrt{n}\right)\left(\frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}}\right)$$

$$= \lim_{n\to\infty} \frac{n+1-n}{\sqrt{n+1}+\sqrt{n}} = \lim_{n\to\infty} \frac{1}{\sqrt{n+1}+\sqrt{n}}$$

$$= 0.$$

If $f(x) = \sqrt{x+1} - \sqrt{x}$,

$$f'(x) = \frac{1}{2\sqrt{x+1}} - \frac{1}{2\sqrt{x}}$$

$$= \frac{\sqrt{x} - \sqrt{x+1}}{2\sqrt{x}\sqrt{x+1}} < 0.$$

(since $x+1 > x$)

10. $\frac{1}{e} + \frac{1}{e^2} + \frac{1}{e^3} + \frac{1}{e^4} + \ldots$

A. converges to $\frac{e}{1-e}$

B. converges to $\frac{1}{1-e}$

C. diverges

D. converges to $\frac{e}{e-1}$

E. $\boxed{\text{converges to } \frac{1}{e-1}}$

$$\frac{a}{1-r} = \frac{1/e}{1-1/e}$$

$$= \frac{1/e}{1-1/e} \cdot \frac{e}{e}$$

$$= \frac{1}{e-1}$$

since $e > 1$, $|r| = \frac{1}{e} < 1$.

11. Only one of these series converges. Which one?

A. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ $\quad p = \frac{1}{2} \leq 1$

B. $\sum_{n=2}^{\infty} \frac{e^n}{\ln n}$ $\quad \lim_{n\to\infty} \frac{e^n}{\ln n} \neq 0$

C. $\boxed{\sum_{n=1}^{\infty} \frac{2^n}{n+3^n}}$ $\quad \boxed{\text{D.C.T.}}\ \frac{2^n}{n+3^n} < \left(\frac{2}{3}\right)^n$ and $\sum \left(\frac{2}{3}\right)^n$ converges $\left(|r| = \frac{2}{3} < 1\right)$.

D. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ $\quad \int_2^{\infty} \frac{1}{x \ln x}\, dx = \lim_{b\to\infty} \ln(\ln x)\Big|_2^b = \infty$

E. $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$ $\quad \boxed{\text{L.C.T.}}\ \lim_{n\to\infty} \frac{\sin\left(\frac{1}{n}\right)}{1/n} = 1$ and $\sum \frac{1}{n}$ diverges.

12. Assume each a_n is positive. Which one of the following conditions guarantees that the series $\sum_{n=1}^{\infty} a_n$ diverges?

A. $\lim_{n\to\infty} \frac{a_n}{1/2^n} = 1$ $\quad \sum a_n$ converges by L.C.T. with $\sum \frac{1}{2^n}$ $\left(|r| = \frac{1}{2} < 1\right)$

B. $a_n > \frac{1}{n^2}$ for all n $\quad$ Inconclusive

C. $a_n < \frac{1}{n}$ for all n $\quad$ Inconclusive

D. $\boxed{\lim_{n\to\infty} \frac{a_n}{\sqrt{2^n}} = 1}$ $\quad \sum \sqrt{2^n} = \sum (\sqrt{2})^n$ diverges $\left(|r| = \sqrt{2} > 1\right)$, so by L.C.T., $\sum a_n$ diverges

E. $\lim_{n\to\infty} \frac{a_n}{1/n^2} = \infty$ $\quad$ Inconclusive