MA 16600
EXAM 3 INSTRUCTIONS
VERSION 01
April 9, 2013

Your name ______________________ Your TA's name ______________________

Student ID # ________________ Section # and recitation time ___________

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your TA's name (NOT the lecturer's name) and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit SECTION NUMBER.

6. Sign the scantron sheet.

7. Write down YOUR NAME and TA's NAME on the exam booklet.

8. Blacken your choice of the correct answer in the spaces provided for each of the questions 1–12. Do all your work on the question sheets. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.

9. There are 12 questions, each worth 8 points. The maximum possible score is
$8 \times 12 + 4$ (for taking the exam) $= 100$ points.

10. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

11. After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.

12. If you finish the exam before 7:25, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 7:25, you should REMAIN SEATED until your TA comes and collects your scantron sheets and exam booklets.

Questions

1. Compute

$$\sum_{n=1}^{\infty} \frac{2^{n-1} - 3^{n+1}}{4^n}.$$

A. -2

B. $-\frac{32}{3}$

C. 4

D. $-\frac{17}{2}$ (correct)

E. 0

2. For what value(s) of p does the series

$$\sum_{n=1}^{\infty} \frac{n^{2p}+1}{\sqrt{n+2}}$$

converge ?

A. $p < -\frac{1}{2}$
B. $p \geq -\frac{1}{2}$
C. $p < 0$
D. $p \geq 0$
E. No values (correct)

3. Which of the following statements are true ?

I. If $\frac{1}{\ln n} \leq a_n$ for $n \geq 2$, then $\sum_{n=2}^{\infty} a_n$ diverges.

II. The alternating series $\sum_{n=1}^{\infty}(-1)^n \sqrt{\frac{n+1}{4n+1}}$ is convergent.

III. If $\frac{1}{n} \leq b_n \leq \frac{1}{\sqrt{n}}$ and $b_n \geq b_{n+1}$, then $\sum_{n=1}^{\infty}(-1)^n b_n$ is conditionally convergent.

A. I and II only.

B. II and III only.

C. I and III only. (correct)

D. I only.

E. III only.

4. We would like to estimate the value of the following series

$$s = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+3}$$

by the sum of the first n terms s_n with an error less than 0.01, i.e., $|s - s_n| < 0.01$, using the Estimation Theorem for Alternating Series.

What is the smallest value of n that gives us the estimate within the required error amount ?

A. 7

B. 8

C. 9 (correct)

D. 10

E. 11

5. Find all values of c for which the following series converges:

$$\sum_{n=1}^{\infty} \left(\frac{c}{n} - \frac{1}{n+1} \right).$$

A. 0

B. 1 (correct)

C. 2

D. All positive numbers

E. No values

6. Given the following series

$$\sum_{n=2}^{\infty} \frac{7n+3}{\sqrt{n^5-4}},$$

Mark, Nancy, and David provide the following reasoning for determining convergence or divergence of the series:

(a) the name of the test to use,

(b) the conclusion for convergence or divergence.

Mark: (a) $b_n = \frac{7n}{\sqrt{n^5}}$, Comparison Test ($0 \leq a_n \leq b_n$) (b) convergent

Nancy: (a) $b_n = \frac{n}{\sqrt{n^5}}$, Limit Comparison Test ($\lim_{n\to\infty} \frac{a_n}{b_n} = 7$) (c) convergent

David: (a) Ratio Test ($\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = 1$) (b) Ratio Test is inconclusive, and we have to use some other test.

Choose the name(s) of the person(s) with correct arguments.

A. Mark only.

B. Nancy only.

C. David only.

D. Nancy and David only. (correct)

E. Mark and Nancy only.

7. Test the following series for convergence or divergence.

$$\text{(a)} \sum_{n=1}^{\infty} \frac{\sqrt{n^3+1}}{3n^3+4n^2+2}$$

$$\text{(b)} \sum_{n=1}^{\infty} ne^{-n^2}$$

$$\text{(c)} \sum_{n=1}^{\infty} \frac{2^n}{n!}$$

A. (a) convergent (b) convergent (c) convergent (correct)

B. (a) convergent (b) divergent (c) convergent

C. (a) divergent (b) convergent (c) convergent

D. (a) divergent (b) convergent (c) divergent

E. (a) divergent (b) divergent (c) divergent

8. Determine whether the following series are absolutely convergent, conditionally convergent, or divergent.

$$\text{(a)} \sum_{n=1}^{\infty} (-1)^n \frac{n}{\sqrt{n^3+9}}$$

$$\text{(b)} \sum_{n=1}^{\infty} (-1)^n \frac{\sin n}{n^2}$$

$$\text{(c)} \sum_{n=1}^{\infty} (-1)^n \frac{n}{n+2}$$

A. (a) absolutely convergent (b) conditionally convergent (c) divergent

B. (a) conditionally convergent (b) absolutely convergent (c) divergent (correct)

C. (a) absolutely convergent (b) absolutely convergent (c) conditionally convergent

D. (a) conditionally convergent (b) conditionally convergent (c) divergent

E. (a) divergent (b) divergent (c) absolutely convergent

9. Suppose the power series

$$\sum_{n=0}^{\infty} c_n (x-2)^n$$

converges when $x = -1$ but diverges when $x = 5$.

Which of the following statements are true ?

I. The radius of convergence is $R = 3$.

II. $\sum_{n=0}^{\infty} (-1)^n c_n 2^n$ converges.

III. $\sum_{n=0}^{\infty} c_n 2^n$ converges.

A. I and II only.

B. I and III only.

C. II and III only.

D. All of them (correct)

E. None of them

10. Find the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{5^n(x-1)^n}{n}.$$

A. $[-\frac{1}{5}, \frac{1}{5})$

B. $[-\frac{1}{5}, \frac{1}{5}]$

C. $[-\frac{1}{5}+1, \frac{1}{5}+1)$ (correct)

D. $[-\frac{1}{5}+1, \frac{1}{5}+1]$

E. $(-\infty, \infty)$

11. Determine

(a) the power series representation (centered at $a = 0$), and

(b) the interval of convergence

for the function

$$f(x) = \frac{x}{9 - x^2}.$$

A. (a) $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{9^{n+1}}$ (b) $(-3, 3)$ (correct)

B. (a) $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{9^{n}}$ (b) $[-3, 3)$

C. (a) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{9^{n+1}}$ (b) $(-3, 3)$

D. (a) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{9^{n}}$ (b) $[-3, 3)$

E. (a) $\sum_{n=0}^{\infty} \frac{x^{2n}}{9^{n}}$ (b) $(-\infty, \infty)$

12. We would like to use the power series representation for

$$f(x) = \frac{1}{1 + x^4}$$

and Estimation Theorem for Alternating Series to evaluate the integral

$$s = \int_0^{0.1} \frac{1}{1 + x^4} dx.$$

If we use the sum of the first 3 terms s_3, then the error $|s - s_3|$ is bounded from above by

A. $(0.1)^4$

B. $(0.1)^{12}$

C. $\frac{1}{4}(0.1)^4$

D. $\frac{1}{(12)!}(0.1)^{12}$

E. $\frac{1}{13}(0.1)^{13}$ (correct)