Solutions & Key

1) The interval which corresponds to the values of x satisfying $|3x-2|<5$ is

$-5 < 3x-2 < 5$

$-3 < 3x < 7$

$-1 < x < \frac{7}{3}$

$(-1, \frac{7}{3})$

A) $(1,3)$

$\longrightarrow$ B) $(-1,7/3)$

C) $(1,3]$

D) $(-1,7/3]$

E) $[1,3]$

2) The center and the radius of the circle represented by the equation $3x^2+3y^2-3x+2y = 1$ are respectively

$3(x^2 - x + \underline{\tfrac{1}{4}} + y^2 + \tfrac{2}{3}y + \underline{\tfrac{1}{9}}) = 1 + \underline{\tfrac{3}{4}} + \underline{\tfrac{1}{3}}$

$3[(x-\tfrac{1}{2})^2 + (y+\tfrac{1}{3})^2] = \frac{12+9+4}{12}$

$= \frac{25}{12}$

$(x-\tfrac{1}{2})^2 + (y+\tfrac{1}{3})^2 = \frac{25}{36} = \left(\frac{5}{6}\right)^2$

Ctr. $(\tfrac{1}{2}, -\tfrac{1}{3})$, $r = 5/6$

A) $(1/2,1/3)$ and $5/6$

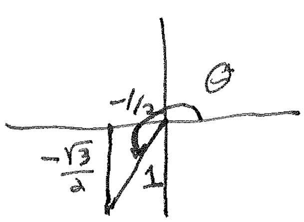
B) $(1,-2)$ and $4/9$

C) $(1/4,1)$ and $5/6$

$\longrightarrow$ D) $(1/2,-1/3)$ and $5/6$

E) $(1/2,-1/3)$ and $7/6$

3) If $\pi<\theta<3\pi/2$ and $\cos\theta = -1/2$ then $\sin\theta$ is equal to



$\sin\theta = -\frac{\sqrt{3}}{2}$

A) $-1/2$

B) $1/2$

C) $\sqrt{3}/2$

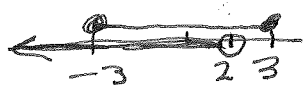
D) $\sqrt{3}/4$

$\longrightarrow$ E) $-\sqrt{3}/2$

4) The domain of the function $f(x) = \frac{\sqrt{9-x^2}}{\sqrt{2-x}}$ is

need $\underbrace{x^2 \le 9}$ and $2 > x$

$-3 \le x \le 3$



$-3 \le x < 2$

$[-3, 2)$

A) $[-3, 3]$

B) $[-3, 2)$

C) $(-3, 2]$

D) $(-3, 2)$

E) $[-3, 2]$

5) Let $f(x) = x^2 + 1$, $g(x) = 2^x$ and $h(x) = x^3$. Then $f \circ g \circ h(x)$ is equal to

$f(g(h(x))) = f(2^{x^3})$

$= (2^{x^3})^2 + 1$

$= 2^{2x^3} + 1$

$= 4^{x^3} + 1$

A) $(2^x)^2 + 1$

B) $2^{6x} + 1$

C) $4^{x^3} + 1$

D) 2^{3x+1}

E) $(x^3 + 2^x)^2 + 1$

6) The graph of $f(x-2) + 5$ can be obtained from the graph of $f(x)$ by

A) Shifting the graph of f to the left by two units and downward by 5 units

B) Shifting the graph of f to the right by two units and downward by 5 units

C) Shifting the graph of f to the right by two units and upward by 5 units

D) Shifting the graph of f to the left by two units and upward by 5 units

E) Shifting the graph of f to along the diagonal by 5 units

7) The quantity $\log_2 3 + 2\log_2 5 + \log_3 9$ is equal to

$= \log_2 3 + \log_2 25 + \underbrace{\log_3 3^2}_{= 2 = \log_2 2^2 = \log_2 4}$

$= \log_2 (3 \cdot 25 \cdot 4)$

$= \log_2 (300)$

→ A) $\log_2 300$

B) $\log_2 450$

C) $\log_2 500$

D) $\log_2 700$

E) $\log_2 900$

8) The inverse of the function $f(x) = \dfrac{5x-3}{3x+7}$ is $f^{-1}(x) =$

Solve: $x = \dfrac{5y-3}{3y+7}$ for y:

$3xy + 7x = 5y - 3$

$7x + 3 = y(5-3x)$

$y = \dfrac{7x+3}{5-3x}$

A) $\dfrac{3x-7}{5-3x}$

B) $\dfrac{3x+5}{7-3x}$

C) $\dfrac{7x-5}{5-3x}$

D) $\dfrac{7x-3}{5-3x}$

→ E) $\dfrac{7x+3}{5-3x}$

9) Evaluate $\lim\limits_{x \to -4} \dfrac{3x^2-48}{x^2+2x-8}$ if it exists. (If it does not exist, choose the answer DNE.)

$= \lim\limits_{x \to -4} \dfrac{3(x^2-16)}{(x+4)(x-2)} = \lim\limits_{x \to -4} \dfrac{3\cancel{(x+4)}(x-4)}{\cancel{(x+4)}(x-2)}$

$= \dfrac{3 \cdot (-8)}{-6} = \dfrac{-24}{-6}$

$= 4$

A) -4

B) -3

C) 3

→ D) 4

E) DNE

10) Evaluate $\lim_{t\to 3} \frac{\sqrt{t+1}-2}{t-3}$, if it exists. (If it does not exist, choose the answer DNE.)

$$= \lim_{t\to 3} \frac{(\sqrt{t+1}-2)}{t-3}\frac{(\sqrt{t+1}+2)}{(\sqrt{t+1}+2)}$$

$$= \lim_{t\to 3} \frac{(t+1-4)}{(t-3)(\sqrt{t+1}+2)}$$

$$= \frac{1}{\sqrt{4}+2} = \frac{1}{4}$$

A) $\frac{1}{5}$

→ B) $\frac{1}{4}$

C) 0

D) ∞

E) DNE

11) Let $a = \lim_{x\to\infty} (x^2 - x)$ and $b = \lim_{x\to 0} (x^2 - x\sin\frac{1}{x})$. Evaluate a and b. (If the limit does not exist, choose DNE.)

$$a = \lim_{x\to\infty} (x^2 - x) = \lim_{x\to\infty} x^2\left(1 - \frac{1}{x^2}\right) = \infty$$

$$b = \lim_{x\to 0} \left(x^2 - x\sin\frac{1}{x}\right) = \lim_{x\to 0} x\left(x - \sin\frac{1}{x}\right)$$

$= 0$ by squeeze thm.

A) $a = 0, b = 0$

B) $a = -\infty$, b DNE

C) $a = 0$, b DNE

D) $a = \infty$, b DNE

→ E) $a = \infty, b = 0$

12) The total number of asymptotes, vertical and horizontal, for the graph of $f(x) = \frac{\sqrt{9x^2+1}}{x}$ is:

<u>horiz. asympt:</u> $\lim_{x\to\infty} \frac{\sqrt{9x^2+1}}{x}$

$$= \lim_{x\to\infty} \frac{\sqrt{x^2(9+\frac{1}{x^2})}}{x} = \lim_{x\to\infty} \sqrt{9+\frac{1}{x^2}} = 3$$

$$\lim_{x\to-\infty} \frac{\sqrt{9x^2+1}}{x} = \lim_{x\to-\infty} \frac{|x|}{x}\sqrt{9+\frac{1}{x^2}} = -3$$

vert. asymptote @ $x=0$.

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A) 0

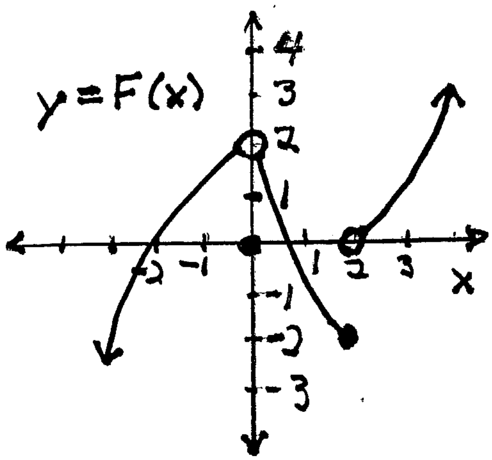
B) 1

C) 2

→ D) 3

E) 4

13) For the function $F(x)$ pictured, which of the following statements are true?



I. $\lim_{x\to 0} F(x) = 2$

II. $\lim_{x\to 2^-} F(x) = 0$

III. F is continuous at x=0

I true
II false
III false

A) I only

B) II only

C) I and II only

D) II and III only

E) All are true

14) The quantity, $\lim_{x\to\frac{\pi}{3}} \dfrac{\cos x - \frac{1}{2}}{x - \frac{\pi}{3}}$, represents the derivative of some function $f(x)$ at some number a. Select an appropriate f(x) and a.

$f(x) = \cos x$, $f(\frac{\pi}{3}) = \cos\frac{\pi}{3} = \frac{1}{2}$

$a = \frac{\pi}{3}$

A) $f(x) = \cos x - \frac{1}{2}, a = \frac{\pi}{3}$

B) $f(x) = \cos x - \frac{1}{2}, a = \pi$

C) $f(x) = \cos x, a = \frac{\pi}{3}$

D) $f(x) = \cos x, a = \pi$

E) $f(x) = 3(\cos x - \frac{1}{2}), a = \pi$