MA 161 EXAM III **Fall 2005**

Name SOLUTIONS

ten–digit Student ID number

Division and Section Numbers

Recitation Instructor

Instructions:

1. Fill in all the information requested above and on the scantron sheet.
2. This booklet contains 14 problems, each worth 7 points. You get 2 points if you fully comply with instruction 1. The maximum score is 100 points.
3. For each problem mark your answer on the scantron sheet and also circle it in this booklet.
4. Work only on the pages of this booklet.
5. Books, notes, calculators are not to be used on this test.
6. At the end turn in your exam and scantron sheet to your recitation instructor.

Key

ccbe acbc dbee cb

$2y = 80 - x$

$\nearrow \rightarrow y = 40 - \frac{x}{2}$

1. Find two positive numbers x and y satisfying $x+2y = 80$ whose product is maximum.

   a. $\{32, 24\}$

   b. $\{28, 26\}$

   c. $\{40, 20\}$

   d. $\{26, 27\}$

   e. none of the above

$P = xy = x\left(40 - \frac{x}{2}\right) = 40x - \frac{1}{2}x^2, \quad x > 0$

$\frac{dP}{dx} = 40 - x = 0 \rightarrow x = 40 \rightarrow y = 40 - \frac{40}{2} = 20$

$0 \quad 40 \quad \infty \rightarrow x$

$40-x \quad + \quad 0 \quad -$

$P \quad \nearrow \searrow \quad P(40)$ is a max

2. A box with a square base has volume $100 in^3$ and dimensions $b \times b \times h$. A formula for its surface area in terms of b is $A(b) =$

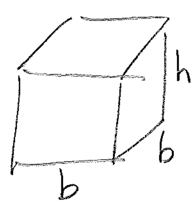
   a. $2b^2 + \dfrac{400}{b^2}$

   b. $2b^2 + \dfrac{200}{b^2}$

   c. $2b^2 + \dfrac{400}{b}$

   d. $2b^2 + \dfrac{200}{b}$

   e. none of these



$A = 2b^2 + 4bh$

$b^2 h = 100 \rightarrow h = \frac{100}{b^2}$

$\Rightarrow A(b) = 2b^2 + 4b\left(\frac{100}{b^2}\right)$

$= 2b^2 + \frac{400}{b}$

3. Let $f(x) = x^2 - 2$ and $x_1 = 3$. Find x_2, the second approximation to $\sqrt{2}$ using Newton's method.

   a. $2\frac{1}{2}$

   b. $1\frac{5}{6}$

   c. $2\frac{1}{7}$

   d. $1\frac{5}{7}$

   e. $2\frac{5}{6}$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_2)}$

$f'(x) = 2x$

$x_2 = x_1 - \frac{x_1^2 - 2}{2x_1}$

$x_1 = 3 \rightarrow x_2 = 3 - \frac{9-2}{6} = 3 - \frac{7}{6} = \frac{18}{6} - \frac{7}{6} = \frac{11}{6}$

$\left(1\frac{5}{6}\right)$

4. Find $f(x)$ if $f'(x) = 3x^2 + \frac{2}{x}$, $x > 0$, $f(1) = 3$.

a. $x^3 + 2\ln x$

b. $x^3 - \frac{1}{x} + 3$

c. $x^3 + 2\ln x + 1$

d. $6x + 2\ln x - 3$

e. $x^3 + 2\ln x + 2$

$f(x) = x^3 + 2\ln|x| + C$

$f(1) = 3 = 1 + 2\ln(1) + C = 1 + C \to C = 2$

$f(x) = x^3 + 2\ln x + 2$

5. Given $\int_1^4 \sqrt{x}\,dx = \frac{14}{3}$ evaluate $\int_1^4 (2 + 3\sqrt{x})dx$.

a. 20

b. $\frac{18}{3}$

c. 14

d. $\frac{26}{3}$

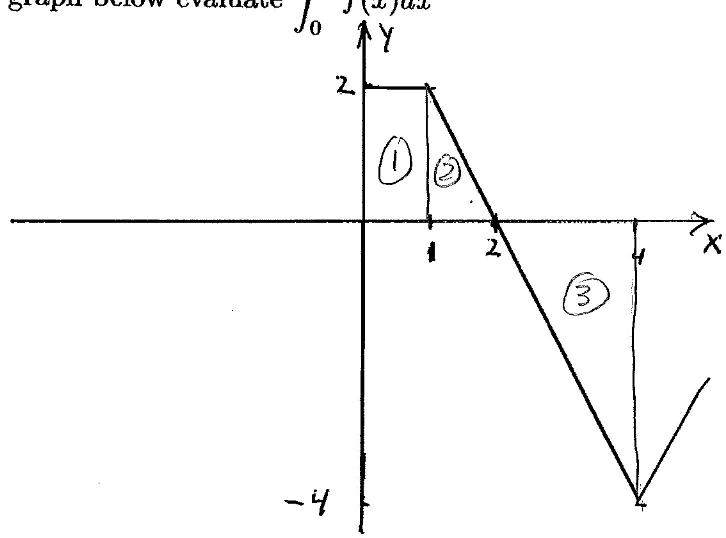
e. 28

$\int_1^4 (2 + 3\sqrt{x})\,dx = 2(4-1) + 3\left(\frac{14}{3}\right)$

$= 6 + 14$

$= 20$

6. From the graph below evaluate $\int_0^4 f(x)dx$



a. -4

b. -2

c. -1

d. 0

e. 2

$\int_0^4 f(x)dx =$ area (1) + area (2) − area (3)

$= (2)(1) + \frac{1}{2}(2)(1) - \frac{1}{2}(2)(4)$

$= 2 + 1 - 4$

3

$= -1$

7. The most general antiderivative to $f(x) = \sin 2x + 2x$ is $F(x) =$

a. $-2\cos 2x + x^2 + C$

b. $-\dfrac{\cos 2x}{2} + x^2 + C$

c. $2\cos 2x + \dfrac{x^2}{2} + C$

d. $\dfrac{\cos 2x}{2} + x^2 + C$

e. $2\cos 2x + x^2 + C$

$\int \sin 2x \, dx$, let $u = 2x$
$\rightarrow du = 2dx \rightarrow \frac{1}{2} du = dx$

$\int \sin 2x \, dx = \int \sin u \left(\frac{1}{2} du\right) = \frac{1}{2}(-\cos u) + C$
$= -\frac{1}{2}\cos 2x + C$

Therefore $\int (\sin 2x + 2x)\, dx = -\frac{1}{2}\cos 2x + x^2 + C$

8. The absolute maximum of $f(x) = \dfrac{x^2 - 4}{x^2 + 2}$ on the interval $[-2, 2]$ is

a. -4

b. 4

c. 0

d. 2

e. -2

$f'(x) = \dfrac{(2x)(x^2+2) - (x^2-4)(2x)}{(x^2+2)^2} = \dfrac{12x}{(x^2+2)^2} = 0 \Rightarrow x = 0$

$f(-2) = 0$

$f(0) = \frac{-4}{2} = -2$

$f(2) = 0$

abs. max value is 0.

9. $\displaystyle\lim_{x \to 1} \frac{\ln x}{\sin \pi x} =$ $\frac{0}{0}$

a. 0

b. -1

c. π

d. $\dfrac{-1}{\pi}$

e. none of the above

$\displaystyle\lim_{x \to 1} \frac{\ln x}{\sin \pi x} \overset{\frac{0}{0}}{=} \lim_{x \to 1} \frac{\frac{1}{x}}{\pi \cos \pi x} = \frac{1}{-\pi}$

10. $\lim_{x\to 0^+} (\sin 3x)^x$ $= 0^0$

Let $y = (\sin 3x)^x$

Then $\ln y = \ln(\sin 3x)^x = x\ln(\sin 3x)$

$= \dfrac{\ln(\sin 3x)}{\frac{1}{x}}$

$\lim_{x\to 0^+} \dfrac{\ln(\sin 3x)}{\frac{1}{x}} \overset{-\infty/\infty}{=} \lim_{x\to 0^+} \dfrac{\frac{3\cos 3x}{\sin 3x}}{-\frac{1}{x^2}}$

$= \lim_{x\to 0^+} (-1)\left(\dfrac{3x}{\sin 3x}\right)(x\cos 3x)$

$= (-1)(1)(0) = 0$

Therefore

$\lim_{x\to 0^+} (\sin 3x)^x = e^0 = 1$

a. 3

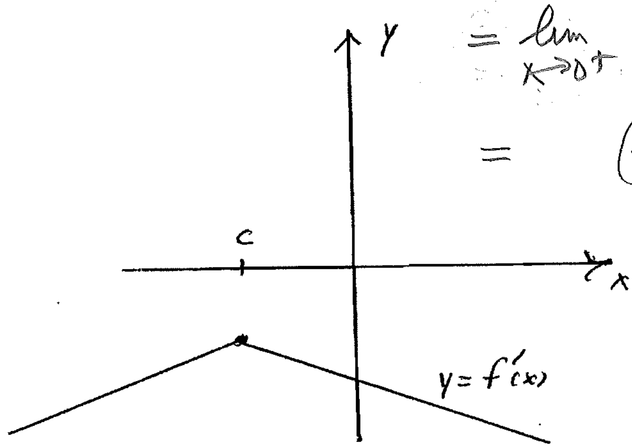
b. 1

c. ∞

d. 0

e. -3

11.



Given the graph of the derivative function, f' above, we may conclude that

a. $f(c) < 0$

b. f has a local maximum at c

c. f is not differentiable at c

d. f is increasing to the left of c

e. f has an inflection point at c

12. The number of points at which $f(x) = x^4 - 8x^2 - 7$ has either a local maximum value, a local minimum value, or an inflection point is

$f'(x) = 4x^3 - 16x = 4x(x^2-4) = 0 \to x = 0, \pm 2$

	$-\infty$ to -2	-2 to 0	0 to 2	2 to ∞
x	$-$	$-$	$+$	$+$
x^2-4	$+$	$-$	$-$	$+$
$f'(x)$	$-$	$+$	$-$	$+$
f				

$f''(x) = 12x^2 - 16 = 0 \to x^2 = \frac{4}{3} \to x = \pm\frac{2}{\sqrt{3}}$

	$-\infty$ to $-\frac{2}{\sqrt3}$	$-\frac{2}{\sqrt3}$ to $\frac{2}{\sqrt3}$	$\frac{2}{\sqrt3}$ to ∞
$12x^2-16$	$+$	$-$	$+$
f	$\cup$	$\cap$	$\cup$

a. 1

b. 2

c. 3

d. 4

e. 5

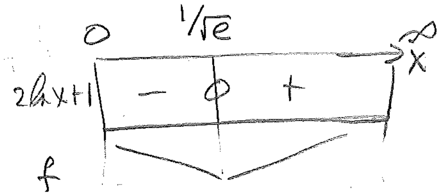
$f(-2)$ and $f(2)$ local mins 5

$f(0)$ local max

$\left(\frac{2}{\sqrt3}, f\left(\frac{2}{\sqrt3}\right)\right)$ and $\left(-\frac{2}{\sqrt3}, f\left(-\frac{2}{\sqrt3}\right)\right)$ are inflection pts

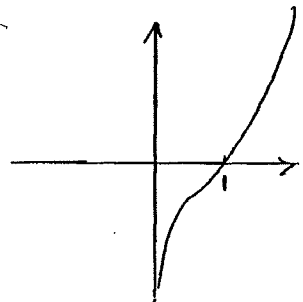
$f'(x) = 2x\ln x + \frac{x^2}{x} = x(2\ln x + 1)$

$f'(x) = 0 \rightarrow x = 0, \quad \ln x = -\frac{1}{2} \rightarrow x = \frac{1}{\sqrt{e}}$

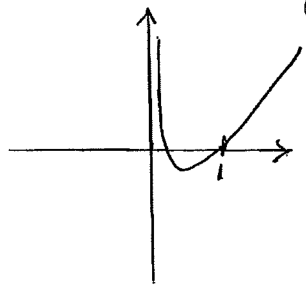


13. The graph of $f(x) = x^2 \ln x$ resembles most

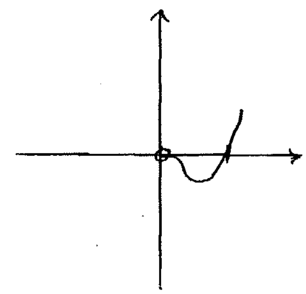
$\cancel{\text{a.}}$



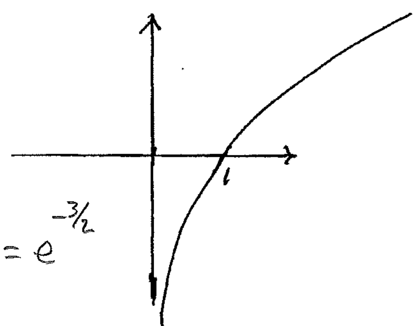
$\cancel{\text{b.}}$



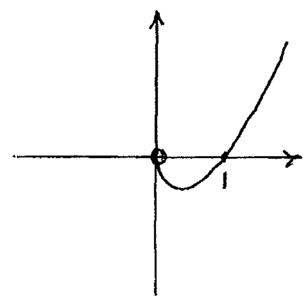
c.



$\cancel{\text{d.}}$



e.



$\lim_{x\to 0^+} x^2 \ln x$

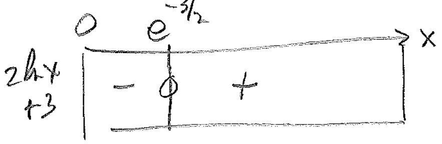
$= \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x^2}}$

$\overset{\frac{-\infty}{\infty}}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-2}{x^3}}$

$= \lim_{x\to 0^+} \frac{x^2}{-2} = 0$

$f''(x) = (2\ln x + 1) + x\left(\frac{2}{x}\right)$

$= 2\ln x + 3 = 0 \rightarrow x = e^{-3/2}$



$\Rightarrow$ graph is c, not e

14. Given that $f(1) = 9$ and $f'(x) \geq 3$ for $1 \leq x \leq 4$, the smallest $f(4)$ can be is

a. 19

b. 18

c. 12

d. cannot be determined

e. none of the above

Mean Value Theorem implies

$$\frac{f(4) - f(1)}{4-1} = f'(c) \geq 3$$

$$\rightarrow \frac{f(4) - 9}{4-1} \geq 3$$

$$\rightarrow f(4) - 9 \geq 9$$

$$\rightarrow f(4) \geq 18$$