Name Solutions

ten–digit Student ID number ______

Lecture Time ______

Recitation Instructor ______

Section Number ______

Instructions:

1. Fill in all the information requested above. On the scantron sheet fill in your name, student ID number, and the section number of your recitation with an extra 0 at the left. See list below. Blacken the correct circles.
2. On the bottom under Test/Quiz Number, write 02 and fill in the little circles.
3. This booklet contains 25 problems, each worth 8 points. The maximum score is 200 points.
4. For each problem mark your answer on the scantron sheet and also circle it in this booklet.
5. Work only on the pages of this booklet.
6. Books, notes, calculators are not to be used on this test.
7. At the end turn in your exam and scantron sheet to your recitation instructor.

TA	Lecture time	Rec. time	Sect. #	TA	Lecture time	Rec. time	Sect. #
Yun Ge	11:30	7:30	0001	Chris Bush	1:30	7:30	0011
		8:30	0002			8:30	0012
Huijie Wang	11:30	9:30	0003	Chi Weng Cheong	1:30	8:30	0031
		10:30	0004	Jing Feng Lau	1:30	9:30	0013
Wei-Nan Lin	11:30	11:30	0005			10:30	0014
		12:30	0006	Raakesh Pankanti	1:30	11:30	0015
Feng Chen	11:30	1:30	0007			12:30	0016
		2:30	0008	Phani Surapaneni	1:30	1:30	0017
Abhijeet Bhalerao	11:30	3:30	0009			2:30	0018
		4:30	0010	Himanshu Markandeya	1:30	3:30	0019
Kevin Mugo	11:30 or 1:30	1:30	0027			4:30	0020

1. The domain of the function $y = \ln(2^{x/3})$ is

$\ln(2^{\frac{x}{3}}) \rightarrow 2^{\frac{x}{3}} > 0$

$\rightarrow$ all real numbers x

A. All real numbers.

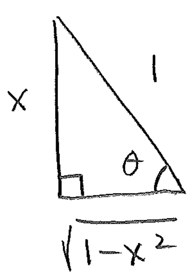
B. $x > 0$

C. $x > \ln\left(\frac{2}{3}\right)$

D. $x > \ln\left(\frac{3}{2}\right)$

E. $x > -\frac{1}{3}$

2. Express $\tan(\sin^{-1}(x))$ as an algebraic function of x.



$\theta = \sin^{-1}(x)$

$\rightarrow \tan(\theta) = \frac{x}{\sqrt{1-x^2}}$

A. $\frac{1}{\sqrt{1-x^2}}$

B. $\frac{1}{\sqrt{x^2-1}}$

C. $\frac{x}{\sqrt{1-x^2}}$

D. $\sqrt{1-x^2}$

E. $\frac{\sqrt{1-x^2}}{x}$

3. $\lim_{x\to -2} \dfrac{\frac{1}{2}+\frac{1}{x}}{2+x} =$ $\dfrac{0}{0}$

Three solutions offered below

A. ∞

B. $-\infty$

C. 0

D. $-\frac{1}{4}$

E. $\frac{1}{2}$

$$\lim_{x\to -2} \frac{\frac{1}{2}+\frac{1}{x}}{2+x} = \lim_{x\to -2} \frac{\frac{x+2}{2x}}{2+x}$$

$$= \lim_{x\to -2} \frac{1}{2x} = -\frac{1}{4}$$

$$\lim_{x\to -2} \frac{\frac{1}{2}+\frac{1}{x}}{2+x} \overset{\frac{0}{0}}{=} \lim_{x\to -2} \frac{0-\frac{1}{x^2}}{0+1} = -\frac{1}{4}$$

$$\lim_{x\to -2} \frac{\frac{1}{2}+\frac{1}{x}}{2+x} = \lim_{x\to -2} \frac{\frac{1}{x}-\left(-\frac{1}{2}\right)}{x-(-2)} = f'(-2) \text{ for } f(x)=\frac{1}{x}.$$

$$f'(x) = -\frac{1}{x^2}. \quad f'(-2) = -\frac{1}{4}$$

4. $\lim_{x\to -\infty} \dfrac{\sqrt{x^2+2}}{x} =$

A. ∞

B. $-\infty$

C. 0

D. 1

E. -1

Note: $x<0 \Rightarrow \sqrt{x^2} = |x| = -x$.

$$\lim_{x\to -\infty} \frac{\sqrt{x^2+2}}{x} = \lim_{x\to -\infty} \frac{\sqrt{x^2\left(1+\frac{2}{x^2}\right)}}{x}$$

$$= \lim_{x\to -\infty} \frac{\sqrt{x^2}\sqrt{1+\frac{2}{x^2}}}{x}$$

$$= \lim_{x\to -\infty} \frac{-x\sqrt{1+\frac{2}{x^2}}}{x}$$

$$= \lim_{x\to -\infty} -\sqrt{1+\frac{2}{x^2}} = -\sqrt{1+0} = -1$$

5. Find an equation of the tangent line to the curve, $y = \dfrac{x}{x+1}$ at the point $\left(1, \dfrac{1}{2}\right)$.

$$\frac{dy}{dx} = \frac{(1)(x+1)-(x)(1)}{(x+1)^2}$$

$$\left.\frac{dy}{dx}\right|_{(1,\frac{1}{2})} = \frac{(1)(2)-(1)(1)}{(1+1)^2} = \frac{1}{4}$$

A. $y = \dfrac{1}{2}$

B. $x + 2y = 2$

C. $x - 2y = 0$

D. $4y + x = 3$

E. $4y - x = 1$

Tangent line : $y - \frac{1}{2} = \frac{1}{4}(x-1)$

$\rightarrow 4y - 2 = x - 1$

$\rightarrow 4y - x = 1$

6. The radius of a sphere is increasing at a rate of 2 mm/sec. How fast is the volume increasing when the radius is 10 mm? ($V = \dfrac{4}{3}\pi r^3$)

Know: $\dfrac{dr}{dt} = 2 \dfrac{mm}{sec.}$

want: $\dfrac{dV}{dt}$ whent $r = 10$ mm.

A. 640π mm^3/ sec

B. 1600π mm^3/ sec

C. 1000π mm^3/ sec

D. 800π mm^3/ sec

E. 1200π mm^3/ sec

$V = \frac{4}{3}\pi r^3 \rightarrow \dfrac{dV}{dt} = 4\pi r^2 \dfrac{dr}{dt}$

$\rightarrow \dfrac{dV}{dt} = 4\pi(10)^2(2) = 800\pi \dfrac{mm^3}{sec.}$

7. Use a linear approximation (or differentials) to estimate $\sqrt[3]{994}$.

A. 9.95

B. 9.8

C. 10.02

D. 9.99

E. 9.98

Let $f(x) = \sqrt[3]{x} = x^{1/3}$, $a = 1000$.

Then $f'(x) = \frac{1}{3} x^{-\frac{2}{3}}$.

$$L(x) = f(1000) + f'(1000)(x-1000)$$
$$= 10 + \frac{1}{300}(x-1000)$$
$$L(994) = 10 + \frac{1}{300}(994-1000)$$
$$= 10 + \frac{1}{300}(-6)$$
$$= 10 - 0.02$$
$$= 9.98$$

8. A population of bacteria doubles every 2 days. How long will it take for the population to triple?

A. $2\ln(\frac{3}{2})$ days

B. $2\,\frac{\ln 3}{\ln 2}$ days

C. 3 days

D. 2.5 days

E. $2\ln 6$ days

$$P(t) = P(0)e^{kt}$$
$$P(2) = 2\cancel{P(0)} = \cancel{P(0)}e^{k\cdot 2}$$
$$\rightarrow 2 = e^{2k} \rightarrow \ln 2 = 2k \rightarrow k = \frac{1}{2}\ln 2$$
$$P(t) = P(0)e^{(\frac{1}{2}\ln 2)t}$$
$$P(t) = 3\cancel{P(0)} = \cancel{P(0)}e^{(\frac{1}{2}\ln 2)t}$$
$$\rightarrow 3 = e^{(\frac{1}{2}\ln 2)t}$$
$$\rightarrow \ln 3 = (\frac{1}{2}\ln 2)t$$
$$\rightarrow t = 2\frac{\ln 3}{\ln 2}$$

9. If $f(x) = \ln(\sin(x^2))$, then $f'(x) =$

A. $\dfrac{\sin(x^2)}{2x\cos(x^2)}$

B. $\dfrac{\cos(x^2)}{2x\sin(x^2)}$

C. $\dfrac{2x\cos(x^2)}{\sin(x^2)}$

D. $\dfrac{2x\sin(x^2)}{\cos(x^2)}$

E. $\dfrac{2x}{\sin(x^2)\cos(x^2)}$

$$f'(x) = \frac{1}{\sin(x^2)} \cdot \left(\cos(x^2)\right)(2x)$$

$$= \frac{2x\cos(x^2)}{\sin(x^2)}$$

10. If $f(x) = x^{\sin x}$, then $f'(x) =$

A. $x^{\cos x}$

B. $x^{\sin x}\cos x$

C. $x^{\sin x}\left(\cos x \ln x + \dfrac{\sin x}{x}\right)$

D. $x^{\sin x}(\cos x \ln x)$

E. $x^{\sin x}\dfrac{\cos x}{x}$

$$f(x) = x^{\sin x} = \left(e^{\ln x}\right)^{\sin x} = e^{(\ln x)(\sin x)}$$

$$\to f'(x) = \left(e^{(\ln x)(\sin x)}\right)\left(\left(\frac{1}{x}\right)(\sin x) + (\ln x)(\cos x)\right)$$

$$= x^{\sin x}\left(\frac{\sin x}{x} + (\ln x)(\cos x)\right)$$

11. Find an equation of the tangent line to the curve $(x^2+y^2)^2 = 4x^2y$ at the point $(x,y) = (1,1)$.

A. $y = 1$

B. $y = x$

C. $y = 2x - 1$

D. $y = -x + 2$

E. $y = -2x+3$

$$\frac{d}{dx} \rightarrow 2(x^2+y^2)\left(2x+2y\frac{dy}{dx}\right) = 8xy + 4x^2\frac{dy}{dx}$$

$$(1,1) \rightarrow 2(2)\left(2+2\frac{dy}{dx}\right) = 8 + 4\frac{dy}{dx}$$

$$\rightarrow 8 + 8\frac{dy}{dx} = 8 + 4\frac{dy}{dx} \rightarrow \frac{dy}{dx} = 0$$

Tangent Line: $y - 1 = 0(x-1) \rightarrow y = 1$

12. The absolute maximum value of the function $f(x) = \dfrac{x}{x^2+1}$ on the interval $[0,2]$ is

A. $\frac{2}{5}$

B. 0

C. 1

D. $\frac{1}{2}$

E. $\frac{3}{4}$

f is continuous on $[0,2]$.

critical numbers:

$$f'(x) = \frac{(1)(x^2+1)-(x)(2x)}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$$

$f'(x) = 0 \rightarrow x = \pm 1$ ($x=-1$ not in $[0,2]$)

check values of f at endpoints and critical number,

$f(0) = \frac{0}{0+1} = 0$

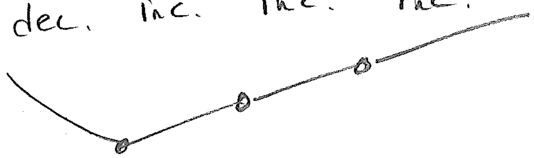
$f(1) = \frac{1}{1+1} = \frac{1}{2}$ ← absolute maximum value of function on $[0,2]$

$f(2) = \frac{2}{4+1} = \frac{2}{5}$

13. Let f be a function whose derivative f' is given by $f'(x) = (x-2)^2x^4(x+2)^5$. Then f has a

	$-\infty$ to -2	-2 to 0	0 to 2	2 to ∞
$(x-2)^2$	+	+	+	0 +
x^4	+	+ 0	+	+
$(x+2)^5$	−	0 +	+	+
$f'(x)$	−	+	+	+
	dec.	inc.	inc.	inc.

f



f has a local minimum at $x = -2$

A. local minimum at $x = 0$

B. local minimum at $x = -2$

C. local maximum at $x = 0$

D. local minimum at $x = 2$

E. local maximum at $x = 2$

14. $\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x) =$ $\infty - \infty$

$$\lim_{x \to \frac{\pi}{2}} \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x} \right)$$

$$= \lim_{x \to \frac{\pi}{2}} \left(\frac{1 - \sin x}{\cos x} \right)$$

$$\overset{\frac{0}{0}}{=} \lim_{x \to \frac{\pi}{2}} \left(\frac{-\cos x}{-\sin x} \right)$$

$$= \frac{0}{-1} = 0$$

A. 0

B. 1

C. ∞

D. -1

E. $\frac{1}{2}$

15. A box with square base and open top must have a volume of 4000 cm^3. If the cost of the material used is \$1/cm^2, the smallest possible cost of the box is

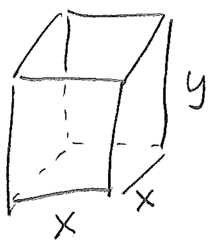
A. \$500

B. \$600

C. \$1000

D. \$1200

E. \$2000



$x^2 y = 4000$

minimize cost

$\text{Cost} = C = \frac{\$1}{cm^2}(\text{surface area})$

$= \frac{\$1}{cm^2}(x^2 + 4xy)$ no top!

$x^2 y = 4000 \rightarrow y = \frac{4000}{x^2}$

$\rightarrow C(x) = \frac{\$1}{cm^2}\left(x^2 + 4x\left(\frac{4000}{x^2}\right)\right) = \frac{\$1}{cm^3}\left(x^2 + \frac{16{,}000}{x}\right)$

$\frac{dC}{dx} = 2x - \frac{16{,}000}{x^2} = 0 \rightarrow \frac{2x^3 - 16{,}000}{x^2} = 0 \rightarrow x^3 = \frac{16{,}000}{2}$

$\rightarrow x = 20$

minimum cost is $C(20) = \left(20^2 + \frac{16{,}000}{20}\right)\$ = 400 + 800 = \$1200$

16. If $\int_{-2}^{2} f(x)dx = 2$ and $\int_{0}^{2} f(x)dx = 3$, then $\int_{-2}^{0} f(x)dx =$

A. -1

B. 1

C. -5

D. 5

E. -3

$\int_{-2}^{0} f(x)\,dx = \int_{-2}^{2} f(x)\,dx + \int_{2}^{0} f(x)\,dx$

$= \int_{-2}^{2} f(x)\,dx - \int_{0}^{2} f(x)\,dx$

$= 2 - 3$

$= -1$

17. $\int_0^1 (x^2 - \sqrt{x} + 1)dx =$

A. $-\frac{1}{6}$

B. $\frac{5}{6}$

C. $\frac{2}{3}$

D. $\frac{1}{3}$

E. 1

$$= \int_0^1 (x^2 - x^{1/2} + 1)\,dx$$

$$= \left(\frac{1}{3}x^3 - \frac{2}{3}x^{3/2} + x\right)\Big|_0^1$$

$$= \frac{1}{3} - \frac{2}{3} + 1$$

$$= \frac{2}{3}$$

18. $\int_0^{\frac{\pi}{2}} \sin(2x)dx =$

A. 1

B. -1

C. 2

D. -2

E. 0

Let $u = 2x$. Then $du = 2\,dx$, so $dx = \frac{1}{2}dx$

Also $u(0) = 0$ and $u(\frac{\pi}{2}) = \pi$

$$\int_0^{\pi/2} \sin(2x)\,dx = \int_0^{\pi} (\sin(u))\left(\frac{1}{2}dx\right)$$

$$= \frac{1}{2}\int_0^{\pi} \sin u\,du$$

$$= \frac{1}{2}(-\cos u)\Big|_0^{\pi}$$

$$= \frac{1}{2}(-\cos \pi) - \frac{1}{2}(-\cos 0)$$

$$= \frac{1}{2}(1) - \frac{1}{2}(-1) = 1.$$

19. $\int_0^1 x^2(x^3+1)^{17}dx =$

Let $u = x^3+1$. Then $du = 3x^2\,dx$

so $x^2 dx = \frac{1}{3} du$.

Also $u(0) = 1$, $u(1) = 2$

$$\int_0^1 x^2(x^3+1)^{17}\,dx = \int_1^2 u^{17}\left(\tfrac{1}{3}du\right)$$

$$= \frac{1}{3}\int_1^2 u^{17}\,du = \frac{1}{3}\,\frac{u^{18}}{18}\Big|_1^2$$

$$= \frac{1}{54}\left(2^{18} - 1^{18}\right) = \frac{2^{18}-1}{54}$$

A. $\frac{2^{18}}{18}$

B. $\frac{2^{18}}{54}$

C. $\frac{2^{18}-1}{18}$

D. $\frac{2^{18}-1}{54}$

E. $\frac{2^{18}-1}{3}$

20. If $g(x) = \int_0^{2x} e^{t^2}\,dt$, then $g'(x) =$ $e^{(2x)^2} \cdot 2$

$= 2e^{4x^2}$

A. e^{2x^2}

B. $2e^{x^2}$

C. $2e^{2x^2}$

D. $2e^{4x^2}$

E. e^{4x^2}

21. The function $2x^3 - 9x^2 - 24x + 1$ is increasing on

A. $(-\infty, -1)$ only

B. $(-1, 4)$ only

C. $(4, \infty)$ only

D. $(-\infty, 1)$ and $(4, \infty)$

E. $(-\infty, -1)$ and $(4, \infty)$

let $f(x) = 2x^3 - 9x^2 - 24x + 1$

Then $f'(x) = 6x^2 - 18x - 24$

$= 6(x^2 - 3x - 4) = 6(x-4)(x+1)$

x	$-\infty$ to -1	-1 to 4	4 to ∞
6	$+$	$+$	$+$
$x-4$	$-$	$-$	0 $+$
$x+1$	$-$	0 $+$	$+$
$f'(x)$	$+$	$-$	$+$
f	inc.	dec.	inc.

f increasing on $(-\infty, -1)$ and $(4, \infty)$.

22. The function $x^4 - 6x^3 + 12x^2 + 1$ is concave down on

A. $(-\infty, 1)$ only

B. $(1, 2)$ only

C. $(2, \infty)$ only

D. $(-\infty, -1)$ and $(2, \infty)$

E. $(-\infty, 1)$ and $(2, \infty)$

let $f(x) = x^4 - 6x^3 + 12x^2 + 1$

Then $f'(x) = 4x^3 - 18x^2 + 24x$

and $f''(x) = 12x^2 - 36x + 24$

$= 12(x^2 - 3x + 2)$

$= 12(x-1)(x-2)$

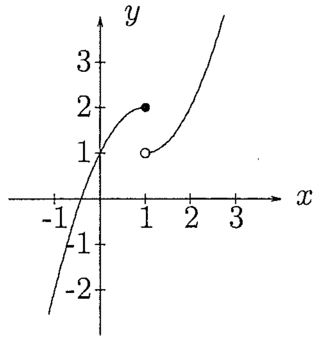
x	$-\infty$ to 1	1 to 2	2 to ∞
12	$+$	$+$	$+$
$x-1$	$-$	0 $+$	$+$
$x-2$	$-$	$-$	0 $+$
$f''(x)$	$+$	$-$	$+$
f	concave up	concave down	concave up

23. The graph of the function f is given below, and $\lim_{x \to 1^+} f(x) = a$, $\lim_{x \to 1^-} f(x) = b$. Which of the following is true?

MA 16100 Fall 2008

1. The graph of $y = f(x)$ is shown below.

$\lim_{x \to 1^-} f(x) = 2 = b$



$\lim_{x \to 1^+} f(x) = 1 = a$

A. $a = 2,\ b = 1$

B. $a = 1,\ b = 2$

C. $a = \text{dne},\ b = \text{dne}$

D. $a = \text{dne},\ b = 2$

E. $a = 1,\ b = \text{dne}$

(dne means "does not exist")

24. The graph of the function $y = \sin(x)$ is shrunk horizontally by a factor of 2, then translated 1 unit to left. The resulting graph is that of

$y = \sin(x) = f(x)$

$y = f(2x) = \sin(2x) = g(x)$

$y = g(x+1) = \sin(2(x+1))$

$= \sin(2x+2)$

A. $y = \sin\left(\frac{1}{2}x + 1\right)$

B. $y = \sin\left(\frac{1}{2}x - 1\right)$

C. $y = \sin\left(\frac{1}{2}x + 2\right)$

D. $y = \sin(2x+1)$

E. $y = \sin(2x+2)$

25. For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} 2cx^2 + 3x & \text{if } x < 2 \\ x^3 + cx^2 & \text{if } x \geq 2 \end{cases}$$

A. $\frac{1}{4}$

B. 4

C. $\frac{1}{2}$

D. 2

E. No value of c makes f continuous on $(-\infty, \infty)$

Want: $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$.

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (2cx^2 + 3x) = 8c + 6$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (x^3 + cx^2) = 8 + 4c$$

$8c + 6 = 8 + 4c$

$\rightarrow \quad 4c = 2$

$\rightarrow \quad c = \frac{1}{2}$