MA 161 EXAM II **Fall 2005**

Name ____

ten–digit Student ID number ________________________________

Division and Section Numbers ________________________________

Recitation Instructor ______________________________________

Instructions:

1. Fill in all the information requested above and on the scantron sheet.
2. This booklet contains 14 problems, each worth 7 points. You get 2 points if you fully comply with instruction 1. The maximum score is 100 points.
3. For each problem mark your answer on the scantron sheet and also circle it in this booklet.
4. Work only on the pages of this booklet.
5. Books, notes, calculators are not to be used on this test.
6. At the end turn in your exam and scantron sheet to your recitation instructor.

1. If $f(x) = \dfrac{x^2 - 2\sqrt{x}}{x}$ then $f'(4) =$

a. $\dfrac{9}{8}$

b. $\dfrac{5}{4}$

c. $\dfrac{7}{8}$

d. $\dfrac{-5}{8}$

e. $\dfrac{-3}{4}$

$f(x) = x - 2x^{-1/2}$

$f'(x) = 1 + x^{-3/2}$

$f'(4) = 1 + 4^{-3/2} = 1 + \frac{1}{8}$

2. If $g(x) = \dfrac{ax + b}{cx + d}$ then $g'(1) =$

a. $\dfrac{a + b - c - d}{c + d}$

b. $\dfrac{ad - bc}{(c + d)^2}$

c. $\dfrac{a + b - c - d}{(c + d)^2}$

d. $\dfrac{ad + bc}{c + d}$

e. $\dfrac{ad + bc}{(c + d)^2}$

$g'(x) = \dfrac{a(cx+d) - (ax+b)c}{(cx+d)^2}$

$g'(1) = \dfrac{ad - bc}{(c+d)^2}$

3. A box with no top has width twice its height and length four times its height. The material for the sides costs \$6/in^2 and for the base \$4/in^2. If its height is s in. find the rate of change of the cost of the box with respect to s in \$/in.

a. $52s\frac{\$}{in}$

b. $62s\frac{\$}{in}$

c. $104s\frac{\$}{in}$

d. $208s\frac{\$}{in}$

e. $86s\frac{\$}{in}$

$w = 2h, \quad l = 4h$

$C = 6(2\cdot w\cdot h + 2 l\cdot h) + 4(w\cdot l)$

$C = 6(2\, 2h\cdot h + 2\cdot 4h\cdot h) + 4(2h\cdot 4h)$

$= 6\cdot 12h^2 + 4\cdot 8\cdot h^2$

$= (72+32)h^2 = 104h^2$

$\frac{dc}{dh} = 208h$

4. If $f(x) = \sec x + \tan x$ then $f'(x) =$ $\sec x \tan x + \sec^2 x$

a. $\sin x + \cos x$

b. $\dfrac{\cos x + 1}{\sin^2 x}$

c. $\tan^2 x \sec x$

d. $\tan x \sec x$

e. $\dfrac{\sin x + 1}{\cos^2 x}$

$= \dfrac{\cancel{\sec x}\sin x}{\cos^2 x} + \dfrac{1}{\cos^2 x}$

$= \dfrac{\sin x + 1}{\cos^2 x}$

5. If $f(x) = (1 + \cos^2 x)^6$ then $f'\left(\frac{\pi}{4}\right) =$

a. $-\left(\frac{3}{2}\right)^6$

b. $-2\left(\frac{3}{2}\right)^5$

c. $-4\left(\frac{3}{2}\right)^6$

d. $4\left(\frac{3}{2}\right)^6$

e. $-4\left(\frac{3}{2}\right)^5$

$f'(x) = 6\,(1+\cos^2 x)^5\, 2\cos x(-\sin x)$

$= -6\left(1+\frac{1}{2}\right)^5 \cdot 2\,\frac{1}{2}$

$= -6\left(\frac{3}{2}\right)^5 = -4\left(\frac{3}{2}\right)^6$

6. $D^{125} xe^{-x} =$

a. $-(x - 124)e^{-x}$

b. $-(x - 125)e^{-x}$

c. $(x - 125)e^{-x}$

d. $(x - 124)e^{-x}$

e. none of the above

$D(xe^{-x}) = e^{-x} - xe^{-x}$

$D^2(\,) = -2e^{-x} + xe^{-x}$

$D^3(\,) = 3e^{-x} - xe^{-x}$

$D^{125}(\,) = 125e^{-x} - xe^{-x}$

7. Find the tangent line to $y = \arcsin(x)$ at $x = \dfrac{\sqrt{3}}{2}$.

a. $y = \dfrac{\pi}{6} + x - 2$

b. $y = \dfrac{\pi}{6} + \dfrac{2\sqrt{3}}{3}\, x - 2$

c. $y = \dfrac{\pi}{3} + 2x - \sqrt{3}$

d. $y = \dfrac{\pi}{6} + 2x - \sqrt{3}$

e. $y = \dfrac{\pi}{3} + \dfrac{2\sqrt{3}}{3}\, x - 2$

$y' = \dfrac{1}{\sqrt{1-x^2}}, \quad y' = 2$

$y\left(\dfrac{\sqrt{3}}{2}\right) = \dfrac{\pi}{3}$

$y = \dfrac{\pi}{3} + 2\left(x - \dfrac{\sqrt{3}}{2}\right)$

8. If $g(u) = \dfrac{e^{2u}}{e^u + e^{-u}}$ then $g'(u) =$

a. $\dfrac{2e^{2u}}{e^{2u} + 2 + e^{-2u}}$

b. $\dfrac{2e^{2u}}{e^u + e^{-u} + 1}$

c. $\dfrac{e^{3u} + 3e^u}{e^{2u} + 2 + e^{-2u}}$

d. $\dfrac{e^{2u} + e^{-2u}}{e^{2u} + 2 + e^{-2u}}$

e. none of the above

$$\frac{2e^{2u}(e^u + e^{-u}) - e^{2u}(e^u - e^{-u})}{(e^u + e^{-u})^2}$$

$$= \frac{e^{3u} + 3e^u}{e^{2u} + 2 + e^{-2u}}$$

9. $\dfrac{d}{dx}(x^{\cos x}) =$

a. $x^{1+\cos x}(\cos x - \sin x \ln x)$

b. $(-\sin x)x^{\cos x}$

c. $(\cos x)x^{(\cos x-1)}$

d. $x^{(\cos x-1)}(\cos x - x\sin x \ln x)$

e. $x^{\cos x} \ln \cos x$

$\left(e^{\cos x \ln x}\right)' = x^{\cos x}$

$\left(-\sin x \ln x + \dfrac{\cos x}{x}\right)$

10. Given $x^y = y^x$, then $\dfrac{dy}{dx} =$

a. $\dfrac{1-\ln x}{1-\ln y}$

b. $x^{y-x}\ln x$

c. $(1-\ln x)\dfrac{y}{x}$

d. $(1-\ln y)\dfrac{y}{x}$

e. $\left(\dfrac{y}{x}\right)^2 \dfrac{1-\ln x}{1-\ln y}$

$y \ln x = x \ln y$

$y' \ln x + \dfrac{y}{x} = \ln y + \dfrac{x y'}{y}$

$y'\left(\dfrac{x}{y} - \ln x\right) = \dfrac{y}{x} - \ln y$

$y' = \dfrac{\frac{y}{x} - \ln y \quad (xy)}{\frac{x}{y} - \ln x \quad (xy)}$

$y' = \dfrac{y^2 - xy \ln y}{x^2 - xy \ln x}$

$= \dfrac{y^2 - y^2 \ln x}{x^2 - x^2 \ln y} = \dfrac{y^2}{x^2}\left(\dfrac{1-\ln x}{1-\ln y}\right)$

11. Given $f(x) = \dfrac{\ln x}{x^2}$, then $f''(x) =$

$= (\ln x)\, x^{-2}$

a. $-\dfrac{1}{2x^2}$

b. $\dfrac{6 \ln x}{x^4}$

c. $\dfrac{1 - 6\ln x}{x^4}$

d. $\dfrac{1 - 2\ln x}{x^3}$

e. none of the above

$f'(x) = x^{-3} - 2 \ln x\, x^{-3}$

$f''(x) = -3x^{-4} - 2x^{-4}$
$+ 6 \ln x \cdot x^{-4}$

$= \dfrac{-5 + 6\ln x}{x^4}$

12. A particle moves along the curve $y = \sqrt[3]{11 + x^4}$. As it reaches the point $(2, 3)$, the y–coordinate is increasing at a rate of 32 $\frac{cm}{s}$. Then, the x–coordinate at that instant is increasing at a rate of

a. 27 $\frac{cm}{s}$

b. 9 $\frac{cm}{s}$

c. 13.5 $\frac{cm}{s}$

d. 6.75 $\frac{cm}{s}$

e. None of the above

$y = (11 + x^4)^{1/3}$

$y' = \frac{1}{3}(11 + x^4)^{-2/3} \cdot 4x^3 \cdot x'$

$\cancel{32} = \frac{1}{3}(27)^{-2/3} \cdot \cancel{4 \cdot 8} \cdot x'$

$1 = \frac{1}{3^3} \cdot x'$

$27 = x'$

13. The minute hand on a watch is 9 cm long and the hour hand is 4 cm long. How fast, in $\frac{cm}{h}$, is the distance between the tips of the hands increasing at ten o'clock?

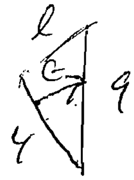
a. $\dfrac{1-8\sqrt{3}}{11\sqrt{61}\pi}$

b. $\dfrac{33\sqrt{3}\pi}{\sqrt{61}}$

c. $\sqrt{61}$

d. $\dfrac{11\sqrt{61}\pi}{6}$

e. $\dfrac{6\sqrt{61}}{11\pi}$



$$\ell = \sqrt{9^2+4^2-2\cdot 4\cdot 9\cdot \cos\theta}$$

$$\ell' = \frac{+4\cdot 9 \sin\theta\cdot\theta'}{\ell}$$

$$\theta = 2\cdot\frac{2\pi}{12} = \frac{\pi}{3}, \quad \sin\theta = \frac{\sqrt{3}}{2}, \quad \ell = \sqrt{9^2+4^2-4\cdot 9} = \sqrt{61}$$

$$\theta' = 2\pi - \frac{2\pi}{12} = \frac{\pi\, 11}{6} \qquad \ell' = \frac{36\cdot\frac{\sqrt{3}}{2}\cdot\frac{11\pi}{6}}{\sqrt{61}} = \frac{33\sqrt{3}\pi}{\sqrt{61}}$$

14. The edge of a cube was found to be 20 cm with a possible error in measurement of 0.1 cm. Using differentials, the percentage error in the surface area of the cube is

a. 1%

b. 2%

c. 0.5%

d. 5%

e. 2.5%

$$A = 6x^2$$

$$dA = 12x\,dx$$

$$\frac{dA}{A}\cdot 100 = \frac{12x\,dx}{6x^2}100 = \frac{2}{x}\,dx\cdot 100$$

$$\frac{2}{20}(.1)100 = 1\%$$