MA 161M EXAM 2 Oct. 22, 2002

Name Key

Differentiate the following:

(8 pts) 1) $y = \dfrac{\sin x + \cos x}{x^2}$

$y' = \dfrac{x^2(\cos x - \sin x) - 2x(\sin x + \cos x)}{x^4}$

$y' = \dfrac{(\cos x)(x-2) - (\sin x)(x+2)}{x^3}$

$y' = \dfrac{x\cos x - x\sin x - 2\sin x - 2\cos x}{x^3}$

$y' = \dfrac{(x-2)\cos x - (x+2)\sin x}{x^3}$

ANSWER ______________________

(7 pts) 2) $y = \sqrt[5]{x}\ \tan x$

$y' = \frac{1}{5}x^{-4/5}\tan x + x^{1/5}\sec^2 x$

$y' = x^{1/5}\left[\dfrac{\tan x}{5x} + \sec^2 x\right]$

ANSWER ______________________

(7 pts) 3) $y = (\ln x)(\tan^{-1} x)$

$y' = \frac{1}{x}\tan^{-1}x + \dfrac{\ln x}{1+x^2}$

ANSWER $y' = \frac{1}{x}\tan^{-1}x + \dfrac{\ln x}{1+x^2}$

(8 pts) 4) $y = e^{x\cos x}$

$y' = e^{x\cos x}[\cos x - x\sin x]$

ANSWER $y' = e^{x\cos x}[\cos x - \sin x]$

(7 pts) 5) $y = \cos(2\pi x^2 + x)$

$y' = \left[-\sin(2\pi x^2 + x)\right]\left[4\pi x + 1\right]$

ANSWER $y' = -(4\pi x + 1)\sin(2\pi x^2 + x)$

6) Differentiate and simplify your answer:

(10 pts) (a) $y = \ln\sqrt{x^2 + 2x}$

$y = \frac{1}{2}\ln(x^2 + 2x)$

$y' = \frac{1}{2}\frac{1}{(x^2+2x)}(2x+2)$

$y' = \frac{(x+1)}{x(x+2)}$

ANSWER $y' = \frac{(x+1)}{x(x+2)}$

(8 pts) (b) $y = x^{\ln x}$

$\ln y = (\ln x)(\ln x) = (\ln x)^2$

$\therefore \frac{1}{y}y' = \frac{2\ln x}{x}$

$\therefore y' = 2\left(\frac{\ln x}{x}\right)x^{\ln x}$

$y' = \left(2\frac{\ln x}{x}\right)x^{\ln x}$

ANSWER $y' = (2\ln x)\, x^{\ln x - 1}$

(5 pts) 7) Find $\lim_{\theta \to 0} \frac{\tan 3\theta}{\theta}$

$\frac{\tan 3\theta}{\theta} = \frac{\sin 3\theta}{\theta} \cdot \frac{1}{\cos 3\theta} = 3\left(\frac{\sin 3\theta}{3\theta}\right) \cdot \frac{1}{\cos 3\theta}$

$\lim_{\theta \to 0} \frac{\sin 3\theta}{3\theta} = 1$

$\lim (\cos 3\theta) = 1$

ANSWER 3

(10 pts) 8) Find the slope of the tangent to the curve defined by $x + \sin y = xy$ at the point $(0,0)$ on the curve.

$x + \sin y = xy$

Differentiating wrt x implicitly gives

$$1 + (\cos y)y' = y + xy'$$

$$y'(\cos y - x) = y - 1$$

$$y' = \frac{y-1}{\cos y - x}$$

$$\therefore\ y'(0,0) = \frac{0-1}{\cos 0 - 0} = \frac{-1}{1} = -1$$

ANSWER -1

(10 pts) 9 The half life of Polonium 210 is 140 days. How many days will it take for a sample to decay to 5% of its original value. Since you are not allowed to use a calculator, give your answer in the form indicated.

Decay eqn is

$$x(t) = x_0 e^{kt} \quad (1)$$

To get k, knowing half life τ

$$x(\tau) = \frac{x_0}{2}.$$

subs into (1) gives

$$\frac{1}{2} = e^{k\tau} \quad \text{and so} \quad k = -\frac{\ln 2}{\tau} = -\frac{\ln 2}{140} \quad (2)$$

Want value of t when $x(t) = .05 x_0$

subs into (1) gives:

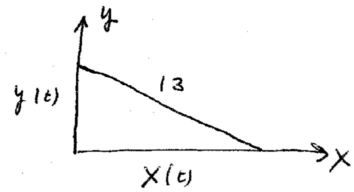
$$(.05)x_0 = x_0 e^{kt}$$

$$\text{or} \quad \ln(.05) = kt$$

$$\therefore\ \cancel{t = \ln(.05)} \text{ or } t = \frac{\ln .05}{k} = \frac{-\ln .05}{\ln 2} \times 140$$

$t = \boxed{-140} \times \dfrac{\ln \boxed{.05}}{\ln \boxed{2}}$

(10 pts) 10) A 13 ft. ladder is leaning against a house when its base starts to slide away. When the base is 12 ft. from the house, the base is moving at the rate of 5 ft./sec. How fast is the top of the ladder sliding down the wall then?



Always:

$[x(t)]^2 + [y(t)]^2 = 169 \quad (1)$

$\therefore 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \quad (2)$

$\therefore$ Prob. is to find $\frac{dy}{dt}$ when $x(t) = 12$, given that $\frac{dx}{dt} = 5$ ft/sec

From (2) we get that

$\frac{dy}{dt} = \frac{-x\frac{dx}{dt}}{y} \quad (3)$ If $x(t) = 12$ then from (1) we get: $144 + y(t)^2 = 169$

$y^2(t) = 25$

$y(t) = 5$

Subs into (3) give

$\frac{dy}{dt} = -\frac{12 \text{ ft}}{5 \text{ ft}} \, 5 \text{ ft/sec} = -12$ ft/sec.

ANSWER 12 ft./sec.

(10 pts) 11) Find the linearization of $f(x) = \sqrt[3]{x}$ at $x = 27$. Use your linearization to approximate $\sqrt[3]{26}$. Leave your answer in the form of an improper fraction.

For any function f, the linearization of f about a is given by

$L(x) = f(a) + f'(a)(x-a)$

for this problem $f(x) = \sqrt[3]{x}$, $a = 27$

$\therefore f(a) = \sqrt[3]{27} = 3 \qquad f'(x) = \frac{1}{3}x^{-2/3}, \quad f'(a) = \frac{1}{3}\left(\frac{1}{27}\right)^{2/3} = \frac{1}{3}\cdot\frac{1}{9} = \frac{1}{27}$

$\therefore L(x) = 3 + \frac{1}{27}(x-27)$

$L(26) = 3 + \frac{1}{27}(26-27) = 3 - \frac{1}{27} = \frac{80}{27}$

=

$L(x) = 3 + \frac{1}{27}(x-27)$

ANSWER $\frac{80}{27}$