1. If $f(x) = \frac{1}{e} + \sqrt{x^3}e^{-x}$, find $f'(1)$.

A. $\frac{3}{2e}$

B. $\frac{1}{e}$

C. $\boxed{\frac{1}{2e}}$

D. $\frac{2}{3e}$

E. $\frac{e-2}{2e^2}$

$$f'(x) = 0 + \frac{3}{2}x^{1/2}e^{-x} - x^{3/2}e^{-x}$$

$$f'(1) = \frac{3}{2}e^{-1} - e^{-1}$$

2. Given the following data:

$f(0) = -3$	$f(1) = 4$
$f'(0) = 2$	$f'(1) = 3$
$g(0) = 1$	$g(1) = 0$
$g'(0) = -1$	$g'(1) = -2$

If $h(x) = \frac{f(g(x))}{g(x)}$, find $h'(0)$.

A. 7

B. 2

C. 4

D. 3

E. $\boxed{1}$

$$h'(0) = \frac{g(0)f'(g(0))g'(0) - f(g(0))g'(0)}{[g(0)]^2}$$

$$= \frac{(1)(3)(-1) - (4)(-1)}{1^2}$$

3. For which θ and x are the following two equations true?

$$\cos^{-1}(\cos\theta) = \theta \qquad \text{and} \qquad \cos(\cos^{-1} x) = x$$

A. $\dfrac{-\pi}{2} \le \theta \le \dfrac{\pi}{2}$ and $-1 \le x \le 1$

B. $0 \le \theta \le \pi$ and $-\infty < x < \infty$

C. $-\infty < \theta < \infty$ and $-1 \le x \le 1$

D. $\boxed{0 \le \theta \le \pi \text{ and } -1 \le x \le 1}$

E. $-1 \le \theta \le 1$ and $\dfrac{-\pi}{2} \le x \le \dfrac{\pi}{2}$

4. The position for the movement of a particle is given by

$$s = \cos(2t - 2) - \sin(3t - 3)$$

where s is measured in feet and t is measured in seconds. Find the acceleration of the particle after 1 second.

A. $\boxed{-4 \text{ feet/sec}^2}$

B. 2 feet/sec^2

C. -1 feet/sec^2

D. 5 feet/sec^2

E. 0 feet/sec^2

$s'(t) = -2\sin(2t-2) - 3\cos(3t-3)$

$s''(t) = -4\cos(2t-2) + 9\sin(3t-3)$

$s''(1) = -4\cos(0) + 9\sin(0)$

5. Find the slope of the tangent line to the curve

$$x^2y^2 + \ln y = x^3 - 4$$

at the point $(2, 1)$.

A. 4/9
B. 0
C. 2/3
D. 1
E. $\boxed{8/9}$

$$2xy^2 + 2x^2y\frac{dy}{dx} + \frac{1}{y}\frac{dy}{dx} = 3x^2 - 0$$

$$2(2)(1)^2 + 2(2)^2(1)\frac{dy}{dx} + \frac{1}{1}\frac{dy}{dx} = 3(2)^2$$

$$4 + 9\frac{dy}{dx} = 12$$

$$\frac{dy}{dx} = \frac{8}{9}$$

6. If $y = \dfrac{1}{\tan^3 x}$, find $\dfrac{dy}{dx}$ at $x = \dfrac{5\pi}{3}$.

A. −36
B. $\boxed{-4/3}$
C. 1/48
D. 1/36
E. −1/12

$$\frac{dy}{dx} = -3\left(\frac{1}{\tan^4 x}\right)\sec^2 x$$

$$= -3\left(\frac{\cos^2 x}{\sin^4 x}\right)$$

at $\frac{5\pi}{3}$: $-3\left(\dfrac{\left(\frac{1}{2}\right)^2}{\left(-\frac{\sqrt{3}}{2}\right)^4}\right) = \dfrac{-3/4}{9/16} = -\dfrac{4}{3}$

7. Suppose a is a positive number and the tangent line to $y = a^{x^2}$ at $x = 1$ has a slope of a. What is a?

A. e

B. $\boxed{\sqrt{e}}$

C. e^2

D. $\ln 2$

E. There is no such number a.

$$\frac{dy}{dx} = \left(a^{x^2}\right)(\ln a)(2x)$$

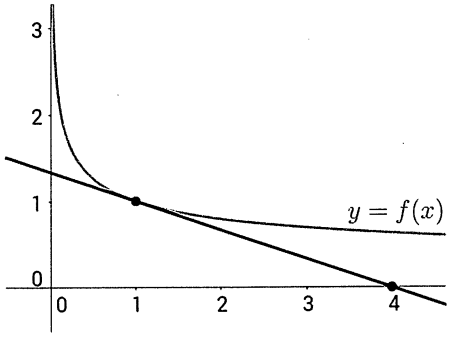
At $x=1$:

$$\left(a^{1^2}\right)(\ln a)(2) = a$$

$$\ln a = \frac{1}{2}$$

$$a = e^{1/2}$$

8. Below is the graph of $y = f(x)$ and its tangent line at $(1, 1)$. The tangent line also passes through $(4, 0)$.



SLOPE $= \dfrac{1-0}{1-4} = -\dfrac{1}{3}$

Which one of these functions could be f?

A. $\boxed{\dfrac{1}{\sqrt[3]{x}}}$

$$\left.\frac{dy}{dx}\right|_{x=1} = -\frac{1}{3}x^{-4/3}\Big|_{x=1} = -\frac{1}{3}$$

B. $\dfrac{1}{x}$

C. $1 - \ln x$

D. $\dfrac{1}{4x}$

E. $\dfrac{1}{x^3}$

9. Find the derivative of $y = x^{\tan^{-1} x}$.

A. $\dfrac{x^{\tan^{-1} x} \ln x}{1+x^2}$

B. $-x^{\tan^{-1} x}(\csc^2 x)(\ln x)$

C. $\dfrac{\tan^{-1} x}{x} + \dfrac{\ln x}{1+x^2}$

D. $\boxed{\dfrac{x^{\tan^{-1} x} \ln x}{1+x^2} + \dfrac{x^{\tan^{-1} x} \tan^{-1} x}{x}}$

E. $\dfrac{\tan^{-1} x}{x} - (\csc^2 x) \ln x$

$\ln y = \tan^{-1} x \ln x$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{1+x^2}\ln x + \frac{\tan^{-1} x}{x}$

$\frac{dy}{dx} = x^{\tan^{-1} x}\left[\frac{\ln x}{1+x^2} + \frac{\tan^{-1} x}{x}\right]$

10. The half-life of strontium-90 is 29 years. Suppose we have a 5 mg sample. Find the mass (in mg) that remains after t years.

A. $0.5e^{29t/\ln(5)}$

B. $0.5e^{t\ln(5)/29}$

C. $\boxed{5e^{t\ln(0.5)/29}}$

D. $5e^{29t\ln(0.5)}$

E. $5e^{29t/\ln(0.5)}$

11. Find the equation of the tangent line to $y = \sinh x$ at $x = \ln 3$.

A. $y = \frac{5}{3}(x - \ln 3) - \frac{4}{3}$

B. $y = \frac{4}{3}(x - \ln 3) - \frac{5}{3}$

C. $y = 3(x - \ln 3)$

D. $y = \frac{4}{3}(x - \ln 3) + \frac{5}{3}$

E. $\boxed{y = \frac{5}{3}(x - \ln 3) + \frac{4}{3}}$

$y' = \cosh x$

$y(\ln 3) = \frac{e^{\ln 3} - e^{-\ln 3}}{2} = \frac{3 - \frac{1}{3}}{2} = \frac{4}{3}$

$y'(\ln 3) = \frac{e^{\ln 3} + e^{-\ln 3}}{2} = \frac{3 + \frac{1}{3}}{2} = \frac{5}{3}$

12. A spherical balloon is inflated at the rate of 10 cubic centimeters per second. Find the rate at which the surface area is increasing when the surface area is 16 square centimeters.

Hint: Volume of a sphere, $V = \frac{S^{3/2}}{6\sqrt{\pi}}$, *where* S *is the surface area.*

A. $\frac{10}{\sqrt{\pi}}$ cm^2/s

B. $\frac{32}{3\sqrt{\pi}}$ cm^2/s

C. $\boxed{10\sqrt{\pi}}$ cm^2/s

D. $\frac{1}{\sqrt{\pi}}$ cm^2/s

E. $40\sqrt{\pi}$ cm/s

$\frac{dV}{dt} = \frac{S^{1/2}}{4\sqrt{\pi}} \frac{dS}{dt}$

$10 = \frac{\sqrt{16}}{4\sqrt{\pi}} \frac{dS}{dt}$

$10\sqrt{\pi} = \frac{dS}{dt}$