MA 16600
FINAL EXAM INSTRUCTIONS
VERSION 01
May 1, 2013

Your name ______________________ Your TA's name ______________________

Student ID # ________________ Section # and recitation time ___________

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your TA's name (NOT the lecturer's name) and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit SECTION NUMBER.

6. Sign the scantron sheet.

7. Write down YOUR NAME and TA's NAME on the exam booklet.

8. There are 25 questions, each worth 8 points. The total is $8 \times 25 = 200$. Blacken your choice of the correct answer in the spaces provided for questions 1–25. Do all your work on the question sheets. Turn in both the scantron sheets and the question sheets when you are finished.

9. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.

10. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

11. After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.

12. If you finish the exam before 12:25, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 12:25, you should REMAIN SEATED until your TA comes and collects your scantron sheets and exam booklets.

Questions

1. Determine (all) the value(s) of t so that the angle θ between the vector $\vec{a} = \langle 1, 0, 1 \rangle$ and $\vec{b} = \langle t, 1, 0 \rangle$ is equal to $\frac{\pi}{3}$, i.e., $\theta = \frac{\pi}{3}$.

A. $t = 0$

B. $t = 1$ (correct)

C. $t = -1$

D. $t = \pm 1$

E. There is no such value for t.

2. Find a unit vector that is perpendicular to the plane passing through the points

$$\begin{cases} P &= (1, 0, 1), \\ Q &= (2, -1, 2), \\ R &= (1, 1, -1). \end{cases}$$

A. $\frac{1}{\sqrt{14}}\,(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$

B. $\frac{1}{\sqrt{3}}\,(2\mathbf{i} - \mathbf{j} + 2\mathbf{k})$

C. $\frac{1}{\sqrt{2}}\,(\mathbf{j} - \mathbf{k})$

D. $\frac{1}{\sqrt{21}}\,(4\mathbf{i} - \mathbf{j} + 2\mathbf{k})$

E. $\frac{1}{\sqrt{6}}\,(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ (correct)

3. Find the area of the region bounded by the curves $y = -x$ and $y = 2 - x^2$.

A. $\frac{1}{2}$

B. 1

C. 2

D. $\frac{5}{2}$

E. $\frac{9}{2}$ (correct)

4. Find the volume of a solid

(i) whose base is the triangle Δ (on the (x, y)-plane) with vertices $(0, 2)$, $(0, 0)$, and $(1, 0)$, and

(ii) whose cross sections, perpendicular to the base and the x-axis, are squares.

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. 1

D. $\frac{4}{3}$ (correct)

E. $\frac{5}{3}$

5. The region bounded by $y = x$ and $y = x^2$ is rotated around the line $x = 3$. Find a formula for the volume of the resulting solid of revolution by using (a) the washer method, and (b) the cylindrical shell method.

A. (a) $\int_0^1 \pi \left\{(3-y)^2 - (3-\sqrt{y})^2\right\} dy$ (b) $2\pi \int_0^1 (3-x)(x-x^2)dx$ (correct)

B (a) $\int_0^1 \pi \left\{y^2 - (\sqrt{y})^2\right\} dy$ (b) $2\pi \int_0^3 x(x-x^2)dx$

C. (a) $\int_0^1 \pi \left\{y^2 - (\sqrt{y})^2\right\} dy$ (b) $2\pi \int_0^3 (3-x)(x-x^2)dx$

D. (a) $\int_0^3 \pi \left\{x^2 - (x^2)^2\right\} dx$ (b) $2\pi \int_0^1 y(\sqrt{y}-y)dy$

E. (a) $2\pi \int_0^1 (3-x)(x-x^2)dx$ (b) $\int_0^1 \pi \left\{(3-y)^2 - (3-\sqrt{y})^2\right\} dy$

6. A tank in the shape of an inverted circular cone is 2 feet high, and has the radius of 1 foot at the top. It is completely filled with water. How much work is needed to pump out all the water through a hose at the top ? (Let **d** represent the density of the water 62.5 lb/ft^3.)

A. $\frac{7\pi}{6}\mathbf{d}$ ft-lb

B. $\pi\mathbf{d}$ ft-lb

C. $\frac{5}{6}\pi\mathbf{d}$ ft-lb

D. $\frac{2\pi}{3}\mathbf{d}$ ft-lb

E. $\frac{\pi}{3}\mathbf{d}$ ft-lb (correct)

7. Compute

$$\int_1^{e^2} \ln x \; dx.$$

A. $\frac{e^2+1}{2}$

B. $\frac{3(e^2-1)}{2}$

C. $\frac{3e^2}{2}$

D. $2e^2 - 2$

E. $e^2 + 1$ (correct)

8. Compute

$$\int_0^{\frac{\pi}{3}} \sec x \tan^3 x \; dx.$$

A. $\frac{4}{3}$ (correct)

B. $\frac{5}{3}$

C. 2

D. $\frac{7}{3}$

E. $\frac{3}{2}$

9. Find a formula for the following indefinite integral

$$\int \sin^2 x \cos^2 x \; dx.$$

A. $\frac{x}{4} + \frac{\sin 4x}{16} + C$
B. $\frac{x}{4} + \frac{\cos 4x}{16} + C$
C. $\frac{3x}{4} - \frac{\cos 4x}{32} + C$
D. $\frac{x}{8} - \frac{\cos 4x}{32} + C$
E. $\frac{x}{8} - \frac{\sin 4x}{32} + C$ (correct)

10. Find a formula for the following indefinite integral

$$\int \frac{dx}{\sqrt{4x^2 - 1}}.$$

A. $\ln|x + \sqrt{4x^2 - 1}| + C$
B. $\ln|2x + \sqrt{4x^2 - 1}| + C$
C. $\ln|\sqrt{4x^2 - 1}| + C$
D. $\frac{1}{2}\ln|2x + \sqrt{4x^2 - 1}| + C$ (correct)
E. $2\ln|2x + \sqrt{4x^2 - 1}| + C$

11. Evaluate

$$\int_3^4 \frac{dx}{x^2 - 3x + 2}.$$

A. $\ln \frac{8}{3}$

B. $\ln \frac{3}{2}$

C. $\ln \frac{4}{3}$ (correct)

D. $\ln \frac{6}{5}$

E. $\ln 3$

12. Use the Trapezoidal rule with $n = 4$ to find the approximate value of the following integral

$$\int_0^{\frac{4\pi}{3}} \sin x \; dx.$$

A. $\frac{\sqrt{3}}{2}\pi$

B. $\frac{\sqrt{3}}{3}\pi$

C. $\frac{\sqrt{3}}{4}\pi$ (correct)

D. $\left(\frac{\sqrt{3}}{4} + \frac{1}{2}\right)\pi$

E. $\left(\frac{\sqrt{3}}{2} - \frac{1}{4}\right)\pi$

13. Indicate convergence or divergence of each of the following improper integrals. If it converges, then evaluate its value.

(a)

$$\int_2^\infty \frac{dx}{x(\ln x)^2}$$

(b)

$$\int_0^1 \frac{dx}{3x-1}$$

A. (a) converges to $\ln 2$ (b) diverges

B. (a) converges to $\frac{1}{\ln 2}$ (b) diverges (correct)

C. (a) converges to $-\frac{1}{\ln 2}$ (b) converges to $\ln 2$

D. (a) diverges (b) converges to $\ln 2$

E. (a) diverges (b) diverges

14. Find the area of the surface obtained by rotating the curve $y = x^2$ for $0 \leq x \leq 1$ about the y-axis.

A. $\frac{\pi}{2}(2\sqrt{2}-1)$

B. $\frac{\pi}{6}(5\sqrt{5}-1)$ (correct)

C. $\frac{\pi}{3}(4\sqrt{4}-1) = \frac{7\pi}{3}$

D. $\frac{\pi}{4}(5\sqrt{5}-1)$

E. $\frac{\pi}{3}$

15. Find the center of mass of the region D bounded by $y = 1 - x^2$ and $y = 0$.

A. $(0, \frac{1}{3})$
B. $(0, \frac{2}{3})$
C. $(0, \frac{1}{2})$
D. $(0, \frac{2}{5})$ (correct)
E. $(0, \frac{3}{8})$

16. Compute the following limit

$$\lim_{n \to \infty} \left(2n \sin(\pi/n) + \frac{n!}{n^n} \right).$$

A. The limit does not exist.
B. $2\pi + 1$
C. 2π (correct)
D. ∞
E. 0

17. Find the coefficient of x^5 in the Maclaurin series for the function $f(x) = \frac{x^2+1}{x-2}$.

A. $-\frac{1}{64}$

B. $\frac{3}{64}$

C. $-\frac{3}{64}$

D. $\frac{5}{64}$

E. - $\frac{5}{64}$ (correct)

18. Using Maclaurin series and the Estimation Theorem for alternating series, we can obtain the approximation

$$\int_0^{0.1} \frac{dx}{1+x^2} \approx 0.1 - \frac{(0.1)^3}{3} \text{ with error } \leq c.$$

The value of c is

A. $(0.1)^5$

B. $(0.1)^6$

C. $(0.1)^7$

D. $\frac{(0.1)^5}{5}$ (correct)

E. $\frac{(0.1)^7}{7}$

19. Find the interval of convergence for the Taylor series

$$\sum_{n=1}^{\infty} \frac{3^n}{n^n}(x-5)^n.$$

A. $(-\frac{1}{3}, \frac{1}{3})$

B. $(5-\frac{1}{3}, 5+\frac{1}{3})$

C. $(5-\frac{e}{3}, 5+\frac{e}{3})$

D. $(5-\frac{e}{3}, 5+\frac{e}{3}]$

E. $(-\infty, \infty)$ (correct)

20. Which of the following equations in polar coordinates represents the circle with center $(0,3)$ (in (x,y)-coordinates) and radius 3 ?

A. $r = 3\sin\theta$

B. $r = 6\sin\theta$ (correct)

C. $r = 3\cos\theta$

D. $r = 6\cos\theta$

E. $r = \cos 3\theta$

21. Which of the following pictures represents the curve given by the equation $r = \cos 3\theta$ with $0 \leq \theta \leq 2\pi$ in polar coordinates ?

A.

B.

C.

D. (correct)

E.

22. Find the range of p (resp. q) such that the following series (a) (resp. (b)) converges.

(a) $\sum_{n=1}^{\infty} \frac{n^{2p}+1}{\sqrt{n+2}}$

(b) $\sum_{n=1}^{\infty} \frac{n^{2q}}{\sqrt{n+2}}$

A. (a) no values for p (b) $q < -\frac{1}{4}$ (correct)

B. (a) no values for p (b) no values for q

C. (a) $p < -\frac{1}{4}$ (b) $q < -\frac{1}{4}$

D. (a) all values for p (b) all values for q

E. (a) $p < -\frac{1}{2}$ (b) $q < -\frac{1}{4}$

23. Which of the following series converge conditionally ?

$$\begin{array}{ll} \text{I.} & \sum_{n=1}^{\infty}(-1)^n \frac{1}{\sqrt{n}} \\ \text{II.} & \sum_{n=2}^{\infty}(-1)^n \frac{1}{\ln n} \\ \text{III.} & \sum_{n=1}^{\infty}(-1)^n \frac{n!}{1\cdot 3\cdot 5\cdots(2n-1)} \end{array}$$

A. I and II only (correct)

B. II and III only

C. I and III only

D. All of them

E. None of them

24. Consider two complex numbers $z = 1 + i\sqrt{3}$ and $w = 1 + i$. Find a polar form for $\frac{z}{w}$.

A. $\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$

B. $2\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$

C. $\sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$ (correct)

D. $\sqrt{2}\left(\cos(-\frac{\pi}{12}) + i\sin(-\frac{\pi}{12})\right)$

E. $\sqrt{2}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

25. Find the TOTAL distance traveled by a particle with position (x, y) as t varies in the given time interval:

$$\begin{cases} x &= \sin^2 t \\ y &= \cos^2 t \end{cases} \qquad 0 \leq t \leq 2\pi.$$

A. 2π

B. 4π

C. $\sqrt{2}$

D. $2\sqrt{2}$

E. $4\sqrt{2}$ (correct)