**MA 16100 Exam II, Spring 2014, March 13**

Name _____

10–digit PUID number _____

Recitation Instructor _____

Recitation Section Number and Time _____

Instructions: MARK TEST NUMBER O1 ON YOUR SCANTRON

1. Do not open this booklet until you are instructed to.
2. Fill in all the information requested above and on the scantron sheet. On the scantron sheet fill in the little circles for your name, section number and PUID.
3. This booklet contains 13 problems, equally weighted.
4. For each problem mark your answer on the scantron sheet and also **circle it in this booklet**.
5. Work only on the pages of this booklet.
6. Books, notes, calculators or any electronic device are not allowed during this test and they should not even be in sight in the exam room. You may not look at anybody else's test, and you may not communicate with anybody else, except, if you have a question, with your instructor.
7. You are not allowed to leave during the first 20 and the last 10 minutes of the exam.
8. When time is called at the end of the exam, put down your writing instruments and remain seated. The TAs will collect the scantrons and the booklets.

1. The rate of change of a certain function $f(x)$ is proportional to $f(x)$. $f(0) = 2$ and $f(3) = 4$. What is $f(9)$?

A. 24
B. 20
C. 16
D. 12
E. 8

$f(x) = y_0 e^{kx}$

$f(0) = 2$ so $y_0 = 2$ so $f(x) = 2e^{kx}$

$f(3) = 4$ so $4 = 2e^{k \cdot 3}$ so $3k = \ln 2$

$f(x) = 2e^{(\frac{\ln 2}{3})x}$

$f(9) = 2e^{(\frac{\ln 2}{3}) \cdot 9} = 2e^{3 \ln 2} = 2 \cdot 8 = 16$

2. What is $\frac{d}{dx}(\tan^{-1}\sqrt{x})$ when $x = 4$?

A. $\frac{1}{40}$
B. $\frac{1}{20}$
C. $\frac{1}{5}$
D. $\frac{1}{80}$
E. $\frac{1}{10}$

$\frac{d}{dx} \tan^{-1}\sqrt{x} = \frac{1}{1+(\sqrt{x})^2} \cdot \frac{1}{2}x^{-1/2}$

$= \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}}$

when $x = 4$ this is

$\frac{1}{5} \cdot \frac{1}{4} = \frac{1}{20}$

3. Use the linear approximation of the function $x^{3/2}$ at 9 to estimate $(9.02)^{3/2}$.

A. 27.07
B. 27.09
C. 27.06
D. 27.08
E. 27.10

$f(9) = 9^{3/2} = 27$

$f'(x) = \frac{3}{2}x^{1/2}$

$f'(9) = \frac{9}{2}$

$L(x) = 27 + \frac{9}{2}(x-9)$

$L(9.02) = 27 + \frac{9}{2}(.02)$

$= 27.09$

4. Find the slope of the tangent line to the curve $x^2 + 2xy - y^2 + x = 2$ at the point $(1, 2)$.

A. $-5/2$
B. $9/2$
C. $7/2$
D. $3/2$
E. $5/2$

$2x + 2y + 2xy' - 2yy' + 1 = 0$

$x = 1$ and $y = 2$

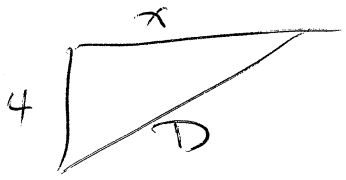
$2 + 4 + 2y' - 4y' + 1 = 0$

$7 - 2y' = 0$

$y' = \frac{7}{2}$

5. A plane flying horizontally at an altitude of 4 mi and a speed of 100 mi/h passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 5 mi away from the station.

A. 50 mi/h
B. 80 mi/h
C. 70 mi/h
D. 60 mi/h
E. 90 mi/h




$D^2 = x^2 + 16$

$2DD' = 2xx' = 2x(100)$

when $D = 5$, $x = 3$

$10D' = 600$

$D' = 60$

6. The volume of a cube is increasing at a rate of 100 cubic inches per minute. How fast is the side of the cube increasing at a time when the length of the side is 10 inches?

A. 1/6 inch per minute
B. 1/3 inch per minute
C. 1/5 inch per minute
D. 1/4 inch per minute
E. 1/2 inch per minute

$V = x^3$

$V' = 3x^2x'$

$100 = 3x^2x'$

when $x = 10$

$100 = 300x'$

$x' = \frac{1}{3}$

7. Suppose that $f(x) = \dfrac{x^2 + x^{3/2} + 8x}{x^{1/2}}$. Then $f'(4) =$

A. 6
B. 9
C. 7
D. 5
E. 8

$f(x) = x^{3/2} + x + 8x^{1/2}$

$f'(x) = \frac{3}{2}x^{1/2} + 1 + 4x^{-1/2}$

$f'(4) = \frac{3}{2} \cdot 2 + 1 + \frac{4}{2} = 6$

8. Find the limit

$$\lim_{t \to 0} \left( \frac{\tan 6t}{\sin 3t} + \cos 2t \right)$$

A. 4
B. 3
C. 2
D. 1
E. 6

$\frac{\tan 6t}{\sin 3t} + \cos 2t$

$= \frac{\sin 6t}{\cos 6t} \cdot \frac{1}{\sin 3t} + \cos 2t$

$= \frac{\sin 6t}{6t} \cdot 6 \cdot \frac{1}{\cos 6t} \cdot \frac{3t}{\sin 3t} \cdot \frac{1}{3} + \cos 2t$

$\xrightarrow{t \to 0} 1 \cdot 6 \cdot 1 \cdot 1 \cdot \frac{1}{3} + 1$

$= 2 + 1 = 3$

9. If $f(x) = (x^2 - 1)\sqrt{x^3 - 4}$, compute $f'(2)$.

A. $\frac{7}{2}$
B. 17
C. 7
D. $\frac{13}{2}$
E. -1

$f'(x) = 2x\sqrt{x^3-4} + (x^2-1) \cdot \frac{1}{2}(x^3-4)^{-1/2} \cdot 3x^2$

$f'(2) = 4\sqrt{8-4} + 3 \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{8-4}} \cdot 12$

$= 8 + 9 = 17$

10. Find an equation for the tangent line to the graph of $y = x^3 \ln x$ at the point $(e, e^3)$.

A. $y = 3e^2x + 2e^3$
B. $y = 3e^2x - 2e^3$
C. $y = 4e^2x + 3e^3$
D. $y = 4e^2x - 3e^3$
E. $y = 3ex - 3e^2 + e^3$

$y' = 3x^2 \ln x + x^3 \cdot \frac{1}{x}$

$= 3x^2 \ln x + x^2$

$y'(e) = 3e^2 + e^2 = 4e^2$

tangent line

$y - e^3 = 4e^2(x - e)$

$y - e^3 = 4e^2x - 4e^3$

$y = 4e^2x - 3e^3$

11. Find $\dfrac{dy}{dx}$ if $y = e^{x \sec x}$.

A. $e^{x \sec x}(x \sec x \tan x + \sec x)$
B. $e^{x \sec x}(x \sec x + \sec^2 x)$
C. $e^{x \sec x}(x \sec x + \sec x \tan x)$
D. $e^{x \sec x}(x \sec^2 x + \sec x \tan x)$
E. $e^{x \sec x}(x \sec^2 x + \sec x)$

$y' = e^{x \sec x}(\sec x + x \sec x \tan x)$

12. $y = \dfrac{e^x}{\sqrt{x}}$ then $\dfrac{dy}{dx} =$

A. $\dfrac{2xe^x - e^x}{2x^{3/2}}$
B. $\dfrac{e^x + \sqrt{x}}{x^{3/2}}$
C. $\dfrac{2e^x}{x^{3/2}}$
D. $\dfrac{xe^x}{x^{3/2}}$
E. $-\dfrac{e^x}{2x^{3/2}}$

$y' = \dfrac{e^x\sqrt{x} - e^x \cdot \frac{1}{2}x^{-1/2}}{(\sqrt{x})^2}$

$= \dfrac{xe^x - \frac{1}{2}e^x}{x^{3/2}}$

$= \dfrac{2xe^x - e^x}{2x^{3/2}}$

13. Find $\frac{d}{dx}(\sin x)^x$

A. $(\cos x)^x(x\cos x + \cot x))$

B. $(\sin x)^x(x\cot x + \ln(\sin x))$

C. $(\sin x)^x(x\cos x + \ln(\cos x))$

D. $(\sin x)^x(x\cos x + \ln(\sin x))$

E. $(\cos x)^x(x(\sin x)^{x-1})$

$$y = (\sin x)^x$$

$$\ln y = x \ln(\sin x)$$

$$\frac{1}{y} \cdot y' = \ln(\sin x) + x \cdot \frac{1}{\sin x} \cdot \cos x$$

$$= \ln(\sin x) + x\cot x$$

$$y' = (\sin x)^x \left(\ln(\sin x) + x\cot x\right)$$