Final Exam MATH 26600 SPRING 2018

$\boxed{\text{MARK TEST 01 ON YOUR SCANTRON!}}$

Name ____

Student ID ____

Section Number (see list below) ______________________________

031	UNIV 103	10:30am TR	Alper, Onur	041	REC 121	3:30pm MWF	Luo, Tao
051	UNIV 103	1:30pm TR	Hora, Raphael	052	UNIV 117	9:00am TR	Kaufmann, Birgit
053	REC 114	12:00pm TR	Petrosyan, Arshak	061	UNIV 101	10:30am MWF	Chen, Min
062	UNIV 103	12:00pm TR	Alper, Onur	063	UNIV 117	1:30pm MWF	Phillips, Daniel
081	REC 114	1:30pm TR	Petrosyan, Arshak	091	REC 121	2:30pm MWF	Luo, Tao
111	UNIV 103	8:30am MWF	Jin, Long	113	UNIV 103	3:00pm TR	Hora, Raphael
121	UNIV 103	9:30am MWF	Jin, Long	122	UNIV 303	12:30pm MWF	Ma, Zheng
141	UNIV 303	1:30pm MWF	Ma, Zheng	151	UNIV 003	10:30am MWF	Bell, Steven
152	UNIV 119	9:30am MWF	Torres, Monica	153	UNIV 117	3:00pm TR	Xu, Jie
154	UNIV 117	4:30pm TR	Xu, Jie	155	UNIV 117	11:30am MWF	Park, Moon Gyu

INSTRUCTIONS:

1. This exam contains 20 problems, worth 5 points each. There is one correct answer for each problem.
2. There is a table of Laplace transforms provided at the end of the exam.
3. Please fill in your name, ID, section number and test number 01 on the scantron.
4. Work only in the space provided, or on the backside of the pages. You must show your work.
5. Mark your answers clearly on the scantron. Also circle your choice for each problem in this booklet.
6. No books, notes, calculators, phones or other electronic devices, please.

ACADEMIC DISHONESTY

Purdue University faculty and students commit themselves towards maintaining a culture of academic integrity and honesty. The students taking this exam are not allowed to seek or obtain any kind of help from anyone to answer questions on this test. If you have questions, consult only an instructor or a proctor. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you finish your exam and hand it in to a proctor or to an instructor. You may not consult notes, books, calculators, cameras, or any kind of communications devices until after you finish your exam and hand it in to a proctor or to an instructor. If you violate these instructions you will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students. Your instructor and proctors will do everything they can to stop and prevent academic dishonesty during this exam. If you see someone breaking these rules during the exam, please report it to the proctor or to your instructor immediately. Reports after the fact are not very helpful.

Anyone who is seen handling any communication device gets automatically a score of 0 for the exam and likely an F in the course

I have read and understood the instructions regarding academic dishonesty:

Student name: Signature:

1. Find the solution to the initial value problem

$$\begin{cases} y' = \dfrac{4x^3+1}{2y-6}, \\ \\ y(1) = 2. \end{cases}$$

A. $y = 3 - \sqrt{x^4 + x - 1}$

B. $y = 2 + \sqrt{x^3 + x - 2}$

C. $y = 1 + \sqrt{x^4 + x - 1}$

D. $y = 4 - \sqrt{4x^3 + x - 1}$

E. $y = \sqrt{4x^3 + x - 1}$

2. Let $y(t)$ be exact solution of the following initial value problem

$$\begin{cases} ty' - 2y = t, \quad t > 0, \\ y(1) = 0. \end{cases}$$

Let $y_{\text{app}}(t)$ be the approximate solution of this initial value problem by using Euler's method with step size $h = 1$. Then $y_{\text{app}}(3) =$

A. 6

B. -4

C. 3

D. -3

E. 4

3. What is the largest open interval in which the solution of the initial value problem

$$\begin{cases} t^2y' + \dfrac{\ln|t-1|}{e^{t-2}}y = \dfrac{t-5}{\sin(t-4)} \\ \\ y(3) = \pi \end{cases}$$

is guaranteed to exist by the Existence and Uniqueness Theorem?

A. $(0, 4)$

B. $(2, 5)$

C. $(4 - \pi, 5)$

D. $(1, 4)$

E. $(4 - \pi, 4)$

4. Identify the differential equation that corresponds to the direction field on the right.

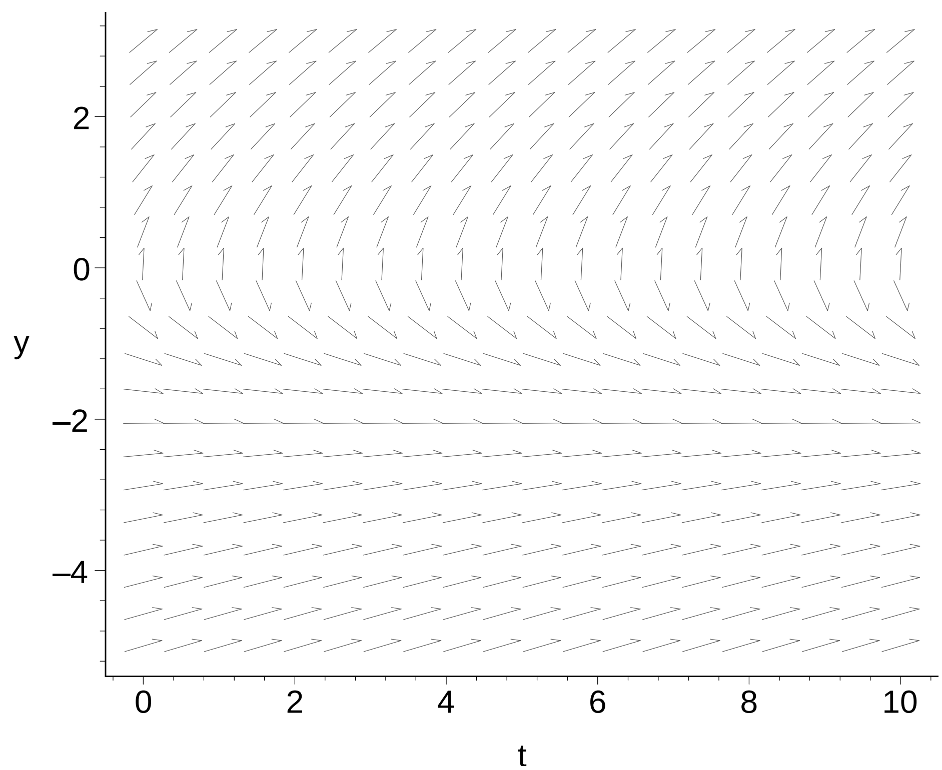
A. $\dfrac{dy}{dt} = y + 2$

B. $\dfrac{dy}{dt} = 2y(y + 2)$

C. $\dfrac{dy}{dt} = \dfrac{y+2}{2y}$

D. $\dfrac{dy}{dt} = \dfrac{2y}{y+2}$

E. $\dfrac{dy}{dt} = y^3(y + 2)^2$



5. Solve the initial value problem for the homogeneous equation

$$\frac{dy}{dx} = \frac{x^4 + y^4}{xy^3}, \quad y(e) = e, \quad x > 0$$

by using a substitution $v = y/x$.

A. $y = \dfrac{x}{\sqrt{3 - 2\ln x}}$

B. $y = \dfrac{x}{\sqrt{2\ln x - 1}}$

C. $y = x(4\ln x - 3)^{1/4}$

D. $y = x\left(\dfrac{8\ln x - 7}{9 - 8\ln x}\right)^{1/4}$

E. $y = x(3 - 2\ln x)^{1/4}$

6. Find the solution of $2ty' - 4y = 4t^4e^{t^2}$, $y(1) = 0$.

A. $t^2e^{t^2} - et^2$

B. $t^2e^{t^2}$

C. $e^{t^2} - e$

D. $te^t - et$

E. $t^2e^{t^2} - 2t^2$

7. Find the solution to the initial value problem

$$\begin{cases} 6xy + y^2 + (3x^2 + 2xy + 2y)\dfrac{dy}{dx} = 0 \\ \\ y(1) = 3 \end{cases}$$

A. $6xy + 2y^2 + x = 37$

B. $x^3y + x^2y^2 + y^2 + x = 22$

C. $3x^2y + 2x^2y + x^3 + 2x^2 + 2y = 24$

D. $x^2y + 2xy^2 + y^2 + x = 31$

E. $3x^2y + xy^2 + y^2 = 27$

8. Consider the initial value problem,

$$y' = -y(y-1)^2(y-2), \quad y(0) = y_0.$$

For which initial value y_0 is $\lim_{t\to\infty} y(t) = 1$?

A. -0.001

B. 0.001

C. 1.001

D. 2.001

E. None of the above

9. Determine a suitable form for $Y(t)$ if the method of undetermined coefficients is to be used on

$$y'' + y = t + t\ \sin(t).$$

A. $Y(t) = At + B + t(Ct + D)\cos(t) + t(Et + F)\sin(t)$

B. $Y(t) = At + B + (Ct + D)\cos(t) + (Et + F)\sin(t)$

C. $Y(t) = At + B + t(Ct + D)(\cos(t) + \sin(t))$

D. $Y(t) = t(At + B) + t(Ct + D)\cos(t) + t(Et + F)\sin(t)$

E. $Y(t) = At + t(Bt + C)\sin(t)$

10. Determine the values of α such that all solutions of the following equation tend to zero as $t \to \infty$.

$$y'' - (2\alpha - 1)y' + (\alpha^2 - \alpha + 1)y = 0.$$

A. $\alpha > 1$

B. $\alpha < 0$

C. $0.5 < \alpha < 1$

D. $0 < \alpha < 0.5$

E. $\alpha < 0.5$

11. Suppose $u(t) = 3\sin 3t + 4\cos 3t$ is a solution to a spring-mass system. The period and the amplitude of $u(t)$ are:

A. 2π, 5

B. $\frac{2}{3}\pi$, 4

C. $\frac{2}{3}\pi$, 5

D. $\frac{2}{3}\pi$, 3

E. 2π, 5

12. The function $y_1 = t^{-1}$ is a solution of

$$t^2y'' + 3ty' + y = 0, \quad t > 0.$$

If we seek a second solution $y_2 = t^{-1}v(t)$ by reduction of order, then $v(t)$ could be

A. $\dfrac{1}{t}$

B. $\ln t$

C. $\dfrac{\ln}{t}$

D. $\dfrac{1}{t^2}$

E. $t\ln t$

13. Which of the following is a fundamental set of solution of

$$4y^{(4)} - 5y'' - 9y = 0.$$

A. $\{e^t, e^{-t}, \cos(3t/2), \sin(3t/2)\}$
B. $\{e^{t/2}, e^{-t/2}, \cos(3t), \sin(3t)\}$
C. $\{e^{3t/2}, e^{-3t/2}, \cos t, \sin t\}$
D. $\{e^{3t}, e^{-3t}, \cos(t/2), \sin(t/2)\}$
E. none of the above

14. Note that $y_1(t) = \sqrt{t}$ and $y_2(t) = t^{-1}$ are solutions of the linear homogeneous differential equation

$$2t^2y'' + 3ty' - y = 0.$$

Use variation of parameters to find the general solution of the nonhomogeneous differential equation

$$2t^2y'' + 3ty' - y = 4t^2 + 4t.$$

A. $C_1\sqrt{t} + C_2t^{-1} + \frac{4}{9}t^2 + 2\,t$
B. $C_1\sqrt{t} + C_2t^{-1} + \frac{4}{9}t^3 + 2\,t^2$
C. $C_1\sqrt{t} + C_2t^{-1} + \frac{8}{35}t^4 + \frac{2}{5}t^3$
D. $C_1\sqrt{t} + C_2t^{-1} + \frac{8}{35}t^2 + \frac{2}{5}t$
E. $C_1\sqrt{t} + C_2t^{-1} + \frac{4}{9}t^2 + \frac{2}{5}t$

15. Find the inverse Laplace transform of the function

$$F(s) \;=\; e^{-3s}\frac{s+1}{s^2-8s+20}.$$

A. $u_3(t)\left(\cos(2t-6)+\sin(2t-6)\right)$

B. $e^{(4t-12)}\left(\cos(2t-3)+\sin(2t-3)\right)$

C. $u_3(t)\left(e^{4t}-e^{5t}\right)$

D. $u_3(t)e^{(4t-12)}\left(\cos(2t-6)+2.5\sin(2t-6)\right)$

E. $u_3(t)e^{4t}\left(\cos(2t-3)+0.5\sin(2t-3)\right)$

16. The solution of the initial value problem

$$y''+4y=g(t); \qquad y(0)=-1, \quad y'(0)=4$$

is

A. $y=\frac{1}{2}g(t)\sin 2t+2\sin 2t-\cos 2t$

B. $y=\int_0^t g(\tau)\sin 2(t-\tau)d\tau+2\sin 2t-\frac{1}{2}\cos 2t$

C. $y=\frac{1}{2}g(t)\sin 2t+2\sin 2t-\frac{1}{2}\cos 2t$

D. $y=\frac{1}{2}\int_0^t g(\tau)\sin 2(t-\tau)d\tau+2\sin 2t-\cos 2t$

E. $y=\frac{1}{2}G(s)\sin 2t+2\sin 2t-\cos 2t$

17. Find the Laplace transform of the function $f(t) = \begin{cases} t, & 0 \le t < 1 \\ 1, & t \ge 1 \end{cases}$.

A. $\dfrac{1}{s} - \dfrac{e^{-s}}{s^2}$

B. $\dfrac{1 - e^{-s}}{s^2}$

C. $\dfrac{1 - e^{-s}}{s}$

D. $\dfrac{1}{s} - \dfrac{e^{-(s-1)}}{s^2}$

E. $1 - \dfrac{e^{-s}}{s^2}$

18. In the phase portrait of the system

$$\mathbf{x}' = \begin{pmatrix} -2 & 3 \\ 1 & -4 \end{pmatrix} \mathbf{x},$$

the origin is a (an)

A. asymptotically stable node

B. unstable node

C. asymptotically stable spiral point

D. saddle point

E. asymptotically unstable spiral point

19. The general solution to

$$\mathbf{x}' = \begin{pmatrix} -4 & 4 \\ -1 & -8 \end{pmatrix} \mathbf{x}$$

can be given by

A. $c_1 e^{-6t} \begin{pmatrix} 2 \\ -1 \end{pmatrix} + c_2 e^{-6t} \left[t \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right]$

B. $c_1 e^{-6t} \begin{pmatrix} 2 \\ -1 \end{pmatrix} + c_2 e^{-6t} \left[t \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right]$

C. $c_1 e^{-6t} \begin{pmatrix} 2 \\ -1 \end{pmatrix} + c_2 e^{-6t} \left[t \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \end{pmatrix} \right]$

D. $c_1 e^{-6t} \begin{pmatrix} 2 \\ -1 \end{pmatrix} + c_2 e^{-6t} \left[t \begin{pmatrix} 4 \\ -2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right]$

E. $c_1 e^{-6t} \begin{pmatrix} 2 \\ -1 \end{pmatrix} + c_2 e^{-6t} \left[t \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right]$

20. The 2×2 matrix $A = \begin{pmatrix} -3 & -1 \\ 2 & -1 \end{pmatrix}$ has complex eigenvalues $r = -2 \pm i$. An eigenvector corresponding to $r = -2 + i$ is $\begin{pmatrix} 1 \\ -1 - i \end{pmatrix}$. The system

$$\mathbf{x}' = A\mathbf{x} + \begin{pmatrix} 3 \\ -4 \end{pmatrix} e^{-2t}$$

has one solution given by $\mathbf{x}(t) = \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-2t}$. What is the general solution to the system?

A. $c_1 \begin{pmatrix} 1 \\ -1 - i \end{pmatrix} e^{(-2+i)t} + c_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{2t}$

B. $c_1 \begin{pmatrix} \cos t \\ \sin t - \cos t \end{pmatrix} e^{-2t} + c_2 \begin{pmatrix} \sin t \\ -\sin t - \cos t \end{pmatrix} e^{-2t} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-2t}$

C. $c_1 \begin{pmatrix} \cos t \\ -\cos t \end{pmatrix} e^{-2t} + c_2 \begin{pmatrix} 0 \\ -\sin t \end{pmatrix} e^{-2t} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-2t}$

D. $c_1 \begin{pmatrix} 1 \\ -1 - i \end{pmatrix} e^{(-2+i)t} + c_2 \begin{pmatrix} -1 \\ 1 + i \end{pmatrix} e^{(-2-i)t} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-2t}$

E. $c_1 \begin{pmatrix} \cos t \\ -\cos t \end{pmatrix} e^{-2t} + c_2 \begin{pmatrix} 0 \\ -\sin t \end{pmatrix} e^{-2t} + c_3 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-2t}$

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\dfrac{1}{s}$
2.	e^{at}	$\dfrac{1}{s-a}$
3.	t^n	$\dfrac{n!}{s^{n+1}}$
4.	$t^p \ (p > -1)$	$\dfrac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sin at$	$\dfrac{a}{s^2+a^2}$
6.	$\cos at$	$\dfrac{s}{s^2+a^2}$
7.	$\sinh at$	$\dfrac{a}{s^2-a^2}$
8.	$\cosh at$	$\dfrac{s}{s^2-a^2}$
9.	$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}$
10.	$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$
11.	$t^n e^{at}$	$\dfrac{n!}{(s-a)^{n+1}}$
12.	$u_c(t)$	$\dfrac{e^{-cs}}{s}$
13.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$
14.	$e^{ct}f(t)$	$F(s-c)$
15.	$f(ct)$	$\dfrac{1}{c}F\left(\dfrac{s}{c}\right), \ c > 0$
16.	$\displaystyle\int_0^t f(t-\tau)\,g(\tau)\,d\tau$	$F(s)\,G(s)$
17.	$\delta(t-c)$	e^{-cs}
18.	$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - sf^{(n-2)}(0) - f^{(n-1)}(0)$
19.	$(-t)^n f(t)$	$F^{(n)}(s)$

Blank page

Blank page