1. Find the radius r of the sphere

$$x^2 + y^2 + z^2 - 2x + 8y + 9 = 0$$

$x^2 - 2x + 1 + y^2 + 8y + 16 + z^2 = -9 + 1 + 16$

$(x-1)^2 + (y+4)^2 + z^2 = 8$

$r = \sqrt{8} = 2\sqrt{2}$

(A) 4

(B) 8

(C) $2\sqrt{2}$

(D) 9

(E) $\sqrt{2}$

2. Find $\text{proj}_{\mathbf{a}}\mathbf{b}$ where $\mathbf{a} = \langle -1, -1, 2 \rangle$ and $\mathbf{b} = \langle 2, 2, -1 \rangle$

$\text{Proj}_{\vec{a}} \vec{b} = \dfrac{\vec{a} \cdot \vec{b}}{\vec{a} \cdot \vec{a}} \vec{a}$

$= \dfrac{-6}{6} (-1, -1, 2)$

$= (1, 1, -2)$

(A) $\langle 1, 1, -2 \rangle$

(B) $\langle 2, 2, -4 \rangle$

(C) $\langle -2, -2, 1 \rangle$

(D) $\langle \frac{1}{2}, \frac{1}{2}, -1 \rangle$

(E) $\langle \frac{1}{3}, \frac{1}{3}, -\frac{2}{3} \rangle$

3. Find the area of the triangle with vertices at

$P(1,1,1) \qquad Q(2,-1,1) \qquad R(1,3,-2)$.

(A) 4

(B) $\frac{\sqrt{3}}{2}$

(C) $\frac{7}{2}$

(D) $\frac{4}{3}$

(E) $3\sqrt{5}$

$A = \frac{1}{2} \left| \overrightarrow{PQ} \times \overrightarrow{PR} \right|$

$\overrightarrow{PQ} = (1,-2,0)$

$\overrightarrow{PR} = (0,2,-3)$

$\overrightarrow{PQ} \times \overrightarrow{PR} = (6,3,2)$

$\left| \overrightarrow{PQ} \times \overrightarrow{PR} \right| = \sqrt{49} = 7$

$A = \frac{7}{2}$

4. For what value of b are the vectors $\mathbf{v} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{w} = -\mathbf{i} + b\mathbf{j} + b^2\mathbf{k}$ orthogonal?

(A) 4

(B) 2

(C) -2

(D) -1

(E) 1

$\vec{v} \cdot \vec{w} = 0$

$-1 + 2b - b^2 = 0$

$b^2 - 2b + 1 = 0$

$(b-1)^2 = 0$

$b = 1$

5. A force $\mathbf{F} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$ is applied to an object that moves from the point $P(1, 2, 0)$ to the point $Q(0, 5, 4)$. Find the work done

$\overrightarrow{PQ} = (-1, 3, 4) = -\vec{i} + 3\vec{j} + 4\vec{k}$

(A) 0

(B) 3

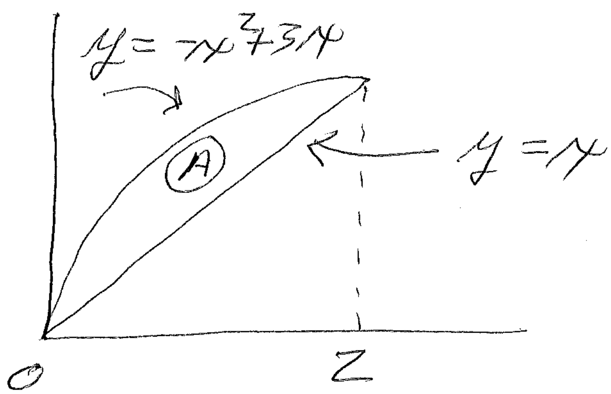
(C) 6

(D) 2

(E) 12

$W = F \cdot \overrightarrow{PQ} = -1 + 12 - 8 = 3$

6. Find the area between the curves $y = -x^2 + 3x$ and $y = x$.



(A) $\frac{9}{2}$

(B) $\frac{4}{3}$

(C) $\frac{8}{3}$

(D) $\frac{13}{2}$

(E) $\frac{14}{3}$

PTS. OF INTERSECTION

$-x^2 + 3x = x$

$x^2 - 2x = 0$

$x(x-2) = 0$

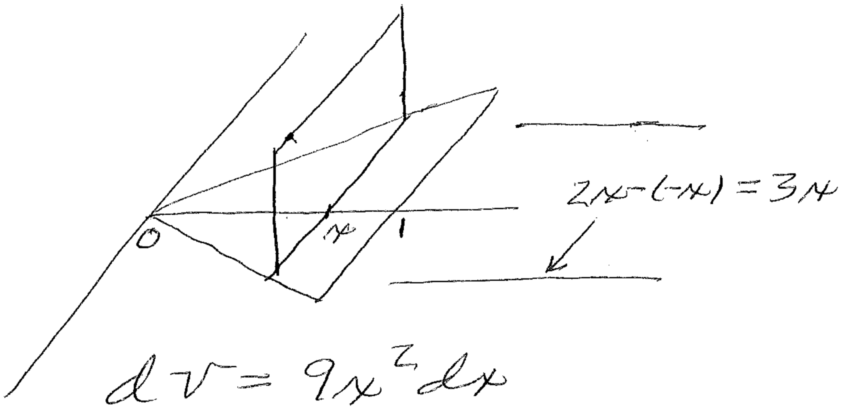
$x = 0, 2)$

$A = \int_0^2 (-x^2 + 3x - x)\,dx$

$= \int_0^2 (-x^2 + 2x)\,dx = \left.\frac{-x^3}{3} + x^2\right]_0^2$

$= -\frac{8}{3} + 4 = \frac{4}{3}$

7. The base of a solid S is the region bounded by the curves $y = -x$, $y = 2x$, and $x = 1$. Cross sections perpendicular to the x–axis are squares. Find the volume of S.



(A) 6

(B) $\frac{4}{3}$

(C) 9

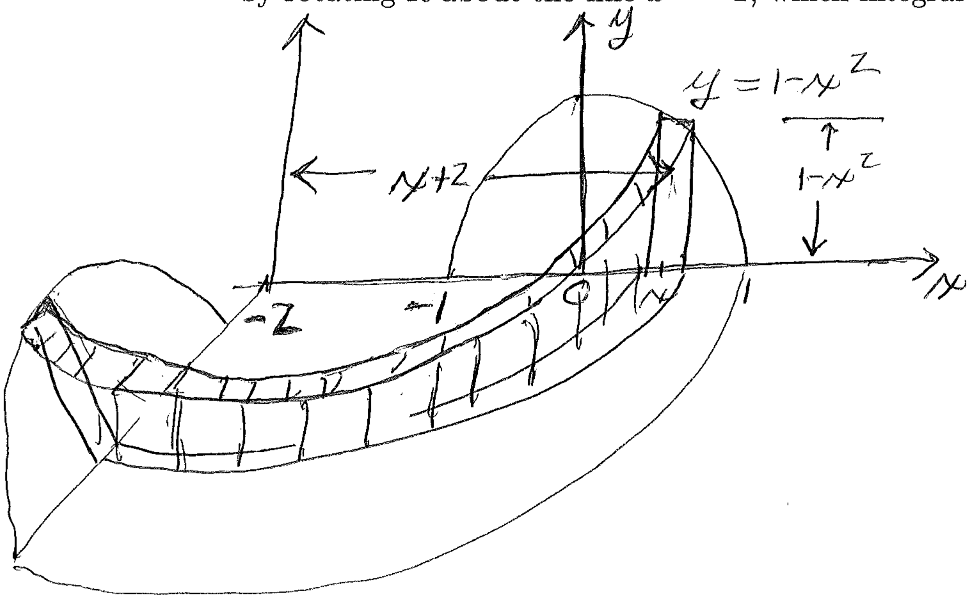
(D) 3

(E) $\frac{8}{3}$

$$dV = 9x^2\,dx$$

$$V = \int_0^1 9x^2\,dx = 9\,\frac{x^3}{3}\Big]_0^1 = 3$$

8. A region R is bounded by the curves $y = 1 - x^2$ and $y = 0$. If a solid S is obtained by rotating R about the line $x = -2$, which integral represents the volume of S.



(A) $\displaystyle\int_{-1}^{1} 2\pi(2-x)(1-x^2)\,dx$

(B) $\displaystyle\int_{-1}^{1} 2\pi x(1-x^2)\,dx$

(C) $\displaystyle\int_{-1}^{1} 2\pi(2-x)(1-x^2)^2\,dx$

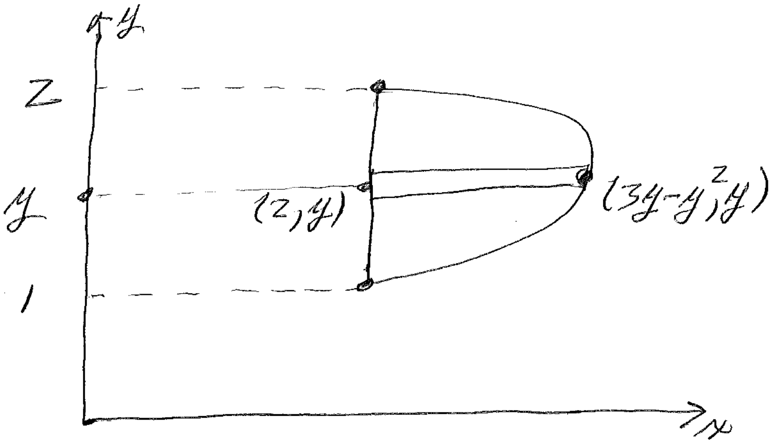
(D) $\displaystyle\int_{-1}^{1} 2\pi(x+2)(1-x^2)\,dx$

(E) $\displaystyle\int_{-1}^{1} \pi(1-x^2)^2\,dx$

$$dV = 2\pi(x+2)(1-x^2)\,dx$$

$$V = \int_{-1}^{1} 2\pi(x+2)(1-x^2)\,dx$$

9. Suppose R is the region bounded by the curves $x = 3y - y^2$ and $x = 2$. Which integral represents the volume of the solid obtained by rotating R about the y–axis?



(A) $\displaystyle\int_1^2 \pi((3y - y^2)^2 - 4)\, dy$

(B) $\displaystyle\int_0^2 \pi(3y - y^2 - 2)\, dy$

(C) $\displaystyle\int_1^2 2\pi y((3y - y^2) - 4)\, dy$

(D) $\displaystyle\int_1^2 2\pi y(3y - y^2 - 2)\, dy$

(E) $\displaystyle\int_0^2 2\pi((3y - y^2)^2 - 4)\, dy$

PTS. INTERSECTION

$2 = 3y - y^2$

$y^2 - 3y + 2 = 0$

$(y-1)(y-2) = 0$

$y = 1, 2$

$dV = \pi (3y - y^2)^2 dy - \pi (2)^2 dy$

$V = \int_1^2 \pi((3y - y^2)^2 - 4)\, dy$

10. A spring has a natural length of 12 inches. It takes 4 lb. to stretch the spring 6 inches. How much work is needed to stretch the spring from a length of 18 inches to a length of 24 inches?

$F(x) = kx$

$4 = k(1/2)$

$k = 8$

$F(x) = 8x$

$W = \int_{1/2}^{1} 8x\, dx = 4x^2 \Big]_{1/2}^{1} = 4 - 1 = 3$

(A) 8 ft–lb.

(B) 4 ft–lb.

(C) 6 ft–lb.

(D) 3 ft–lb.

(E) 2 ft–lb.

11. Compute $\int_1^e x \ln x \, dx.$ $= \left.\frac{x^2}{2}\ln x - \frac{x^2}{4}\right]_1^e = \frac{e^2}{2} - \frac{e^2}{4} + \frac{1}{4} = \frac{e^2}{4} + \frac{1}{4}$

$\int x \ln x \, dx$

$u = \ln x \quad du = \frac{1}{x}\,dx$

$dv = \ln x\, dx \quad v = \frac{x^2}{2}$

$= \frac{x^2}{2}\ln x - \int \frac{x}{2}\,dx$

$= \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$

(A) $\frac{e^2}{4} + \frac{1}{2}$

(B) $\frac{e^2}{2} + \frac{1}{2}$

(C) $\frac{e^2}{4} + \frac{1}{4}$

(D) $\frac{e^2}{4} - \frac{1}{2}$

(E) $\frac{e^2}{2} - \frac{1}{4}$

12. Compute $\int_0^{\pi/4} \sec^4 x \tan^2 x \, dx.$ $= \left.\frac{\tan^3 x}{3} + \frac{\tan^5 x}{5}\right]_0^{\pi/4} = \frac{1}{3} + \frac{1}{5} = \frac{8}{15}$

$\int \sec^4 x \tan^2 x \, dx$

$= \int \sec^2 x \tan^2 x \sec^2 x \, dx$

$= \int (1+\tan^2 x)\tan^2 x \sec^2 x \, dx$

$u = \tan x$
$du = \sec^2 x\, dx$

$= \int (1+u^2)u^2\,du = \frac{u^3}{3} + \frac{u^5}{5} + C$

$= \frac{\tan^3 x}{3} + \frac{\tan^5 x}{5} + C$

(A) $\frac{2}{5}$

(B) $\frac{2}{15}$

(C) $\frac{3}{4}$

(D) $\frac{4}{5}$

(E) $\frac{8}{15}$