MA26600 Final Exam

GREEN VERSION 01

NAME: ____________________

INSTRUCTOR: ____________________ SECTION/TIME: ____________

- You must use a **#2 pencil** on the mark-sense answer sheet.
- Fill in the **ten digit PUID** (starting with two zeroes) and your **Name** and blacken in the appropriate spaces.
- Fill in the correct **Test/Quiz number** (GREEN is **01**, ORANGE is **02**)
- Fill in the **four digit section number** of your class and blacken the numbers below them. Here they are:

0313	MWF	10:30AM	Gayane Poghotanyan	0376	MWF	1:30PM	Chen Liu
0314	MWF	11:30AM	Gayane Poghotanyan	0901	MWF	8:30AM	Krishnendu Khan
0325	MWF	9:30AM	Gayane Poghotanyan	0902	MWF	9:30AM	Krishnendu Khan
0326	MWF	3:30PM	Min Chen	0903	MWF	9:30AM	Yilong Zhang
0337	MWF	2:30PM	Min Chen	0904	MWF	10:30AM	Yilong Zhang
0338	MWF	10:30AM	Mahesh Sunkula	0905	MWF	12:30PM	Guang Yang
0340	MWF	11:30AM	Mahesh Sunkula	0906	MWF	11:30AM	Guang Yang
0341	MWF	8:30AM	Heejin Lee	0907	TR	12:00PM	Qing Zhan
0352	MWF	9:30AM	Heejin Lee	0908	TR	10:30AM	Qing Zhan
0353	MWF	11:30AM	Michelle Michelle	0909	MWF	10:30AM	Ke Wu
0364	MWF	12:30PM	Michelle Michelle	0910	MWF	11:30AM	Ke Wu
0365	MWF	12:30PM	Chen Liu	0912	TR	4:30PM	Plamen Stefanov

- Sign the mark-sense sheet.
- Fill in your name and your instructor's name and the time of your class meeting on the exam booklet above.
- There are 20 multiple-choice questions, each worth 10 points. **Blacken in** your choice of the correct answer in the spaces provided for questions 1–20 in the answer sheet. Do all your work on the question sheets, in addition, also **Circle** your answer choice for each problem on the question sheets in case your mark-sense sheet is lost.
- Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.
- **No calculators, books, electronic devices, or papers are allowed.** Use the back of the test pages for scratch paper.
- Pull off the **table of Laplace transforms** on the last page of the exam for reference. Do not turn it in with your exam booklet at the end.

1. Find the general solution of the equation

$$y^3 \frac{dy}{dx} = (y^4 + 1)\cos x.$$

A. $\ln(y^4 + 1) = \sin x + C$

B. $\ln(y^4 + 1) = \sin x + C$

C. $\ln(y^4) = 4\sin x + C$

D. $\ln(y^4 + 1) = 4\sin x + C$

E. $\ln(y^4 + 1) = 4\sin x$

2. The general solution $y(x)$ to

$$x^2 y' + 5xy = \frac{e^{4x}}{x^3}, \qquad x > 0$$

is

A. $y(x) = \dfrac{e^{4x}}{x^3} + C$

B. $y(x) = \dfrac{e^{4x}}{4x^5} + \dfrac{C}{x^5}$

C. $y(x) = \dfrac{e^{x}}{x^3} + \dfrac{C}{x^2}$

D. $y(x) = Cx^{-5}$

E. $y(x) = \dfrac{Ce^{4x}}{4}$

3. Consider the autonomous differential equation

$$y' = (1 - y)^3 (y + 2)(y - 4).$$

Classify the stability of each equilibrium solution.

A. $y = -2, 1, 4$ are all stable

B. $y = -2$ is stable; $y = 1, 4$ are unstable

C. $y = 1$ is stable; $y = -2, 4$ are unstable

D. $y = 1, 4$ are stable; $y = -2$ is unstable

E. $y = -2, 4$ are stable; $y = 1$ is unstable

4. Let $y_1 = e^{3t}$ and $y_2 = te^{3t}$. Which of the following statements is true about $W(y_1, y_2)$, the Wronskian of y_1 and y_2?

A. $W(y_1, y_2) = e^{6t}$, therefore y_1 and y_2 are linearly dependent.

B. $W(y_1, y_2) = 0$, therefore y_1 and y_2 are linearly independent.

C. $W(y_1, y_2) = 0$, therefore y_1 and y_2 are linearly dependent.

D. $W(y_1, y_2) = e^{6t}$, therefore y_1 and y_2 are linearly independent.

E. $W(y_1, y_2) = te^{3t}$, therefore y_1 and y_2 are linearly independent.

5. Let $y(x)$ be the solution to the initial value problem

$$y'' - 10y' + 25y = 0, \quad y(0) = 1, \ y'(0) = 3.$$

Then $y(1) =?$

A. $-e^5$

B. 1

C. e^5

D. $\frac{4}{5}e^5 + \frac{1}{5}e^{-5}$

E. e^{-5}

6. The solution of the initial value problem

$$(y^2 - 1)dx + (2xy + 3y^2)dy = 0, \quad y(0) = 2$$

is given implicitly by

A. $\frac{y^3}{3} - y + xy^2 + y^3 = \frac{24}{3}$

B. $xy^2 - x + y^2 = 4$

C. $xy^2 - x + y^3 = 8$

D. $x^2y + 3x - y^2 = 4$

E. $3xy^2 - x + y^3 = 8$

7. Which of the following differential equations has

$$y(t) = c_1 e^{-t} + c_2 t e^{-t} + c_3 \cos t + c_4 \sin t$$

as a general solution?

A. $y^{(4)} - 2y'' + y = 0$
B. $y^{(4)} + 2y''' + 2y'' + 2y' + y = 0$
C. $y^{(4)} - 2y''' - 2y' + y = 0$
D. $y^{(4)} + 2y''' - 2y' - y = 0$
E. $y^{(4)} - 2y''' + 2y' - y = 0$

8. Select the system of first-order differential equations that is equivalent to

$$3x'' - 2x' + x = \sin t.$$

A. $x_1' = x_2, \quad x_2' = 2x_1 - x_2 + \dfrac{1}{3}\sin t$
B. $x_1' = 3x_2, \quad x_2' = 2x_2 - x_1 + \sin t$
C. $x_1' = x_2, \quad x_2' = \dfrac{1}{3}(2x_2 - x_1 + \sin t)$
D. $x_1' = 3(2x_1 - x_2 + \sin t), \quad x_2' = x_1$
E. $x_1' = -x_2, \quad x_2' = -2x_2 + x_1 - \sin t$

9. The general solution of the differential equation

$$y''' - 2y'' + y' - 2y = 2e^t + 4$$

is

A. $e^{2t} + \cos t + \sin t - e^t + 2$

B. $c_1 e^t + c_2 \cos 2t + c_3 \sin 2t + e^t - 2$

C. $c_1 e^{2t} + c_2 \cos 2t + c_3 \sin 2t + e^t + 2$

D. $c_1 e^{2t} + c_2 \cos t + c_3 \sin t - e^t - 2$

E. $c_1 e^t + c_2 \cos t + c_3 \sin t$

10. Write $u = -\sin t - \cos t$ in the form $u = C\cos(t - \alpha)$ with $C > 0$ and $0 \le \alpha < 2\pi$.

A. $u = \sqrt{2}\cos(t - \pi/4)$

B. $u = \cos(t - \pi/4)$

C. $u = \cos(t - 5\pi/4)$

D. $u = 2\cos(t - \pi/4)$

E. $u = \sqrt{2}\cos(t - 5\pi/4)$

11. Find the general solution of the equation

$$y'' - 2y' + 2y = \frac{e^t}{\cos t}, \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}.$$

A. $c_1 e^t \cos t + c_2 e^t \sin t + e^t \cos t \ln(\cos t) + e^t t \sin t$

B. $c_1 e^t \cos t + c_2 e^t \sin t + \dfrac{e^t}{\cos t}$

C. $c_1 \cos t + c_2 \sin t - e^t \ln(\cos t)$

D. $c_1 e^t \cos t + c_2 e^t \sin t - e^t \cos t \ln(\cos t) + e^t t \sin t$

E. $c_1 e^t \cos t + c_2 e^t \sin t + e^t \cos t \ln(\cos t) - e^t t \sin t$

12. Consider the system $\mathbf{x}' = \begin{bmatrix} \alpha & 1 \\ 1 & \alpha \end{bmatrix} \mathbf{x}$. For what value of α, the origin is a saddle point?

A. No value of α

B. $\alpha < -1$

C. $-1 < \alpha < 1$

D. $\alpha > 1$

E. Any real α

13. Find the general solution of the given system.

$$\mathbf{x}' = \begin{bmatrix} 7 & 1 \\ -4 & 3 \end{bmatrix} \mathbf{x}.$$

A. $C_1 e^{-5t} \begin{bmatrix} 1 \\ -2 \end{bmatrix} + C_2 e^{-5t} \begin{bmatrix} t \\ -2t+1 \end{bmatrix}$

B. $C_1 e^{5t} \begin{bmatrix} 1 \\ -2 \end{bmatrix} + C_2 e^{5t} \begin{bmatrix} t \\ -2t+1 \end{bmatrix}$

C. $C_1 e^{5t} \begin{bmatrix} 1 \\ -2 \end{bmatrix} + C_2 e^{5t} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

D. $C_1 e^{-5t} \begin{bmatrix} 1 \\ -2 \end{bmatrix} + C_2 e^{-5t} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

E. $C_1 e^{-5t} \begin{bmatrix} -2 \\ 1 \end{bmatrix} + C_2 e^{-5t} \begin{bmatrix} t \\ -2t+1 \end{bmatrix}$

14. Find the solution of the following initial value problem

$$\mathbf{x}' = \begin{bmatrix} -4 & 2 \\ 2 & -4 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

A. $\mathbf{x}(t) = \begin{bmatrix} 2 \\ 2 \end{bmatrix} e^{-2t} - \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-6t}$

B. $\mathbf{x}(t) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{-2t}$

C. $\mathbf{x}(t) = \frac{1}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-6t} + \frac{1}{2} \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-2t}$

D. $\mathbf{x}(t) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{-6t}$

E. $\mathbf{x}(t) = \frac{1}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-2t} + \frac{1}{2} \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-6t}$

15. The general solution to the nonhomogeneous linear system $\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{g}$, where

$$\mathbf{A} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad \mathbf{g} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

is

A. $c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cos t + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} \sin t + \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

B. $c_1 \begin{bmatrix} \cos t \\ \sin t \end{bmatrix} + c_2 \begin{bmatrix} \sin t \\ -\cos t \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

C. $c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cos t + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} \sin t + \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

D. $c_1 \begin{bmatrix} \cos t \\ \sin t \end{bmatrix} + c_2 \begin{bmatrix} \sin t \\ -\cos t \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

E. $c_1 \begin{bmatrix} \cos t \\ \sin t \end{bmatrix} + c_2 \begin{bmatrix} \sin t \\ \cos t \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

16. Find the inverse Laplace transform of

$$F(s) = \frac{4(1+s)}{s^2(s^2+4)}.$$

A. $f(t) = t - \cos(2t) - \frac{1}{2}\sin(2t)$

B. $f(t) = 1 + t - \cos(2t) - \sin(2t)$

C. $f(t) = t - \cos(2t) - \sin(2t)$

D. $f(t) = 1 + t - \cos(2t) - \frac{1}{2}\sin(2t)$

E. $f(t) = 1 + t + \cos(2t) + \frac{1}{2}\sin(2t)$

17. The solution of the initial value problem

$$x'' + 8x' + 16x = f(t); \quad x(0) = 0, \quad x'(0) = 0$$

is

A. $\int_0^t \tau e^{-4\tau} f(t-\tau)d\tau$

B. $\int_0^t \tau e^{4\tau} f(t-\tau)d\tau$

C. $\int_0^t \tau e^{-4\tau} f(t-\tau)d\tau - \cos 4t$

D. $\int_0^t \tau e^{4\tau} f(t-\tau)d\tau + \sin 4t$

E. $\int_0^t \tau e^{4\tau} f(\tau - t)d\tau$

18. Find the Laplace transform of $f(t) = te^{3t}\cos 6t$

A. $\dfrac{(s-3)^2-36}{(s-3)^2+36}$

B. $\dfrac{s^2-36}{\left((s-3)^2+36\right)^2}$

C. $\dfrac{(s-3)^2-36}{\left((s-3)^2+36\right)^2}$

D. $\dfrac{s-36}{\left((s-3)^2+36\right)^2}$

E. $\dfrac{3s}{\left((s-3)^2+36\right)^2}$

19. Solve the initial value problem

$$y'' + 2y' + 5y = \delta(t-3), \quad y(0) = 0,\ y'(0) = 2.$$

A. $y = e^{-t}\cos(2t) + \dfrac{1}{2}u_3(t)e^{3-t}\sin(2t-6)$

B. $y = \dfrac{1}{2}e^{-t}\sin(2t) + \dfrac{1}{2}u_3(t)e^{3-t}\cos(2t-6)$

C. $y = e^{-t}\sin(2t) + u_3(t)e^{3-t}\sin(2t-6)$

D. $y = e^{-t}\cos(2t) + \dfrac{1}{2}u_3(t)e^{3-t}\sin(2t-6)$

E. $y = e^{-t}\sin(2t) + \dfrac{1}{2}u_3(t)e^{3-t}\sin(2t-6)$

20. Find the Laplace transform of $f(t) = \begin{cases} 0, & \text{if } 0 \le t < 1, \\ t^2, & \text{if } 1 \le t < 2, \\ 0, & \text{if } 2 \le t < \infty. \end{cases}$

A. $e^{-s}\left(\frac{2}{s^3} - \frac{2}{s^2} + \frac{1}{s}\right) - e^{-2s}\left(\frac{2}{s^3} - \frac{4}{s^2} + \frac{4}{s}\right)$

B. $e^{-s}\left(\frac{2}{s^3} + \frac{2}{s^2} + \frac{1}{s}\right) - e^{-2s}\left(\frac{2}{s^3} + \frac{4}{s^2} + \frac{4}{s}\right)$

C. $e^{-s}\left(\frac{2}{s^3}\right) - e^{-2s}\left(\frac{2}{s^3}\right)$

D. $e^{s}\left(\frac{2}{s^3} - \frac{2}{s^2} + \frac{1}{s}\right) - e^{2s}\left(\frac{2}{s^3} - \frac{4}{s^2} + \frac{4}{s}\right)$

E. $e^{-s}\left(\frac{2}{s^3} + \frac{2}{s^2} + \frac{1}{s}\right) + e^{-2s}\left(\frac{2}{s^3} + \frac{4}{s^2} + \frac{4}{s}\right)$

Table of Laplace Transforms

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\dfrac{1}{s}, \quad s > 0$
2.	e^{at}	$\dfrac{1}{s-a}, \quad s > a$
3.	$t^n, \quad n = \text{positive integer}$	$\dfrac{n\,!}{s^{n+1}}, \quad s > 0$
4.	$t^p, \quad p > -1$	$\dfrac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$
5.	$\sin at$	$\dfrac{a}{s^2+a^2}, \quad s > 0$
6.	$\cos at$	$\dfrac{s}{s^2+a^2}, \quad s > 0$
7.	$\sinh at$	$\dfrac{a}{s^2-a^2}, \quad s > \lvert a \rvert$
8.	$\cosh at$	$\dfrac{s}{s^2-a^2}, \quad s > \lvert a \rvert$
9.	$e^{at} \sin bt$	$\dfrac{b}{(s-a)^2+b^2}, \quad s > a$
10.	$e^{at} \cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}, \quad s > a$
11.	$t^n e^{at}, \quad n = \text{positive integer}$	$\dfrac{n\,!}{(s-a)^{n+1}}, \quad s > a$
12.	$u(t-c)$	$\dfrac{e^{-cs}}{s}, \quad s > 0$
13.	$u(t-c)f(t-c)$	$e^{-cs}F(s)$
14.	$e^{ct}f(t)$	$F(s-c)$
15.	$f(c\,t)$	$\dfrac{1}{c}\,F\left(\dfrac{s}{c}\right), \quad c > 0$
16.	$\displaystyle\int_0^t f(t-\tau)\,g(\tau)\,d\tau$	$F(s)\,G(s)$
17.	$\delta(t-c)$	e^{-cs}
18.	$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - sf^{(n-2)}(0) - f^{(n-1)}(0)$
19.	$t^nf(t)$	$(-1)^nF^{(n)}(s)$