MA26600 FINAL EXAM INSTRUCTIONS Spring 2017

GREEN - Test Version 01

NAME ______________________ INSTRUCTOR ______________________

1. You must use a **#2 pencil** on the mark–sense sheet (answer sheet).
2. On the mark–sense sheet, fill in the **instructor's** name (if you do not know, write down the class meeting time and location) and the **course number** which is **MA266**.
3. Fill in your **NAME** and blacken in the appropriate spaces.
4. Fill in the **SECTION Number** boxes with section number of your class (see below if you are not sure) and blacken in the appropriate spaces.

0031	TR	3:00PM	Arindam Banerjee	0091	TR	12:00PM	Samy Tindel
0041	MWF	12:30PM	Yong Suk Moon	0111	MWF	3:30PM	Yanghui Liu
0051	MWF	8:30AM	Long Jin	0113	MWF	9:30AM	Long Jin
0052	MWF	11:30AM	Yong Suk Moon	0121	MWF	4:30PM	Yanghui Liu
0053	TR	4:30PM	Jie Xu	0122	TR	3:00PM	Jie Xu
0061	MWF	2:30PM	Min Chen	0141	TR	10:30AM	Samy Tindel
0062	TR	4:30PM	Arindam Banerjee	0151	MWF	1:30PM	Min Chen
0063	TR	1:30PM	Yang Yang	0152	MWF	11:30AM	Sai Kee Yeung
0081	TR	12:00PM	Yang Yang	0153	MWF	10:30AM	Ying Zhang

5. Fill in the correct TEST/QUIZ NUMBER (**GREEN** is 01).
6. Fill in the 10-DIGIT PURDUE ID and blacken in the appropriate spaces.
7. Sign the mark–sense sheet.
8. Fill in your name and your instructor's name on the question sheets (above).
9. There are 20 questions, each worth 10 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–20 in the answer sheet. Do all your work on the question sheets, in addition, also **CIRCLE** your answer choice for each problem on the question sheets in case your scantron is lost. **Turn in both the mark-sense sheets and the question sheets to your instructor when you are finished**.
10. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.
11. NO CALCULATORS, BOOKS, NOTES, PHONES OR CAMERAS ARE ALLOWED on this exam. Turn off or put away all electronic devices. Use the back of the test pages for scrap paper.
12. **The Laplace transform table is provided at the end of the question sheets**.

1. Let $y(t)$ be the solution to the initial value problem

$$ty' + y = 3t^2 - 2t + 2, \quad y(1) = 2.$$

Then $y(2) =$

A. 0

B. 2

C. 4

D. 6

E. 8

2. Find the solution to the initial value problem

$$y' = \frac{3x^2 + 4x - 4}{2y - 4}, \quad y(1) = 3.$$

A. $y^2 - 4y = x^3 + x^2 - x - 2$

B. $y^2 - 4y = x^3 + 2x^2 - 4x - 1$

C. $y^2 - 4y = x^3 + 2x^2 - 4x$

D. $y^2 - 4y - 2 = x^3 + 2x^2 - 4x$

E. $y^2 - 4y = x^3 + 2x^2 - 4x - 2$

3. Initially a tank holds 40 gallons of water with 10 lb of salt in solution. A salt solution containing $\frac{1}{2}$ lb of salt per gallon runs into the tank at the rate of 4 gallons per minute. The well mixed solution runs out of the tank at a rate of 2 gallons per minute. Let $y(t)$ be the amount of salt in the tank after t minutes. Then $y(20) =$

A. 15 lb

B. 25 lb

C. 35 lb

D. 45 lb

E. 55 lb

4. If $y = y(x)$ is a solution of

$$y' = \frac{y}{x} + \frac{x}{y}, \quad x > 0 \quad \text{and} \quad y(1) = 2,$$

then $y(e) =$

A. 0

B. $6\sqrt{e}$

C. $e\sqrt{6}$

D. $\sqrt{6}$

E. $\sqrt{6} - e$

5. If the following differential equation is exact, select the implicit solution to the initial value problem

$$(e^x \sin y - 2y \sin x) + (e^x \cos y + 2\cos x + 2y)y' = 0; \qquad y(0) = \pi.$$

If it is not exact, select "NOT EXACT".

A. $e^x \cos y - 2y \sin x = -1$

B. $e^x \sin y + 2y \cos x = 2\pi$

C. $e^x \sin y + 2y \cos x + y^2 = 2\pi + \pi^2$

D. $e^x \sin y - 2y \cos x + y^2 = -2\pi + \pi^2$

E. NOT EXACT

6. Which of the following statements is true about the equilibrium solutions of the autonomous equation

$$\frac{dy}{dt} = y(9 - y^2).$$

A. $y = -3$ is (asymptotically) stable, $y = 0$ is unstable, $y = 3$ is (asymptotically) stable

B. $y = -3$ is unstable, $y = 0$ is (asymptotically) stable, $y = 3$ is unstable

C. All of $y = -3$, $y = 0$ and $y = 3$ are semistable

D. $y = -3$ is unstable, $y = 0$ is semistable, $y = 3$ is (asymptotically) stable

E. $y = -3$ is (asymptotically) stable, $y = 0$ is semistable, $y = 3$ is unstable

7. Use Euler's method to find approximate value of $y(0.2)$ for the following initial value problem with step size $h = 0.1$,

$$y' = y^2 + t^2, \qquad y(0) = 1.$$

A. 1.1

B. 1.1122

C. 1.2

D. 1.22

E. 1.222

8. The largest interval in which the solution of the initial value problem

$$\begin{cases} (5-t)y'' + (t-4)y' + 2y = \ln t \\ y(7) = 8 \end{cases}$$

is guaranteed to exist by the Existence and Uniqueness Theorem is:

A. $(-\infty, 5)$

B. $(0, \infty)$

C. $(0, 5)$

D. $(0, 4)$

E. $(5, \infty)$

9. Given that the function $y_1 = t$ is a solution to the differential equation

$$t^2y'' - ty' + y = 0, \quad t > 0,$$

choose a function y_2 from the list below so that the pair $\{y_1, y_2\}$ is a fundamental set of the solutions to the differential equation above.

A. $y_2 = t^3$

B. $y_2 = t \ln t$

C. $y_2 = t \sin t$

D. $y_2 = t \cos t$

E. $y_2 = te^t$

10. Which of the following is a particular solution to the differential equation

$$y'' + 4y = \frac{1}{\cos(2t)} \quad ?$$

A. $t\cos(2t) + \ln|\cos(2t)|\sin(2t)$

B. $\frac{1}{2}t\cos(2t) - \frac{1}{2}\ln|\sin(2t)|\sin(2t)$

C. $-\frac{1}{2}t\cos(2t) + \frac{1}{4}\ln|\sin(2t)|\sin(2t)$

D. $\frac{1}{2}t\sin(2t) + \frac{1}{4}\ln|\cos(2t)|\cos(2t)$

E. $-\frac{1}{2}t\sin(2t) - \ln|\cos(2t)|\cos(2t)$

11. In an electric circuit, a capacitor and an inductor are connected in series. The charge $Q(t)$ in the capacitor satisfies the equation

$$9Q'' + Q = 0,$$

with the initial condition $Q(0) = 1,\ Q'(0) = 1$. If we write $Q(t) = R\cos(\omega t - \delta)$, then find R and δ.

A. $R = 10,\ \delta = \arctan(2)$

B. $R = \sqrt{10},\ \delta = \arctan(3)$

C. $R = \sqrt{5},\ \delta = \dfrac{\pi}{4}$

D. $R = \sqrt{10},\ \delta = \arctan(\dfrac{1}{3})$

E. $R = 5,\ \delta = -\dfrac{\pi}{3}$

12. A particular solution of the equation

$$y^{(4)} + 2y'' + y = 9\cos(t) + 2e^{-t} - 5t$$

is of the form

A. $Y(t) = At\cos(t) + Bt\sin(t) + Ce^{-t} + Dt + E$

B. $Y(t) = A\cos(t) + B\sin(t) + Cte^{-t} + Dt^2 + Et$

C. $Y(t) = At\cos(t) + Bt\sin(t) + Ce^{-t} + Dt^2 + Et$

D. $Y(t) = At^2\cos(t) + Bt^2\sin(t) + Ce^{-t} + Dt + E$

E. $Y(t) = At^2\cos(t) + Bt^2\sin(t) + Cte^{-t} + Dt + E$

13. A spring system with external forcing term is represented by the equation

$$y'' + 2y' + 2y = 4\cos(t) + 2\sin(t).$$

Then the steady state solution of the system is given by

A. $Y(t) = 2\sin(t)$

B. $Y(t) = \cos(t) - 2\sin(t)$

C. $Y(t) = -2\cos(t) + \sin(t)$

D. $Y(t) = -\cos(t)$

E. None of the above

14. Find the inverse Laplace transform of

$$F(s) = \frac{2s^2 + 5s + 7}{(s+2)(s^2 + 2s + 5)}.$$

A. $e^{-2t} + e^{-t}\sin(2t)$

B. $e^{-2t} + e^{-t}\cos(2t)$

C. $e^{-2t} - e^{-t}\cos(2t)$

D. $e^{-2t} - e^{-t}\sin(2t)$

E. $e^{2t} + e^{-t}\cos(2t)$

15. Find the Laplace transform of $f(t) = \begin{cases} 0, & 0 \le t < 1 \\ te^t, & t \ge 1. \end{cases}$

A. $\dfrac{e}{(s-1)^2} + \dfrac{e}{s-1}$

B. $\dfrac{e^{1-s}}{(s-1)^2} + \dfrac{e^{1-s}}{s-1}$

C. $\dfrac{e^{-s}}{(s-1)^2}$

D. $\dfrac{e^{-s}}{s^2(s-1)}$

E. $\dfrac{e^{1-s}}{s^2(s-1)}$

16. If $y(t)$ is the solution of the initial value problem

$$y'' + 2y' - 15y = \delta(t-1), \quad y(0) = 0, \quad y'(0) = 0,$$

then $y(t) =$

A. $\dfrac{u_1(t)}{8}\left(e^{3t} - e^{-5t}\right)$

B. $\dfrac{u_1(t)}{8}\left(e^{-3t-1} - e^{-5t-1}\right)$

C. $\dfrac{u_1(t)}{8}\left(e^{3t-1} - e^{-5t-1}\right)$

D. $\dfrac{u_1(t)}{8}\left(e^{5t-5} - e^{-3t+3}\right)$

E. $\dfrac{u_1(t)}{8}\left(e^{3t-3} - e^{-5t+5}\right)$

17. Find the Laplace transform of

$$e^t \int_0^t \sin\tau \cos(t-\tau)d\tau.$$

A. $\dfrac{s-1}{[(s-1)^2+1]^2}$

B. $e^s \dfrac{s}{(s^2+1)^2}$

C. $\dfrac{s^2}{(s^2+1)^2}$

D. $\dfrac{1}{s(s^2+1)}$

E. None of the above

18. Consider the system

$$\mathbf{x}' = \begin{pmatrix} 0 & 8 \\ -2 & \alpha \end{pmatrix} \mathbf{x},$$

find all the value of α so that the origin is an asymptotically unstable spiral point.

A. $\alpha > 0$

B. $\alpha > 8$

C. $\alpha < -8$

D. $0 < \alpha < 8$

E. $0 > \alpha > -8$

19. Let $\mathbf{x}(t)$ be the solution of the initial value problem

$$\mathbf{x}'(t) = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} \mathbf{x}(t), \qquad \mathbf{x}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

What is $\mathbf{x}(1)$?

A. $\begin{pmatrix} 3e \\ e \end{pmatrix}$

B. $\begin{pmatrix} 2e \\ e \end{pmatrix}$

C. $\begin{pmatrix} e \\ 0 \end{pmatrix}$

D. $\begin{pmatrix} 3e \\ 0 \end{pmatrix}$

E. $\begin{pmatrix} e \\ e \end{pmatrix}$

20. Let $\Phi(t) = \begin{pmatrix} e^t & e^{3t} \\ -e^t & e^{3t} \end{pmatrix}$ be a fundamental matrix of the homogeneous system

$$\mathbf{x}'(t) = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \mathbf{x}(t).$$

Which of the following is a particular solution to the nonhomogeneous system

$$\mathbf{x}'(t) = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \mathbf{x}(t) + \begin{pmatrix} 0 \\ 4e^t \end{pmatrix} \quad ?$$

A. $\begin{pmatrix} 2t \\ -2t+1 \end{pmatrix} e^t$

B. $\begin{pmatrix} 2t+1 \\ 2t-1 \end{pmatrix} e^t$

C. $\begin{pmatrix} 2t+1 \\ -2t \end{pmatrix} e^t$

D. $\begin{pmatrix} -2t-1 \\ 2t-1 \end{pmatrix} e^t$

E. $\begin{pmatrix} 2t+2 \\ 2t-2 \end{pmatrix} e^t$

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\dfrac{1}{s}$
2.	e^{at}	$\dfrac{1}{s-a}$
3.	t^n	$\dfrac{n!}{s^{n+1}}$
4.	$t^p \ (p > -1)$	$\dfrac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sin at$	$\dfrac{a}{s^2+a^2}$
6.	$\cos at$	$\dfrac{s}{s^2+a^2}$
7.	$\sinh at$	$\dfrac{a}{s^2-a^2}$
8.	$\cosh at$	$\dfrac{s}{s^2-a^2}$
9.	$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}$
10.	$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$
11.	$t^n e^{at}$	$\dfrac{n!}{(s-a)^{n+1}}$
12.	$u_c(t)$	$\dfrac{e^{-cs}}{s}$
13.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$
14.	$e^{ct}f(t)$	$F(s-c)$
15.	$f(ct)$	$\dfrac{1}{c}F\left(\dfrac{s}{c}\right), \ c > 0$
16.	$\int_0^t f(t-\tau)\,g(\tau)\,d\tau$	$F(s)\,G(s)$
17.	$\delta(t-c)$	e^{-cs}
18.	$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - sf^{(n-2)}(0) - f^{(n-1)}(0)$
19.	$(-t)^n f(t)$	$F^{(n)}(s)$