MA162 — EXAM I — FALL 2016 — SEPTEMBER 22, 2016
TEST NUMBER 01

INSTRUCTIONS:

1. **Do not open the exam booklet until you are instructed to do so.**
2. **Before you open the booklet fill in the information below and use a # 2 pencil to fill in the required information on the scantron.**
3. MARK YOUR TEST NUMBER ON YOUR SCANTRON
4. Once you are allowed to open the exam, make sure you have a complete test. There are 7 different test pages (including this cover page).
5. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers on this test booklet.
6. Each problem is worth 100/12 points. The maximum possible score is 100 points. No partial credit.
7. Do not leave the exam room during the first 20 minutes of the exam.
8. If you do not finish your exam in the first 50 minutes, you must wait until the end of the exam period to leave the room.
9. After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

DON'T BE A CHEATER:

1. Do not give, seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only your instructor.
2. Do not look at the exam or scantron of another student.
3. Do not allow other students to look at your exam or your scantron.
4. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
5. Do not consult notes or books.
6. **Do not handle** phones or cameras, calculators or any electronic device until after you have finished your exam, handed it in to your instructor and left the room.
7. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs collect the scantrons and the exams.
8. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty include an F in the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME: SOLUTIONS.

STUDENT SIGNATURE: ______________________

STUDENT ID NUMBER: ______________________

SECTION NUMBER AND RECITATION INSTRUCTOR: ______________________

1. Find the center and the radius of the sphere $4x^2 + 4y^2 + 4z^2 - 5x + 10y - 6z = 0$

A. Center $(\frac{5}{8}, -\frac{5}{4}, \frac{3}{4})$ and radius $\frac{\sqrt{135}}{8}$

B. Center $(\frac{5}{8}, -\frac{5}{4}, \frac{3}{4})$ and radius $\frac{\sqrt{161}}{8}$

C. Center $(\frac{5}{8}, -\frac{5}{4}, -\frac{3}{4})$ and radius $\frac{\sqrt{161}}{8}$

D. Center $(-\frac{5}{8}, \frac{5}{4}, -\frac{3}{4})$ and radius $\frac{\sqrt{135}}{8}$

E. Center $(-\frac{5}{8}, \frac{5}{4}, -\frac{3}{4})$ and radius $\frac{\sqrt{161}}{8}$

Divide the equation by 4.

$x^2 + y^2 + z^2 - \frac{5}{4}x + \frac{5}{2}y - \frac{3}{2}z = 0$

But

$x - \frac{5}{4}x = (x - \frac{5}{8})^2 - \frac{25}{64}$

$y^2 + \frac{5}{2}y = (y + \frac{5}{4})^2 - \frac{25}{16}$

$z^2 - \frac{3}{2}z = (z - \frac{3}{4})^2 - \frac{9}{16}$.

So the equation becomes.

$(x - \frac{5}{8})^2 + (y + \frac{5}{4})^2 + (z - \frac{3}{4})^2 = \frac{25}{64} + \frac{25}{16} + \frac{9}{16} = \frac{161}{64}$

Radius $\frac{\sqrt{161}}{8}$, Center $(\frac{5}{8}, -\frac{5}{4}, \frac{3}{4})$

2. The measure of the angle (in radians) between the vectors

$$\vec{u} = 2\vec{i} + \vec{j} + \vec{k} \text{ and } \vec{v} = -\sqrt{\frac{6}{5}}\,\vec{i} - \vec{j} + \vec{k} \quad \text{is equal to}$$

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. $\frac{2\pi}{3}$

D. $\frac{5\pi}{6}$

E. $\frac{3\pi}{4}$

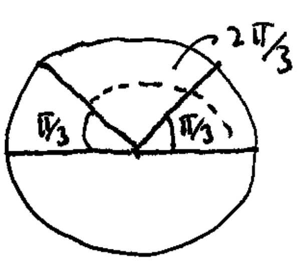
$\vec{u} \cdot \vec{v} = |\vec{u}| \cdot |\vec{v}| \cdot \cos\theta$.

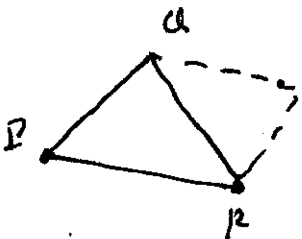
$\vec{u} \cdot \vec{v} = -2\sqrt{\frac{6}{5}} - 1 + 1 = -2\sqrt{\frac{6}{5}}$.

$|\vec{u}| = \sqrt{4+1+1} = \sqrt{6}$; $|\vec{v}| = \sqrt{\frac{6}{5} + 1 + 1} = \frac{4}{\sqrt{5}}$.

So we have : $-2\frac{\sqrt{6}}{\sqrt{5}} = \sqrt{6} \cdot \frac{4}{\sqrt{5}} \cos\theta$.

$\boxed{\cos\theta = -\frac{1}{2}}$ $\quad \theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$





3. The area of the triangle with vertices $P(1,0,2)$, $Q(2,-1,1)$ and $R(0,1,-1)$ is equal to

$\boxed{\text{A. } 2\sqrt{2}}$

B. $\sqrt{5}$

C. $4\sqrt{2}$

D. $2\sqrt{3}$

E. $3\sqrt{2}$

$\vec{PQ} = Q - P = \langle 1, -1, -1 \rangle$

$\vec{PR} = R - P = \langle -1, 1, -3 \rangle$

Area of the triangle $= \frac{1}{2} |\vec{PQ} \times \vec{PR}|$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & -1 \\ -1 & 1 & -3 \end{vmatrix}$$

$$= \vec{i}(4) - \vec{j}(-4) + \vec{k}(0) = 4\vec{i} + 4\vec{j}$$

$|\vec{PQ} \times \vec{PR}| = \sqrt{16+16} = \sqrt{2 \times 16} = 4\sqrt{2}$

Area of the triangle $= 2\sqrt{2}$

4. The area of the region bounded by $y = x(2-x)$ and $y = x^2$ is equal to

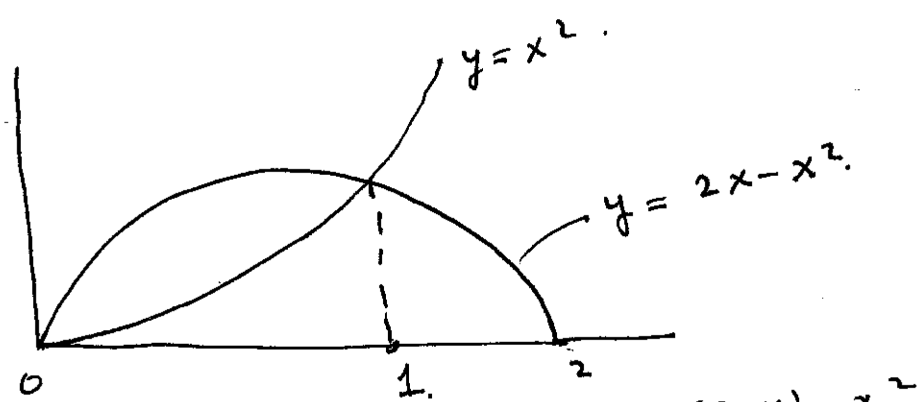
A. $\frac{1}{4}$

B. $\frac{2}{3}$

C. $\frac{1}{2}$

D. $\frac{2}{5}$

$\boxed{\text{E. } \frac{1}{3}}$



$x(2-x) = x^2$

$x = 0$; $2 - x = x$

$x = 1$

$$A = \int_0^1 (2x - x^2 - x^2)\, dx = \int_0^1 (2x - 2x^2)\, dx$$

$$= 2\left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_0^1 = 2\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{1}{3}$$

5. Find the volume of the solid whose base is the region bounded by $y = x^2$ and $y = 1$, and whose cross-sections are squares that are perpendicular to the y-axis.

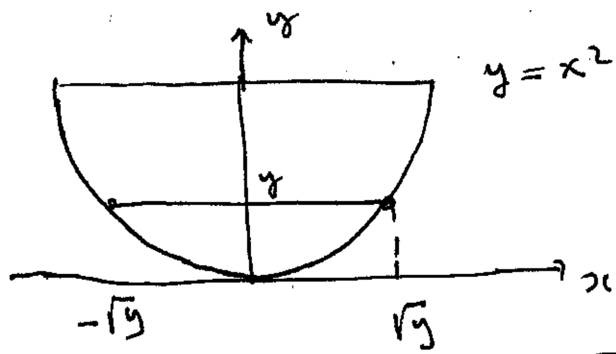
A. 1

$\boxed{\text{B. } 2}$

C. 4

D. 6

E. 3



length of the base: $2\sqrt{y}$.

Area of the cross section $= 4y$.

Volume of the solid $= \int_0^1 4y\,dy = 2$.

6. The volume of the solid obtained by rotating the region bounded by $y = x + 1$, $x = 0$ and $y = 3$ about the y-axis is equal to

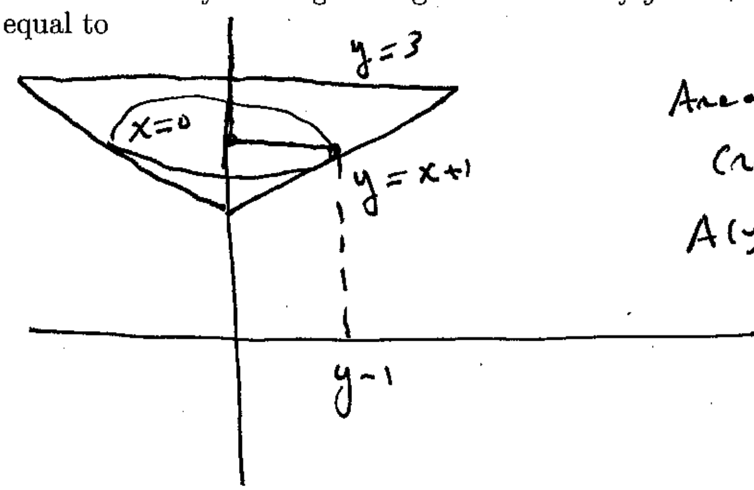
A. $\frac{2\pi}{3}$

B. $\frac{4\pi}{3}$

C. $\frac{5\pi}{3}$

D. $\frac{7\pi}{3}$

$\boxed{\text{E. } \frac{8\pi}{3}}$



Area of the cross section

$A(y) = \pi (y-1)^2$

$$V = \int_1^3 \pi (y-1)^2\,dy = \pi \int_0^2 y^2\,dy$$

$$= \frac{\pi}{3} \cdot y^3 \Big|_0^2 = \frac{8\pi}{3}$$

7. If one uses the method of cylindrical shells to find the volume of the region bounded by the curves $y = \sqrt{x}$ and $y = x^2$ rotated about the x-axis, one arrives at the following integral

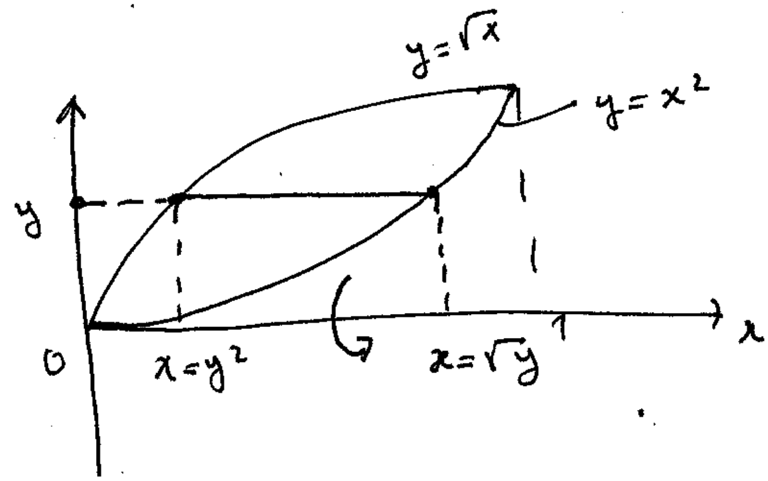
A. $2\pi \int_0^1 (y^{\frac{3}{2}} - \frac{1}{2}y^2)\, dy$

B. $2\pi \int_0^1 (y^{\frac{5}{2}} - \frac{1}{2}y^{\frac{3}{2}})\, dy$

$\boxed{\text{C.}}$ $2\pi \int_0^1 (y^{\frac{3}{2}} - y^3)\, dy$

D. $2\pi \int_0^1 (y^{\frac{3}{2}} - \frac{1}{2}y)\, dy$

E. $2\pi \int_0^1 y^2(y^{\frac{1}{2}} - \frac{1}{2}y)\, dy$



$$V = 2\pi \int_0^1 y(\sqrt{y} - y^2)\, dy$$

$$= 2\pi \int_0^1 (y^{3/2} - y^3)\, dy$$

8. A conical tank T is 5 meters high and the radius of its base is 4 meters long. The base of tank rests on the ground. If the tank is filled with a liquid of density ρ kg/m^3 (g is the acceleration of gravity), the work necessary to empty it by pumping the liquid through its top is

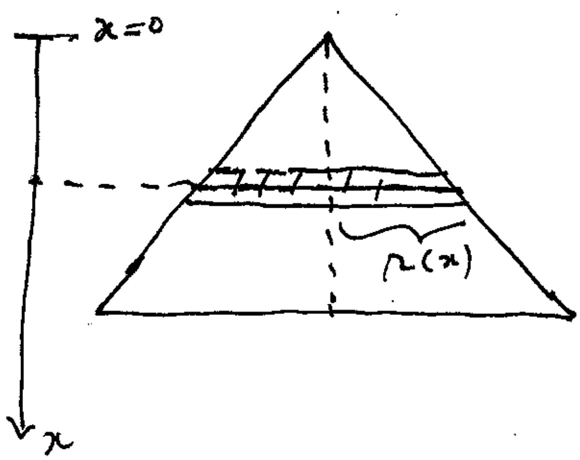
A. $50\rho\pi g$

B. $75\rho\pi g$

C. $80\rho\pi g$

—D $\boxed{\text{D}}$ $100\rho\pi g$

E. $120\rho\pi g$



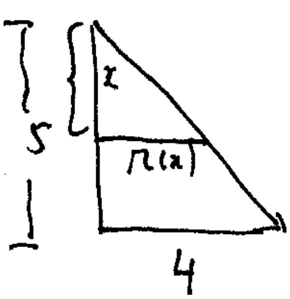
Weight of the slab with width dx

$= \rho\pi\, r(x)^2 \cdot dx \cdot g$

Work to move the slab to the top

$= x \cdot \pi\rho\, r(x)^2\, g \cdot dx.$

$= (4/5)^2 \pi\rho g\, x^3\, dx$



$\frac{r(x)}{x} = \frac{4}{5}$

$r = \frac{4}{5}x$

Total work:

$W = \frac{16}{25}\rho\pi g \int_0^5 x^3\, dx = \frac{16}{25}\pi g \times \frac{(5)^4}{4}$

$= 100\, \rho\pi g.$

9. The integral

$$\int_1^2 x \ln x dx \text{ is equal to}$$

$$\int_1^2 x \ln x\, dx = \int_1^2 \underbrace{\ln x}_{u} \cdot \underbrace{x\, dx}_{dV}$$

$u = \ln x$, $du = \frac{1}{x} dx$

$dV = x\, dx$; $V = x^2/2$

$$= \frac{x^2}{2}\ln x - \int \frac{x}{2}\, dx$$

$$= \frac{x^2}{2}\ln x - \frac{x^2}{4}.$$

A. $4 \ln 2 + 3/2$

$\boxed{\text{B.}}$ $2 \ln 2 - 3/4$

C. $4 \ln 2 - 3$

D. $2 \ln 2 - 3/2$

E. $\frac{1}{2} \ln 2 - 2$

$$\int_1^2 x \ln x\, dx = \frac{x^2}{2}\ln x - \frac{x^2}{4}\Big| = (2\ln 2 - 2) - (-1/4)$$

$$= 2\ln 2 - 3/4.$$

10. A force of 4 pounds stretches a spring with natural length of 12 inches to 18 inches. Find the total work by stretching the spring from a length of 18 inches to 24 inches:

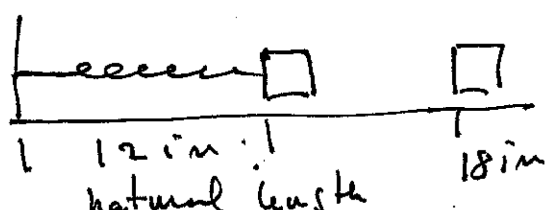
A. 5 ft-lb

$\boxed{\text{B.}}$ 3 ft-lb

C. 8 ft-lb

D. 6 ft-lb

E. $\frac{3}{2}$ ft-lb



6 in = ½ ft

$F = kx$

$4 = k/2$

$\boxed{k = 8}$

$$W = \int_{0.5}^{1} 8x\, dx = 4x^2\Big|_{0.5}^{1}$$

$$= 4(1 - (0.5)^2) = 4 - 1 = 3$$

11. Compute the value of the following definite integral

$$\int_0^{\frac{1}{\sqrt{2}}} x^2\sqrt{1-x^2}dx$$

$x = \sin\theta$

$dx = \cos\theta\, d\theta$

$1 - x^2 = \cos^2\theta$

A. $\frac{\pi}{12}$

B. $\frac{\pi}{8}$

$\boxed{\text{C.}}$ $\frac{\pi}{32}$

D. $\frac{2\pi}{16}$

E. $\frac{3\pi}{4}$

$$= \int_0^{\pi/4} \sin^2\theta \cdot \cos^2\theta \, d\theta$$

$$= \frac{1}{4}\int_0^{\pi/4} (\sin 2\theta)^2 \, d\theta$$

$$= \frac{1}{4}\int_0^{\pi/4} \frac{1-\cos 4\theta}{2} \, d\theta = \frac{1}{8} \cdot \pi/4 = \pi/32.$$

12. The integral $\int_0^{\pi/4} \tan^3 x \sec^2 x \, dx$ is equal to

Set $u = \tan x$

$du = \sec^2 x \, dx$

$\boxed{\text{A}}$ $\frac{1}{4}$

B. $\frac{\pi}{3}$

C. $\frac{1}{2}$

D. $\frac{2\pi}{5}$

E. $\frac{1}{3}$

$$= \int_0^1 u^3 \, du = \frac{u^4}{4}\Big|_0^1$$

$$= 1/4$$

MA162 — EXAM I — FALL 2016 — SEPTEMBER 22, 2016
TEST NUMBER 02

INSTRUCTIONS:

1. **Do not open the exam booklet until you are instructed to do so.**
2. **Before you open the booklet fill in the information below and use a # 2 pencil to fill in the required information on the scantron.**
3. MARK YOUR TEST NUMBER ON YOUR SCANTRON
4. Once you are allowed to open the exam, make sure you have a complete test. There are 7 different test pages (including this cover page).
5. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers on this test booklet.
6. Each problem is worth 100/12 points. The maximum possible score is 100 points. No partial credit.
7. Do not leave the exam room during the first 20 minutes of the exam.
8. If you do not finish your exam in the first 50 minutes, you must wait until the end of the exam period to leave the room.
9. After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

DON'T BE A CHEATER:

1. Do not give, seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only your instructor.
2. Do not look at the exam or scantron of another student.
3. Do not allow other students to look at your exam or your scantron.
4. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
5. Do not consult notes or books.
6. **Do not handle** phones or cameras, calculators or any electronic device until after you have finished your exam, handed it in to your instructor and left the room.
7. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs collect the scantrons and the exams.
8. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty include an F in the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME: SOLUTIONS.

STUDENT SIGNATURE: ______

STUDENT ID NUMBER: ______

SECTION NUMBER AND RECITATION INSTRUCTOR: ______

1. Find the center and the radius of the sphere $3x^2 + 3y^2 + 3z^2 - 6x - 5y + 2z = 0$

A. Center $(-1, -\frac{5}{6}, \frac{1}{3})$ and radius $\frac{\sqrt{75}}{6}$

B. Center $(1, \frac{5}{6}, -\frac{1}{3})$ and radius $\frac{\sqrt{75}}{6}$

C. Center $(1, \frac{5}{6}, \frac{1}{3})$ and radius $\frac{\sqrt{65}}{6}$

D. Center $(-1, \frac{5}{6}, -\frac{1}{3})$ and radius $\frac{\sqrt{65}}{8}$

$\boxed{\text{E.}}$ Center $(1, \frac{5}{6}, -\frac{1}{3})$ and radius $\frac{\sqrt{65}}{6}$

Divide the equation by 3:

$$x^2+y^2+z^2-2x-\tfrac{5}{3}y+\tfrac{2}{3}z=0$$

But

$$x^2-2x = (x-1)^2-1$$

$$y^2-\tfrac{5}{3}y = (y-\tfrac{5}{6})^2-\frac{25}{36}$$

$$z^2+\tfrac{2}{3}z = (z+\tfrac{1}{3})^2-\tfrac{1}{9}$$

So we obtain:

$$(x-1)^2+(y-\tfrac{5}{6})^2+(z+\tfrac{1}{3})^2 = 1+\frac{25}{36}+\frac{1}{9} = \frac{65}{36}$$

Center: $(1, \tfrac{5}{6}, -\tfrac{1}{3})$ Radius: $\frac{\sqrt{65}}{6}$

2. The measure of the angle (in radians) between the vectors

$$\vec{u} = 2\vec{i} + \vec{j} + \vec{k} \text{ and } \vec{v} = -\sqrt{6}\,\vec{i} - \vec{j} + \vec{k} \quad \text{is equal to}$$

$\boxed{\text{A}}$ $\frac{3\pi}{4}$

B. $\frac{\pi}{3}$

C. $\frac{2\pi}{3}$

D. $\frac{5\pi}{6}$

E. $\frac{\pi}{4}$

$$\vec{u}\cdot\vec{v} = |\vec{u}|\,.\,|\vec{v}|\,.\,\cos\theta.$$

$$\vec{u}\cdot\vec{v} = -2\sqrt{6}-1+1 = -2\sqrt{6}$$

$$|\vec{u}| = \sqrt{4+1+1} = \sqrt{6}$$

$$|\vec{v}| = \sqrt{6+1+1} = \sqrt{8}$$

$$-2\sqrt{6} = \sqrt{6}\,.\,\sqrt{8}\,.\,\cos\theta \qquad \cos\theta = -\frac{2}{\sqrt{8}}$$

$$\cos\theta = -\frac{1}{\sqrt{2}} \qquad \theta = \pi-\tfrac{\pi}{4} = \tfrac{3\pi}{4}.$$

3. The area of the triangle with vertices $P(1,0,2)$, $Q(2,-1,1)$ and $R(0,-1,-1)$ is equal to

A. $2\sqrt{2}$

$\boxed{\text{B.}}$ $\sqrt{6}$

C. $4\sqrt{2}$

D. $2\sqrt{6}$

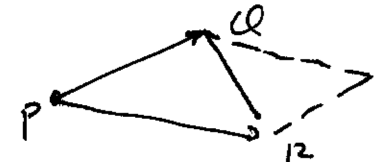
E. $4\sqrt{2}$

$\vec{PQ} = \langle 1, -1, -1 \rangle$

$\vec{PR} = \langle -1, -1, -3 \rangle$

$\vec{PQ} \times PR = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & -1 \\ -1 & -1 & -3 \end{vmatrix}$

$= i(2) - \vec{j}(-4) + \vec{k}(-2)$



Area of the triangle

$= \frac{1}{2} | \vec{PQ} \times \vec{PR} |$

; $|\vec{PQ} \times \vec{PR}| = \sqrt{4+4+16}$

$= \sqrt{24} = 2\sqrt{6}.$

Area of the triangle $= \sqrt{6}$

4. The area of the region bounded by $y = x(3-x)$ and $y = x^2$ is equal to

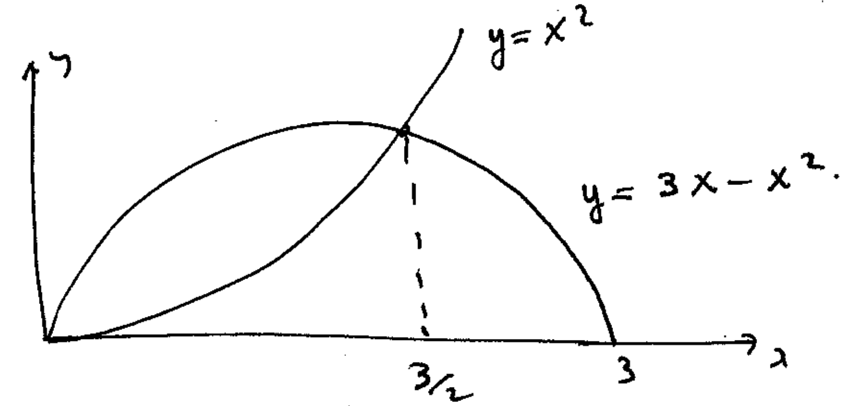
A. $\frac{2}{3}$

B. $\frac{5}{3}$

C. $\frac{1}{6}$

$\boxed{\text{D.}}$ $\frac{9}{8}$

E. $\frac{1}{3}$



$x^2 = x(3-x)$ $x = 0$

$x = 3 - x$; $x = 3/2$

$$A = \int_0^{3/2} (-x^2 - x^2 + 3x)\, dx$$

$$= -2\frac{x^3}{3} + \frac{3x^2}{2} = -\frac{2}{3}\cdot\left(\frac{3}{2}\right)^3 + \left(\frac{3}{2}\right)^3$$

$$= \left(3/2\right)^3 - \left(3/2\right)^2 = \frac{27}{8} - \frac{9}{4} = \frac{27-18}{8}$$

$$= 9/8$$

5. Find the volume of the solid whose base is the region bounded by $y = 4x^2$ and $y = 4$, and whose cross-sections are squares that are perpendicular to the y-axis.

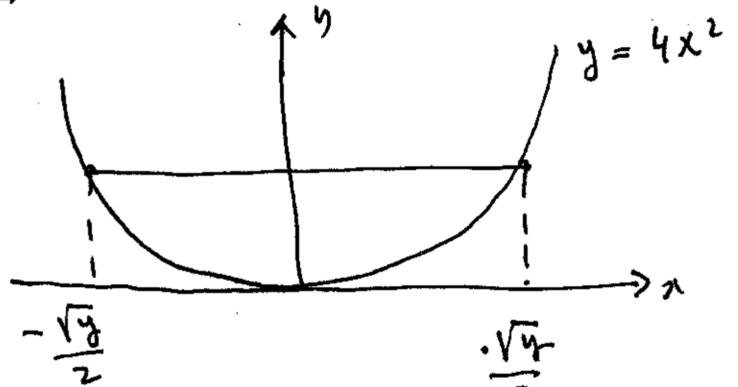
A. 2

B. 4

$\boxed{\text{C.}}$ 8

D. 6

E. 10



length of the side of the base $= \sqrt{y}$

Area of the cross section y.

$$V = \int_0^4 y\,dy = 8.$$

6. The volume of the solid obtained by rotating the region bounded by $y = x^2 + 1$, $x = 0$ and $y = 3$ about the y-axis is equal to

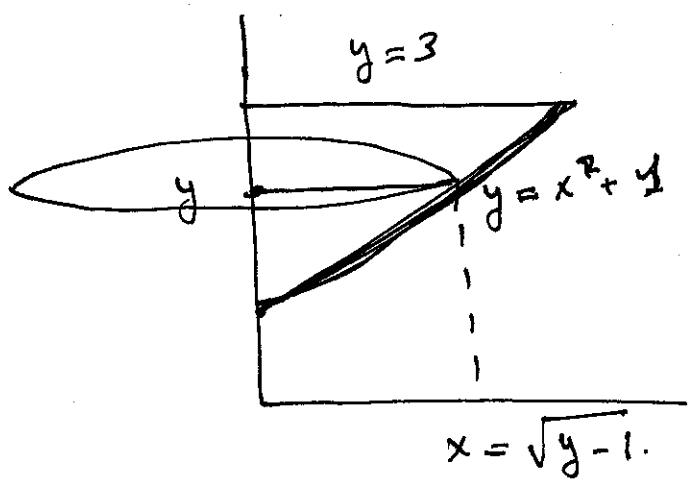
$\boxed{\text{A.}}$ 2π

B. 3π

C. 4π

D. 6π

E. 8π



Area of the cross section $A(y) = \pi(y-1)$

$$V = \int_1^3 \pi(y-1)\,dy = \pi \int_0^2 y\,dy = 2\pi.$$

7. If one uses the method of cylindrical shells to find the volume of the region bounded by the curves $y = 2x$ and $y = x^2$ rotated about the x-axis, one arrives at the following integral

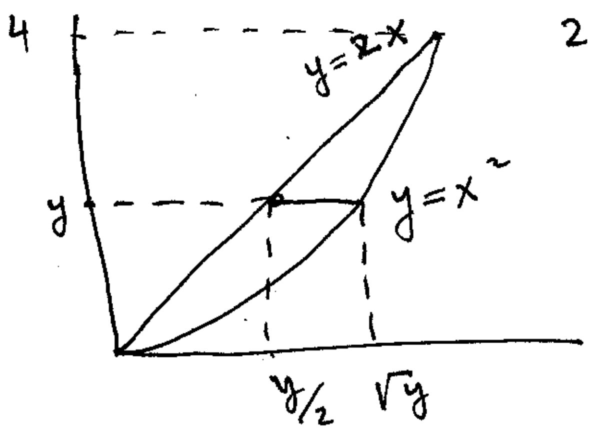
A. $2\pi \int_0^4 (y^{\frac{3}{2}} - \frac{1}{2}y^2)\, dy$

B. $2\pi \int_0^4 (y^{\frac{5}{2}} - \frac{1}{2}y^{\frac{3}{2}})\, dy$

C. $2\pi \int_0^4 (y^{\frac{3}{2}} - \frac{1}{3}y^3)\, dy$

D. $2\pi \int_0^4 (y^{\frac{1}{2}} - \frac{1}{2}y)\, dy$

E. $2\pi \int_0^4 y^2(y^{\frac{1}{2}} - \frac{1}{2}y)\, dy$



$2x = x^2$

$x = 0$

$x = 2$

$$V = 2\pi \int_0^4 y(\sqrt{y} - y/2)\,dy = 2\pi \int_0^4 (y^{3/2} - \tfrac{1}{2}y^2)\,dy$$

8. A conical tank T is 3 meters high and the radius of its base is 2 meters long. The base of tank rests on the ground. If the tank is filled with a liquid of density ρ kg/m^3 and g denotes the acceleration of gravity, the work necessary to empty it by pumping the liquid through its top is

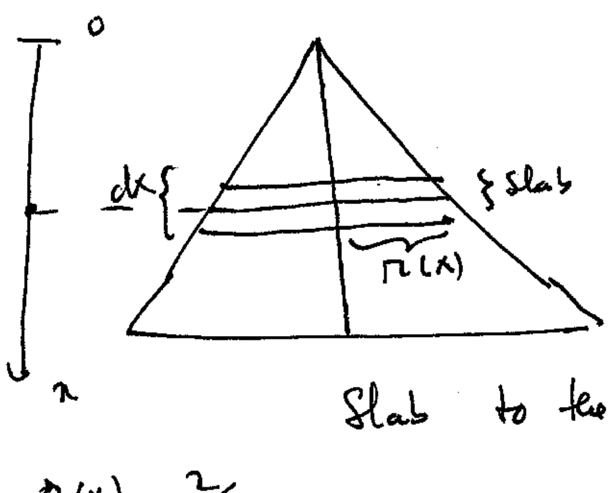
A. $7\rho\pi g$

B. $9\rho\pi g$

C. $15\rho\pi g$

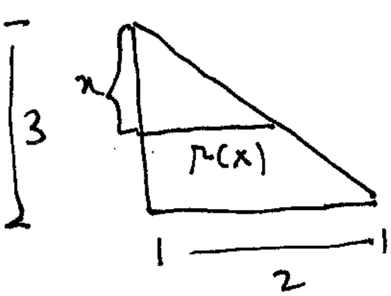
D. $12\rho\pi g$

E. $8\rho\pi g$



Weight of the slab
$= \text{Volume} \cdot \rho \cdot g$
$= \pi (r(x))^2 dx\, \rho \cdot g$

Work to move the slab to the top
$= \pi \rho g\, x (r(x))^2 dx$



$\frac{r(x)}{x} = \frac{2}{3}$

$r(x) = \frac{2x}{3}$

$\text{Work} = \pi \rho g \frac{4}{9} \int_0^3 x^3\, dx$

$\boxed{\text{Work} = 9\pi g \rho}$

9. The integral $\int_1^3 x \ln x dx$ is equal to

A. $\frac{9}{2}\ln 3 - \frac{9}{4}$

B. $2\ln 3 - 3/2$

C. $\frac{9}{2}\ln 3 - 3$

$\boxed{\text{D.}}$ $\frac{9}{2}\ln 3 - 2$

E. $9\ln 3 - 2$

$$\int \underbrace{\ln x}_{u} \underbrace{x\,dx}_{dV} = \frac{x^2}{2}\ln x - \int \frac{x}{2}\,dx$$

$$u = \ln x, \quad du = \frac{1}{x}dx \qquad = \frac{x^2}{2}\ln x - \frac{x^2}{4}$$

$$dV = x\,dx, \quad V = \frac{x^2}{2}$$

$$\int_1^3 x \ln x\,dx = \left(\frac{9}{2}\ln 3 - \frac{9}{4}\right) - \left(-\frac{1}{4}\right)$$

$$= \frac{9}{2}\ln 3 - 2.$$

10. A force of 10 pounds stretches a spring with natural length of 6 inches to 12 inches. Find the total work by stretching the spring from a length of 12 inches to 18 inches:

A. $\frac{5}{2}$ ft-lb

B. 9 ft-lb

C. 10 ft-lb

D. 15 ft-lb

$\boxed{\text{E.}}$ $\frac{15}{2}$ ft-lb

$6 \text{ in} = \frac{1}{2}$ ft

$$F = kx \qquad 10 = k \cdot \frac{1}{2} \qquad k = 20$$

$$W = 20 \int_{0.5}^{1} x\,dx = 10\,x^2 \Big|_{0.5}^{1}$$

$$= 10\left(1 - (0.5)^2\right) = 10 \cdot \frac{3}{4} = \frac{15}{2} \text{ ft-lb}$$

11. Compute the value of the following definite integral

$$\int_0^1 x^2\sqrt{1-x^2}dx$$

$x = \sin x,\ dx = \cos x\, dx$

A. $\frac{\pi}{12}$

B. $\frac{\pi}{4}$

$\boxed{C}$ $\frac{\pi}{16}$

D. $\frac{\pi}{8}$

E. $\frac{\pi}{2}$

$$= \int_0^{\pi/2} \sin^2 x \cos^2 x\, dx = \frac{1}{4}\int_0^{\pi/2} \sin^2 2x\, dx$$

$$= \frac{1}{4}\int_0^{\pi/2} \left(\frac{1-\cos 4x}{2}\right) dx$$

$$= \frac{1}{4}\cdot \pi/4 = \pi/16.$$

12. The integral $\int_0^{\pi/4} \tan^5 x \sec^2 x\, dx$ is equal to

$\tan x = u. \quad du = \sec^2 x\, dx$

$\boxed{A.}$ $\frac{1}{6}$

B. $\frac{1}{4}$

C. $\frac{9}{4}$

D. $\frac{2}{5}$

E. $\frac{1}{3}$

$$= \int_0^1 u^5\, du = 1/6.$$