

MA 16600
EXAM 3 INSTRUCTIONS
VERSION 01
April 9, 2024

Your name _____ Your TA's name _____

Student ID # _____ Section # and recitation time _____

1. You must use a #2 pencil on the scantron sheet (answer sheet).
2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.
3. On the scantron sheet, fill in your **TA's name, i.e., the name of your recitation instructor (NOT the lecturer's name)** and the course number.
4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.
5. Fill in the four-digit **SECTION NUMBER**. Your section number is a 3 digit number. Put 0 at the front to make it a 4 digit number, and then fill it in.
6. **Sign the scantron sheet.**
7. Blacken your choice of the correct answer in the space provided for each of the questions 1–12. While mark all your answers on the scantron sheet, you should show your work on the exam booklet. Although no partial credit will be given, any disputes about the grade or grading will be settled by examining your written work on the exam booklet.
8. There are 12 questions, each of which is worth 8 points. You get 4 free points for trying. The maximum possible score is
 $12 \text{ questions} \times 8 \text{ points} + 4 \text{ free points} = 100 \text{ points}.$
9. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.
10. After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.
11. If you finish the exam before 7:25, you may leave the room after turning in the scantron sheet and the exam booklet. If you don't finish before 7:25, you should REMAIN SEATED until your TA comes and collects your scantron sheet and exam booklet.

Exam Policies

1. There is no individual seating. Just follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs/proctors will collect the scantron sheet and the exam booklet.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor/proctor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: _____

STUDENT SIGNATURE: _____

Recitation Section numbers by TA:

7:30 am Lecture		
TA name	Time & Location	Section #
Yannis Yicong Wu-Yip	4:30pm in PHYS 110	0208
Yannis Yicong Wu-Yip	3:30pm in PHYS 110	0215
Yannis Yicong Wu-Yip	2:30pm in PHYS 110	0222
Deanna Marie Wightman	3:30pm in PHYS 202	0229
Deanna Marie Wightman	2:30pm in PHYS 202	0236
Deanna Marie Wightman	4:30pm in PHYS 202	0243
Barath Balasubramanian Ramakrishna	9:30pm in PHYS 333	0250
Barath Balasubramanian Ramakrishna	8:30am in PHYS 333	0257
Barath Balasubramanian Ramakrishna	10:30am in PHYS 333	0264
Mei Lam Chan	1:30pm in Schm 307	0296
Mei Lam Chan	2:30pm in Schm 307	0297
Mei Lam Chan	12:30pm in Schm 307	0298

1:30pm Lecture		
TA name	Time & Location	Section #
Shengwei Qiu	2:30pm in PHYS 333	0271
Shengwei Qiu	1:30pm in PHYS 333	0278
Shengwei Qiu	12:30pm in PHYS 333	0285
Giancarlo Thomas Arcese	2:30pm in HAMP 2101	0287
Giancarlo Thomas Arcese	1:30pm in HAMP 2101	0288
Giancarlo Thomas Arcese	12:30pm in HAMP 2101	0289
Jiarui Huang	10:30am in PHYS 202	0290
Jiarui Huang	12:30pm in PHYS 202	0291
Jiarui Huang	11:30am in PHYS 202	0292
Asini Anuradhika Konpola	9:30am in PHYS 202	0293
Asini Anuradhika Konpola	7:30am in PHYS 202	0294
Asini Anuradhika Konpola	8:30am in PHYS 202	0295

Questions

1. (8 Points) Test the following series for convergence:

I. $\sum_{k=1}^{\infty} \frac{3^k}{k^2}$

II. $\sum_{k=1}^{\infty} \frac{5}{k^3}$

III. $\sum_{k=1}^{\infty} e^{-2k}$

- A. I and II are convergent, and III is divergent.
- B. I, II, and III are convergent.
- C. II and III convergent, and I is divergent. **(Correct)**
- D. II is convergent, and I and III are divergent.
- E. I, II, and III are divergent.

2. (8 Points) The series $\sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}$ is convergent by the Alternating series test. According to the Alternating Series Estimation Test, what is the smallest number of terms needed to find the sum of the series with error less than $\frac{1}{5}$?

- A. 1
- B. 2 (**Correct**)
- C. 3
- D. 4
- E. 5

3. (8 Points) Which of the following series converge ?

$$\text{I. } \sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^2}$$

$$\text{II. } \sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$$

$$\text{III. } \sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln(n)}$$

- A. I and II only.
- B. II and III only.
- C. All.
- D. I and III only. **(Correct)**
- E. III only.

4. (8 Points) Determine whether the following series are absolutely convergent, conditionally convergent, or divergent.

I.
$$\sum_{k=1}^{\infty} \frac{\cos(k\pi)}{k^{3/2}}$$

II.
$$\sum_{k=1}^{\infty} \frac{(-1)^k}{3k}$$

- A. I and II are conditionally convergent.
B. I is absolutely convergent, II is conditionally convergent. **(Correct)**
C. II is conditionally convergent, I is divergent.
D. I and II are divergent.
E. I and II are absolutely convergent.

5. (8 points) Consider the series

$$\sum_{k=1}^{\infty} a_k = \sum_{k=1}^{\infty} \left(\frac{1}{k^3 + 2k + 4} \right) \cos(k).$$

We conclude

A. the series is absolutely convergent by Comparison Test with $\sum_{k=1}^{\infty} \frac{1}{k^3}$.

(Correct)

B. the series is divergent by Ratio Test since $\lim_{k \rightarrow \infty} \left| \frac{a_{k+1}}{a_k} \right| = 1$.

C. the series is convergent since $\lim_{k \rightarrow \infty} a_k = 0$.

D. the series is convergent by Limit Comparison Test with $\sum_{k=1}^{\infty} \frac{1}{k^3}$.

E. the series is divergent by Divergence test since $\lim_{k \rightarrow \infty} a_k$ does not exist.

6. (8 Points) Determine which of the following statements are true and which are false.

I. The series $\sum_{k=1}^{\infty} \frac{1}{k^4 + 1}$ is convergent by the limit comparison test with $\sum_{k=1}^{\infty} \frac{1}{k^4}$.

II. The series $\sum_{k=1}^{\infty} \frac{1}{k^4 + 1}$ is convergent by the comparison test with $\sum_{k=1}^{\infty} \frac{1}{k^4}$.

III. The series $\sum_{k=1}^{\infty} \frac{1}{k^4 + 1}$ is convergent by the ratio test.

A. I and III are true, II is false.

B. I, II, and III are true.

C. II and III are true, I is false.

D. I and II are true, III are false. (**Correct**)

E. II is true, I and III are false.

7. (8 Points) Which of the following are **always true**?

I. If $\lim_{k \rightarrow \infty} k^2 |a_k| = 1$, then $\sum_{k=1}^{\infty} (-1)^{k+1} a_k$ converges absolutely.

II. If $\lim_{k \rightarrow \infty} |a_k| = 0$, then $\sum_{k=1}^{\infty} a_k$ converges absolutely.

III. If $\sum_{k=1}^{\infty} |a_k|$ converges, then $\sum_{k=1}^{\infty} \frac{a_k}{k}$ converges.

A. II and III true.

B. I and II are true.

C. I is true.

D. I and III are true. **(Correct)**

E. I, II and III true.

8. (8 Points) Find the radius of convergence R for the power series

$$\sum_{n=1}^{\infty} \frac{(2x - 7)^n}{6^n + n^3}.$$

A. $R = 3$ (Correct)

B. $R = 6$

C. $R = 12$

D. $R = 7$

E. $R = 13$

9. (8 Points) Find the interval of convergence of the power series

$$\sum_{k=1}^{\infty} \frac{3^k (x+1)^k}{\sqrt{k}}$$

- A. $[-4, 2)$
- B. $[-4/3, -2/3]$
- C. $(-4, 2)$
- D. $[-4/3, -2/3)$ **(Correct)**
- E. $[-4, 2]$

10. (8 Points) Which of the following is the power series representation of the function $\frac{x^3}{1+x^2}$ for $|x| < 1$?

A. $\sum_{n=0}^{\infty} (-1)^n x^{2n+3}$ **(Correct)**

B. $\sum_{n=0}^{\infty} (-1)^n x^{2n+5}$

C. $\sum_{n=0}^{\infty} (-1)^{n+1} x^{2n+5}$

D. $\sum_{n=0}^{\infty} x^{2n+3}$

E. $\sum_{n=0}^{\infty} x^{2n+2}$

11. (8 Points) We compute the Taylor series for $f(x) = \frac{1}{x^4}$ centered at $a = 10$.

What is the coefficient of $(x - 10)^2$ in the Taylor series ?

A. $\frac{2!}{10^5}$

B. $-\frac{10^5}{2!}$

C. $-\frac{1}{2!10^5}$

D. $\frac{1}{10^5}$ (Correct)

E. $\frac{1}{2!10^6}$

12. (8 Points) Compute value of the series

$$\sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n}}{(2n)! 3^{2n}}.$$

HINT: find the power series expansion of $\cos x$.

- A. $\frac{1}{2}$ (Correct)
- B. $\frac{1}{\sqrt{2}}$
- C. 0
- D. $\frac{\pi}{4}$
- E. The series diverges.