MA 16600
FINAL EXAM INSTRUCTIONS
VERSION 01
May 7, 2014

Your name ____________________ Your TA's name ____________________

Student ID # _______________ Section # and recitation time __________

**1.** You must use a #2 pencil on the scantron sheet (answer sheet).

**2.** Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

**3.** On the scantron sheet, fill in your TA's name (NOT the lecturer's name) and the course number.

**4.** Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

**5.** Fill in the four-digit SECTION NUMBER.

**6.** Sign the scantron sheet.

**7.** Write down YOUR NAME and TA's NAME on the exam booklet.

**8.** There are 25 questions, each worth 8 points. The total is $8 \times 25 = 200$. Blacken your choice of the correct answer in the spaces provided for questions 1–25. Do all your work on the question sheets. Turn in both the scantron sheets and the question sheets when you are finished.

**9.** Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.

**10.** NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

**11.** After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.

**12.** If you finish the exam before 12:25, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 12:25, you should REMAIN SEATED until your TA comes and collects your scantron sheets and exam booklets.

# Exam Policies

1. Students must take pre-assigned seats and/or follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of the above rules may result in score of zero.

# Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: ______________________________

STUDENT SIGNATURE: ______________________________

## Questions

**1.** It is given that $x^2 + y^2 + z^2 + 4x - 6y + 2z = b$ is the equation of a sphere passing through the point $(1, 1, 1)$. What is the radius of the sphere?

A. $\sqrt{29}$

B. $\sqrt{17}$ (correct)

C. 3

D. $\sqrt{8}$

E. . $\sqrt{7}$

**2.** Find the value(s) of $x$ such that the two vectors $\langle x, 2, 1\rangle$ and $\langle x, -2x, 4\rangle$ are perpendicular to each other.

A. $x = 1$

B. $x = 3$

C. $x = 2$ (correct)

D. $x = 1$ or 4

E. $x = 4$

**3.** Find the area of the triangle with vertices

$$\begin{cases} P &= (1,0,1), \\ Q &= (2,-1,2), \\ R &= (2,2,-1). \end{cases}$$

A. $3$

B. $\sqrt{5}$

C. $\frac{\sqrt{5}}{2}$

D. $2\sqrt{3}$

E. $\frac{3\sqrt{2}}{2}$ (correct)

**4.** Find the area of the region bounded by the curves $x = 8 - 2y^2$ and $x = 2y^2 + 4$.

A. $\frac{16}{3}$ (correct)

B. $5$

C. $2$

D. $\frac{5}{2}$

E. $\frac{8}{3}$

**5.** The region bounded by $y = \ln x$, $y = 0$ and $x = e$ is rotated around the $y$-axis. Find a formula for the volume of the resulting solid of revolution by using

(a) the washer method, and

(b) the cylindrical shell method.

A. (a) $\int_1^e \pi(\ln x)^2 dx$ (b) $2\pi \int_0^1 (e - e^y) dy$

B. (a)$\int_1^e 2\pi x(\ln(x)) dx$ (b) $\int_0^1 \pi \left\{e^2 - e^{2y}\right\} dx$

C. (a) $\int_0^1 \pi \left\{y^2 - e^y\right\} dy$ (b) $2\pi \int_1^e (\ln x)^2 dx$

D. (a) $\int_0^1 \pi \left\{e^2 - e^{2y}\right\} dx$ (b) $\int_1^e 2\pi x(\ln(x)) dx$ (correct)

E. (a) $2\pi \int_0^1 (e - e^y) dy$ (b) $\int_1^e \pi(\ln x)^2 dx$.

**6.** A tank in the shape of an inverted circular cone is 4 feet high, and has the radius of 2 foot at the top. Water is filled up to 2 ft high. How much work is needed to pump out all the water through a hose at the top ? (Let **d** represent the density of the water 62.5 lb/ft$^3$.)

A. $\frac{7\pi}{6}\mathbf{d}$ ft-lb

B. $\pi\mathbf{d}$ ft-lb

C. $\frac{5\pi}{3}\mathbf{d}$ ft-lb (correct)

D. $\frac{4\pi}{3}\mathbf{d}$ ft-lb

E. $\frac{\pi}{3}\mathbf{d}$ ft-lb

**7.** Compute the integral

$$\int_0^{\pi/2} \sin^2 x \cos^5 x \; dx.$$

A. $\frac{13}{35}$
B. $\frac{31}{105}$
C. $\frac{3}{35}$
D. $\frac{5}{21}$
E. $\frac{8}{105}$ (correct)

**8.** Compute the integral

$$\int_0^{\frac{\pi}{4}} (\sec x)^4 (\tan x)^2 \; dx.$$

A. $\frac{8}{3}$
B. $\frac{8}{15}$(correct)
C. $\frac{4}{5}$
D. $\frac{10}{3}$
E. $\frac{3}{2}$

**9.** Compute the integral

$$\int_{1/2}^{1} \frac{x dx}{\sqrt{4x^2 - 1}}.$$

A. $\frac{\sqrt{2}}{4}$

B. $\frac{4}{3}$

C. $\frac{\sqrt{3}}{4}$ (correct)

D. $\frac{21}{4}$

E. $\frac{\sqrt{22}}{3}$.

**10.** When one makes a suitable trigonometric substitution to evaluate

$$\int \frac{x^5}{\sqrt{x^2 - 4}} dx,$$

which of the following arises?

A. $16 \int \frac{\sec^4 \theta}{\tan \theta} d\theta$

B. $64 \int \frac{\sec^5 \theta}{\tan \theta} d\theta$

C. $16 \int \sec^5 \theta \tan \theta d\theta$

D. $32 \int \sec^6 \theta d\theta$ (correct)

E. $64 \int \frac{\sec^5 \theta}{\tan^2 \theta} d\theta$

**11.** It is known that $\int \frac{6}{x^2(x-2)}dx = a\ln|x| + \frac{b}{x} + c\ln|x-2| + C$ for some constants $a, b$ and $c$, where $C$ is the integration constant.

What is $b$?

A. $-6$

B. $-3$

C. $\frac{3}{2}$

D. 3 (correct)

E. 6.

**12.** Evaluate

$$\int_3^4 \frac{6x-8}{x^2-3x+2}dx.$$

A. $\ln\frac{3}{2}$

B. $\ln 12$

C. $\ln 10$

D. $\ln\frac{6}{2}$

E. $\ln 36$ (correct)

**13.** Use the Trapezoidal rule with $n = 4$ to find the approximate value of the following integral

$$\int_0^{\pi} (\cos x)^2 \; dx.$$

A. $\frac{\pi}{2}$ (correct)

B. $\pi$

C. $\frac{\sqrt{3}}{2}\pi$

D. $\left(\frac{3}{\sqrt{2}} + \frac{1}{2}\right)\pi$

E. $\left(\frac{3}{2} - \frac{1}{4}\right)\pi$

**14.** Indicate convergence or divergence of each of the following improper integrals. If it converges, evaluate its value.

(a)

$$\int_2^{\infty} \frac{2}{x^2} dx$$

(b)

$$\int_0^1 \frac{x^2}{(1-x^3)^2} dx$$

A. (a) converges to 1 (b) diverges (correct)

B. (a) converges to $\frac{1}{2}$ (b) diverges

C. (a) converges to $-\frac{1}{2}$ (b) converges to $\frac{1}{3}$

D. (a) diverges (b) converges to $\frac{1}{3}$

E. (a) diverges (b) diverges

**15.** The length of the arc of the curve $y = 2x^2, 0 \le x \le \frac{1}{4}$ is represented by the integral

A. $4\int_0^{\frac{\pi}{4}} \sec^2 \tan\theta d\theta$

B. $\int_0^{\frac{\pi}{4}} \tan^3\theta d\theta$

C. $\frac{1}{4}\int_0^{\frac{\pi}{3}} \tan^3\theta d\theta$

D. $\int_0^{\frac{\pi}{3}} \sec^3\theta d\theta$

E. $\frac{1}{4}\int_0^{\frac{\pi}{4}} \sec^3\theta d\theta$ (correct)

**16.** Which of the following represent the area of the surface obtained by rotating the curve $y = \sqrt{1+x^2}$ for $0 \le x \le 2$ about the $x$-axis.

A. $2\pi\int_0^2 \frac{\sqrt{1+x^2}}{\sqrt{1+2x^2}} dx$

B. $\pi\int_0^2 \sqrt{1+x^2} dx$

C. $2\pi\int_0^2 \sqrt{1+2x^2} dx$ (correct)

D. $\pi\int_0^2 \frac{\sqrt{1+2x^2}}{\sqrt{1+x^2}} dx$

E. $2\pi\int_0^2 x\frac{\sqrt{1+2x^2}}{\sqrt{1+x^2}} dx$

**17.** Denote by $(\overline{x}, \overline{y})$ the center of mass of the region $D$ bounded by $y = 2\sqrt{x}$, $x = 0$, $x = 1$ and $y = 0$ with uniform density $\rho$.

What is $\overline{y}$?

A. $\frac{3}{4}$ (correct)

B. $\frac{2}{3}$

C. $\frac{1}{3}$

D. $\frac{2}{5}$

E. $\frac{3}{10}$.

**18.** Which of the following sequences are convergent?

$$\begin{aligned} &\text{(I)} \quad a_n = (-1)^n \frac{\sin n}{n} \\ &\text{(II)} \quad b_n = \frac{\ln n}{\ln(4 \ln n)} \\ &\text{(III)} \quad c_n = \frac{n+1}{(n^2+1)^{\frac{1}{3}}} \end{aligned}$$

A. (I) only (correct)

B. (II) only

C. (III) only

D. (II) and (III) only

E. (I) and (II) only

**19.** Which of the following statements are always true?

(I) If $\lim_{n\to\infty} a_n = 0$, then the series $\sum_{i=1}^{\infty} a_n$ converges.
(II) If a series $\sum_{i=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$
(III) Suppose $a_n > 0$ for all integer $n$ and $\lim_{n\to\infty} n^3 a_n$ converges to a real number, then $\sum_{i=1}^{\infty} a_n$ converges.

A. None

B. All

C. (I) and (II) only

D. (II) and (III) only (correct)

E. (I) and (III).

**20.** Which of the following series converge?

$$\text{(I)} \quad \sum_{n=1}^{\infty} (\frac{n^2 - n}{3n^2 - n})^n$$

$$\text{(II)} \quad \sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1/2}}$$

$$\text{(III)} \quad \sum_{n=2}^{\infty} \frac{n^2}{\sqrt{n^6 - 1}}$$

A. None

B. All

C. (I) and (II) only (correct)

D. (I) and (III)

E. (II) and (III) only

**21.** Find the interval of convergence for the following power series centered at $-2$,

$$\sum_{n=1}^{\infty} \frac{1}{n9^n}(x+2)^{2n}.$$

A. $(-3-2, 3-2)$ (correct)

B. $[-3-2, 3-2)$

C. $(-9-2, 9-2)$

D. $[-9-2, 9-2]$

E. $(-9-2, 3-2)$

**22.** Which of the following is the length of the curve $x = e^{2t} + e^{-2t}, y = 2 - 4t$ for $0 \le t \le 1$ ?

A. $(e + e^{-1})^2$

B. $e^2 + e^{-2} + 4$

C. $\frac{1}{2}(e^2 + e^{-2})$

D. $e^2 - e^{-2}$ (correct)

E. $\frac{1}{4}(e^2 - e^{-2} + 4)$

**23.** The following equations in polar coordinates

(a) $r = 2\cos\theta$

(b) $3\sin\theta - 5\cos\theta = \frac{3}{r}$

represent:

A. (a) a circle (b) a straight line (correct)

B. (a) a cycloid (b) straight line

C. (a) a parabola (b) a cycloid

D. (a) a hyperbola (b) a cycloid

E. (a) a straight line (b) an ellipse

**24.** Let $z$ be the complex number $\frac{1}{\sqrt{2}}(1+i)$. Which of the following is $z^{2014}$?
(Hint: Use polar coordinates.)

A. $\frac{1}{\sqrt{2}}(1-i)$

B. $\frac{1}{\sqrt{2}}(1+i)$

C. $i$

D. 1

E. $-i$. (correct)

**25.** We use the notations $s = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3+1}$ and $s_N = \sum_{n=1}^{N} \frac{(-1)^n}{n^3+1}$.

What is the smallest $N$ such that the Alternating Series Estimation Test implies that $|s - s_N| < 0.01$?

A. 7

B. 3

C. 6

D. 5

E. 4. (correct)