Name GRADING KEY

10-digit PUID

RECITATION Section Number and time

Recitation Instructor

Lecturer

Instructions:

1. Fill in all the information requested above and on the scantron sheet. On the scantron sheet also fill in the little circles for your name, section number and PUID.
2. This booklet contains 16 problems, each worth 6 points (except problems 1, 5, 13 and 15 are worth 7 points each). The maximum score is 100 points. The test booklet has 9 pages, including this one.
3. For each problem mark your answer on the scantron sheet and also circle it in this booklet.
4. Work only on the pages of this booklet.
5. Books, notes, calculators or any electronic devices are not to be used on this test.
6. At the end turn in your exam and scantron sheet to your recitation instructor.

1. The graph of $f(x) = \tan x$ is shrunk horizontally by a factor of 3, then translated to the left by 2 units and finally translated down 4 units to obtain the graph of $h(x)$. Then $h(x) =$

A. $\tan(3x+2) - 4$

B. $\tan\left(\frac{1}{3}x+2\right) + 4$

C. $\tan\left(\frac{1}{3}x+4\right) - 2$

D. $\tan(3x-2) - 4$

E. $\tan(3x+6) - 4$

$\tan x \rightarrow \tan(3x)$

$\rightarrow \tan(3(x+2))$

$\rightarrow \tan(3(x+2)) - 4$

$= \tan(3x+6) - 4$

2. The center and radius of the circle given by $2x^2+2y^2-4x+8y+1=0$ are respectively

A. $(2,-4)$, 3

B. $(2,-4)$, $\sqrt{3}$

C. $(-1,2)$, $\frac{3}{\sqrt{2}}$

D. $(1,-2)$, 3

E. $(1,-2)$, $\frac{3}{\sqrt{2}}$

$2(x^2-2x+\underline{1}) + 2(y^2+4y+\underline{\underline{4}})$

$= -1 + \underline{2} + \underline{\underline{8}}$

$\rightarrow 2(x-1)^2 + 2(y+2)^2 = 9$

$\rightarrow (x-1)^2 + (y+2)^2 = \frac{9}{2}$

$\rightarrow$ center is $(1,-2)$ and radius is $\frac{3}{\sqrt{2}}$

form: $(x-h)^2 + (y-k)^2 = r^2$

3. An equation of the line perpendicular to $2x + 6y = 2$ and containing the point $(1, 2)$ is

A. $3x + y = 1$

B. $3x - y = 1$

C. $\frac{1}{3}x + 2y = -\frac{11}{3}$

D. $-\frac{1}{3}x + 2y = \frac{11}{3}$

E. $\frac{1}{3}x - y = -\frac{5}{3}$

$2x+6y=2 \rightarrow y = -\frac{2}{6}x + \frac{2}{6}$

slope $= -\frac{1}{3} \rightarrow$ perpendicular slope $= 3$

Line $\perp$ has equation: $y-2 = 3(x-1)$
with slope 3 and containing $(1,2)$

$\rightarrow y - 2 = 3x - 3$

$\rightarrow 1 = 3x - y$

4. Let $f(x) = \frac{1}{x+2}$ and $g(x) = \frac{1}{x}$. What is the domain of $f \circ g$?

A. $x \neq -2$

B. $x \neq 0$

C. $x \neq 0$ and $x \neq -2$

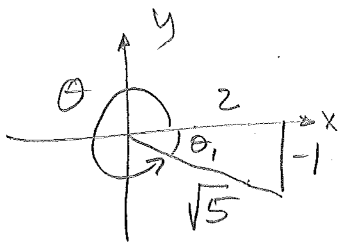
D. $x \neq 0$ and $x \neq -\frac{1}{2}$

E. $x \neq -\frac{1}{2}$ and $x \neq -2$

$g(x) = \frac{1}{x} \rightarrow x \neq 0$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x}+2} = \frac{1}{\frac{1+2x}{x}} = \frac{x}{1+2x} \rightarrow x \neq -\frac{1}{2}$$

5. If $\frac{3\pi}{2} < \theta < 2\pi$ and $\tan\theta = -\frac{1}{2}$, then $\cos\theta =$



$\Rightarrow \cos\theta_1 = \frac{2}{\sqrt{5}}$

A. $\frac{1}{\sqrt{5}}$

B. $\frac{2}{\sqrt{5}}$

C. $-\frac{2}{\sqrt{5}}$

D. $-\frac{1}{\sqrt{5}}$

E. $\frac{\sqrt{5}}{2}$

6. Which of the following statements are true for all values of x and y?

False I. $2^x \cdot 2^y = 2^{xy}$

False II. $\frac{2^x}{2^y} = 2^{y-x}$

False III. $2^x + 2^y = 2^{x+y}$

A. None are true.

B. Only I.

C. Only I and III.

D. Only II.

E. All are true.

I. $x = 2, \; y = 3$

$2^2 \cdot 2^3 = 4 \cdot 8 = 32$

$2^{2\cdot 3} = 2^6 = 64$

II $\frac{2^x}{2^y} = 2^{x-y}$

III $x = 1, \; y = 2$

$2^1 + 2^2 = 5$

$2^{1+2} = 2^3 = 8$

7. Solve for x: $x^2 - x - 6 > 0$.

$x^2 - x - 6 = (x-3)(x+2)$

sign of $x^2 - x - 6$:

x: $-\infty$ … -2 … 3 … ∞			
$x+2$	$-$ 0	$+$	$+$
$x-3$	$-$	$-$ 0	$+$
x^2-x-6	$+$	$-$	$+$

$(-\infty, -2) \cup (3, \infty)$

A. $(-\infty, \infty)$

B. $(-\infty, -3) \cup (2, \infty)$

C. $(-\infty, -2) \cup (3, \infty)$

D. $(-\infty, 3)$

E. $(-2, 3)$

8. If $f(x) = \sqrt{4 - 2x}$, what is the range of f^{-1} ?

range of f^{-1} is domain of f.

domain of f is $4 - 2x \geq 0$

$\rightarrow -2x \geq -4$

$\rightarrow x \leq 2$

$\rightarrow (-\infty, 2]$

A. $[2, \infty)$

B. $[0, \infty)$

C. $(-\infty, 2]$

D. $[0, 2]$

E. $(0, 2)$

9. Solve the equation $e^{4-2x} = \frac{1}{e^2}$.

$$e^{4-2x} = e^{-2}$$

$$\rightarrow \ln(e^{4-2x}) = \ln(e^{-2})$$

$$\rightarrow 4-2x = -2$$

$$\rightarrow -2x = -6$$

$$\rightarrow x = 3$$

A. $x = 3$

B. $x = 2 - 2\ln 2$

C. $x = 1 + \ln 2$

D. $x = 1$

E. $x = 2 + \ln 2$

10. Evaluate $\lim_{x\to -1} \frac{x^2 - x - 2}{x+1} = \frac{1+1-2}{-1+1} = \frac{0}{0}$

$$= \lim_{x\to -1} \frac{(x+1)(x-2)}{x+1}$$

$$= \lim_{x\to -1} x-2$$

$$= -1-2$$

$$= -3$$

A. -3

B. 2

C. -2

D. 1

E. -1

11. Evaluate $\lim_{x \to 1^+} \frac{2x}{3x-3} =$

$$= \lim_{x \to 1^+} \frac{2x}{3(x-1)} = \frac{2}{0^+} = +\infty$$

$(x > 1 \Rightarrow x - 1 > 0)$

A. $\frac{2}{3}$

B. $-\frac{2}{3}$

C. ∞

D. $-\infty$

E. 0

12. Compute $\lim_{x \to \infty} \frac{x+3}{2x^2-4x} =$

$$= \lim_{x \to \infty} \frac{\cancel{x}\left(1 + \frac{3}{x}\right)}{\cancel{x}\left(2x + 4\right)}$$

$$= \frac{1+0}{\infty} = \frac{1}{\infty} = 0$$

A. $\frac{1}{2}$

B. $-\frac{1}{4}$

C. $\frac{3}{2}$

D. $-\frac{3}{4}$

E. 0

13. Evaluate $\lim_{x\to 1} \dfrac{\sqrt{3x+1}-2}{x-1} = \dfrac{\sqrt{4}-2}{1-1} = \dfrac{0}{0}$

$= \lim_{x\to 1} \dfrac{\sqrt{3x+1}-2}{x-1} \cdot \dfrac{\sqrt{3x+1}+2}{\sqrt{3x+1}+2}$

$= \lim_{x\to 1} \dfrac{3x+1-4}{(x-1)(\sqrt{3x+1}+2)}$

$= \lim_{x\to 1} \dfrac{3\cancel{(x-1)}}{\cancel{(x-1)}(\sqrt{3x+1}+2)}$

$= \dfrac{3}{\sqrt{4}+2} = \dfrac{3}{4}$

A. $\dfrac{3}{\sqrt{2}}$

B. $\dfrac{3}{2}$

C. $\dfrac{3}{4}$

D. $\dfrac{1}{4}$

E. $-\dfrac{3}{2}$

14. If

$$f(x) = \begin{cases} 3x+a, & \text{if } x \le 1 \\ \sqrt{2x-1}, & \text{if } x > 1 \end{cases}$$

and if f is continuous at $x = 1$, what is a ?

A. 2

B. 1

C. -1

D. -2

E. 3

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (3x+a) = 3+a$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \sqrt{2x-1} = \sqrt{2-1} = 1$

$\lim_{x\to 1} f(x)$ exists if left- and right-hand limits are equal.

$\to 3+a = 1 \to a = -2$

Then $\lim_{x\to 1} f(x) = 1 = 3(1)-2 = f(1)$

and f is continuous at $x=1$.

15. If $A = \lim_{x\to\infty} \frac{-x}{\sqrt{x^2+4}} + 1$ and $B = \lim_{x\to-\infty} \frac{-x}{\sqrt{x^2+4}} + 1$, then

A. $A = 1,\ B = -1$

B. $A = -1,\ B = 1$

C. $A = 2,\ B = 0$

D. $A = 0,\ B = 0$

E. $A = 0,\ B = 2$

Note: $\frac{-x}{\sqrt{x^2+4}} = \frac{-x}{\sqrt{x^2(1+\frac{4}{x^2})}} = \frac{-x}{\sqrt{x^2}\sqrt{1+\frac{4}{x^2}}}$

$$= \begin{cases} \frac{-x}{-x\sqrt{1+\frac{4}{x^2}}} = \frac{1}{\sqrt{1+\frac{4}{x^2}}} & \text{if } x<0 \\ \frac{-x}{x\sqrt{1+\frac{4}{x^2}}} = -\frac{1}{\sqrt{1+4/x^2}} & \text{if } x>0 \end{cases}$$

$$A = \lim_{x\to\infty}\left(-\frac{1}{\sqrt{1+4/x^2}}+1\right) = -\frac{1}{1}+1 = 0$$

$$B = \lim_{x\to-\infty}\left(\frac{1}{\sqrt{1+4/x^2}}+1\right) = \frac{1}{1}+1 = 2$$

16. Evaluate $\lim_{x\to2} \frac{1+\sqrt{x+2}}{1-\sqrt{2x}}$.

A. $-\frac{3}{4}$

B. -3

C. -5

D. $1+\sqrt{2}$

E. The limit does not exist.

$$= \frac{1+\sqrt{2+2}}{1-\sqrt{2(2)}} = \frac{1+\sqrt{4}}{1-\sqrt{4}} = \frac{3}{-1}$$

$$= -3$$

note: $\frac{1+\sqrt{x+2}}{1-\sqrt{2x}}$ is continuous at $x=2$