MA 16500
FINAL EXAM INSTRUCTIONS
VERSION 01
DECEMBER 9, 2013

Your name ______________________ Your TA's name ______________________

Student ID # ________________ Section # and recitation time ___________

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your question booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your TA's name (NOT the lecturer's name) and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit SECTION NUMBER.

6. Sign the scantron sheet.

7. Blacken your choice of the correct answer in the spaces provided for each of the questions 1–25. Do all your work on the question sheets. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.

8. There are 25 questions, each worth 8 points. The maximum possible score is

8×25 (for taking the exam) $= 200$ points.

9. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

10. After you finish the exam, turn in BOTH the scantron sheets and the exam booklets.

11. If you finish the exam before 8:55, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 8:55, you should REMAIN SEATED until your TA comes and collects your scantron sheets and exam booklets.

Questions

1. Let $f(x) = (x+2)^3$. Evaluate

$$\lim_{x \to 1} \left(\frac{f(x) - f(1)}{x - 1} \right).$$

A. 27 (correct)

B. 8

C. 3

D. 1

E. ∞

2. Evaluate

$$\lim_{x \to 0^+} x \sin(\frac{1}{x}).$$

.

A. 0 (correct)

B. 1

C. -1

D. $\frac{\pi}{2}$

E. The limit does not exist.

3. Let $h(x)$ be the function defined by

$$\begin{cases} \cos \frac{\pi}{x} & \text{if } \ 0 < x < 3 \\ x^2 - a & \text{if } \ 3 \leq x < 2\pi \end{cases}$$

Determine the value of a so that the function h is continuous for all $0 < x < 2\pi$.

A. 9

B. $\frac{17}{2}$ (correct)

C. 8

D. $\frac{15}{2}$

E. There is no such value of a.

4. Let $y = \ln(\tan x)$. Then $\frac{dy}{dx} =$

A. $\frac{\sec^2 x}{\tan x}$ (correct)

B. $\frac{1}{\tan x \sec^2 x}$

C. $\sec x$

D. $\frac{1}{\tan x}$

E. $\frac{\sec x}{\tan x}$

5. Compute
$$\lim_{x\to 0^+}\left(\frac{x+\sqrt{x}}{2x-4\sqrt{x}}\right).$$

A. $\frac{1}{2}$

B. $-\frac{1}{4}$ (correct)

C. $-\frac{1}{2}$

D. -1

E. The limit does not exist.

6. Given $f(x)=\frac{x^3-1}{x^3+1}$, find the formula for its inverse function $f^{-1}(x)$.

A. $f^{-1}(x)=\frac{\sqrt[3]{x}-1}{\sqrt[3]{x}+1}$

B. $f^{-1}(x)=\frac{\sqrt[3]{x}+1}{\sqrt[3]{x}-1}$

C. $f^{-1}(x)=\frac{x^3+1}{x^3-1}$

D. $f^{-1}(x)=\sqrt[3]{\frac{1-x}{1+x}}$

E. $f^{-1}(x)=\sqrt[3]{\frac{1+x}{1-x}}$ (correct)

7. Assume that y is defined implicitly as a differentiable function of x by the equation

$$xy^2 + xy + x = 6.$$

Find $\frac{dy}{dx}$ at the point $(2, 1)$.

A. $-\frac{1}{2}$ (correct)
B. $-\frac{1}{6}$
C. $\frac{1}{6}$
D. $\frac{1}{2}$
E. $-\frac{7}{2}$

8. Use the linear approximation of the function $f(x) = \sqrt[4]{x}$ at $a = 16$ to estimate the number $\sqrt[4]{16.2}$.

A. $2 + \frac{1}{320}$
B. $2 + \frac{1}{160}$ (correct)
C. $2 + \frac{1}{80}$
D. $2 + \frac{1}{40}$
E. $2 + \frac{1}{20}$

9. Find the absolute maximum "Max" and absolute minimum "Min" of the function

$$f(x) = x^6 - 2x^3 \text{ on the interval } [-1, 2].$$

A. $\text{Max} = 48, \text{Min} = -1$ (correct)

B. $\text{Max} = 48, \text{Min} = 0$

C. $\text{Max} = 3, \text{Min} = -1$

D. $\text{Max} = 3,\ \text{Min} = 0.$

E. $\text{Max} = 0, \text{Min} = -1$

10. Find the interval where the function $f(x) = 4x^3 - 6x^2 + 3x + 1$ takes the value 3.

A. $(-2, -1)$

B. $(-1, 0)$

C. $(0, 1)$

D. $(1, 2)$ (correct)

E. $(2, 3)$

11. Evaluate

$$\lim_{x \to 0^+} x \ln x.$$

A. 0 (correct)

B. $\frac{1}{2}$

C. 1

D. $-\infty$

E. ∞

12. Which of the following is/are true about the function $f(x) = x^3 - 3x^2 - 9x$?

(1) The function f is increasing on the interval $(-1, 3)$.

(2) 5 is a local maximum value of f.

(3) The graph of f is concave down when $x < 1$.

A. (2) and (3) only (correct)

B. (1) and (2) only

C. (1) and (3) only

D. (3) only

E. All are true

13. Choose the one which describes best the graph of the function

$$f(x) = \frac{\sin x}{1 + \cos x} \text{ on } [0, \pi) \cup (\pi, 2\pi].$$

A.

B.

C.

D. (correct)

E.

14. Assume $3 \leq f'(x) \leq 6$ for all values of x.

What are the

(a) minimum possible value, and

(b) maximum possible value

of $f(8) - f(2)$?

A. (a) 24 (b) 48

B. (b) 18 (b) 36 (correct)

C. (a) 9 (b) 16

D. (a) 6 (b) 10

E. (a) $\frac{1}{2}$ (b) 1

15. Water is withdrawn from a conical reservoir, 4 feet in diameter and 4 feet deep (vertex down) at the constant rate of 1 ft^3/min. How fast is the water level falling when the depth of the water in the reservoir is 3 feet ?

A. $\frac{4}{9\pi}$ ft/min (correct)

B. $\frac{1}{9\pi}$ ft/min

C. $\frac{9\pi}{4}$ ft/min

D. 9π ft/min

E. $\frac{\pi}{4}$ ft/min

16. The points on the ellipse $x^2 + 2y^2 = 5$ that are closest to the point $(\frac{1}{2}, 0)$ are:

A. $(1, \pm\sqrt{2})$ (correct)

B. $(\sqrt{2}, \pm\sqrt{\frac{3}{2}})$

C. $(\sqrt{\frac{1}{2}}, \pm\sqrt{\frac{3}{2}})$

D. $(2, \pm\sqrt{\frac{1}{2}})$

E. $(\sqrt{3}, \pm 1)$

17. Evaluate $\frac{d}{dx}\left(\arcsin(4x+1)^2\right)$.

A. $\frac{1}{\sqrt{1-(4x+1)^2}}$

B. $\frac{2(4x+1)}{\sqrt{1-(4x+1)^2}}$

C. $\frac{1}{\sqrt{1-(4x+1)^4}}$

D. $\frac{4(4x+1)}{\sqrt{1-(4x+1)^4}}$

E. $\frac{8(4x+1)}{\sqrt{1-(4x+1)^4}}$ (correct)

18. Let $f(x) = x^{\ln x}$. Evaluate $f'(x)$.

A. $(\ln x)x^{(\ln x - 1)}$

B. $2(\ln x)x^{(\ln x - 1)}$ (correct)

C. $(\ln x)x^{\ln x}$

D. $x^{(\ln x - 1)}$

E. $x^{\frac{1}{x}}$

19. The area under the graph of $y = 9e^{3x}(1 + e^{3x})^5$ and above the y axis between $x = 0$ and $x = 3$ is:

A. $(1 + e^9)^6 - 64$

B. $\frac{3}{4}(1 + e^9)^6 - 48$

C. $\frac{(1+e^9)^6}{2} - 32$ (correct)

D. $\frac{(1+e^9)^6}{2}$

E. $\frac{3}{4}(1 + e^9)^6$

20. Compute

$$\int_0^{\frac{\pi}{6}} \frac{\sin(2t)}{\sqrt{\cos(2t)}} dt.$$

A. $1 - \frac{1}{\sqrt{2}}$ (correct)

B. $1 - \frac{\sqrt{3}}{2}$

C. 1

D. $1 - \frac{\pi}{2}$

E. $2 - \frac{\sqrt{3}}{2}$

21. The half-life of a certain material is 20 years. If we start with a sample of 100 grams, how much remains after 50 years?

A. $\frac{25}{\sqrt{2}}$ grams (correct)

B. $25\sqrt{2}$ grams

C. $\frac{50}{\sqrt{2}}$ grams

D. $50\sqrt{2}$ grams

E. $18 + \frac{3}{4}$ grams

22. Evaluate

$$\frac{d}{dx}\left(\int_0^{x^2}(t^2+3)^{10}dt\right)$$

at $x = 2$.

A. 7^{10}

B. $2 \cdot 7^{10}$

C. $4 \cdot 7^{10}$

D. 19^{10}

E. $4 \cdot 19^{10}$ (correct)

23. The directrix of the parabola $y - 1 = 2x^2$ is:

A. $y = \frac{7}{8}$ (correct)

B. $y = \frac{1}{2}$

C. $y = -1$

D. $x = \frac{1}{2}$

E. $x = -\frac{1}{4}$

24. Find an equation of the ellipse with the following conditions:

(a) the foci are $(-2,-2)$ and $(4,-2)$, and

(b) one of the vertices is $(-4,-2)$.

A. $\frac{x^2}{5^2}+\frac{y^2}{4^2}=1$

B. $\frac{(x-1)^2}{5^2}+\frac{(y+2)^2}{3^2}=1$

C. $\frac{(x-1)^2}{5^2}+\frac{(y+2)^2}{4^2}=1$ (correct)

D. $\frac{(x+1)^2}{5^2}+\frac{(y-2)^2}{3^2}=1$

E. $\frac{(x+1)^2}{5^2}+\frac{(y-2)^2}{4^2}=1$

25. One of the foci for the hyperbola $9x^2-4y^2-36x-8y-4=0$ is:

A. $(2+\sqrt{13},-1)$ (correct)

B. $(-2+\sqrt{13},1)$

C. $(\sqrt{13},0)$

D. $(2,-1+\sqrt{13})$

E. $(-2,1+\sqrt{13})$