MA 266
FINAL EXAM INSTRUCTIONS
May 8, 2010

NAME ______________________ INSTRUCTOR ______________________

1. You must use a #2 pencil on the mark–sense sheet (answer sheet).

2. On the mark-sense sheet, fill in the instructor's name (if you do not know, write down the class meeting time and location) and the course number which is MA266.

3. Fill in your NAME and blacken in the appropriate spaces.

4. Fill in the SECTION NUMBER boxes with section number of your class (see below if you are not sure) and blacken in the appropriate spaces.

- 0001 meets MWF 8:30- 9:20 REC 313 Yu, Haijun
- 0002 meets MWF 9:30-10:20 REC 313 Yu, Haijun
- 0003 meets MWF 10:30-11:20 UNIV 019 Li, Fang
- 0004 meets MWF 11:30-12:20 UNIV 019 Li, Fang
- 0006 meets MWF 12:30- 1:20 UNIV 019 Scheiblechner, Peter
- 0007 meets MWF 1:30- 2:20 UNIV 019 To, Tung Tran
- 0008 meets MWF 1:30- 2:20 UNIV 017 Lee, Kyungyong
- 0005 meets MWF 2:30- 3:20 REC 313 Liu, Tong
- 0009 meets MWF 4:30- 5:20 UNIV 019 To, Tung Tran
- 0010 meets TTh 12:00- 1:15 REC 122 Tohaneanu, Mihai H
- 0011 meets TTh 12:00- 1:15 REC 123 Momin, Al Saeed
- 0012 meets TTh 1:30- 2:45 UNIV 103 Tohaneanu, Mihai H
- 0013 meets TTh 1:30- 2:45 REC 123 Momin, Al Saeed
- 0014 meets TTh 3:00- 4:15 REC 302 Chen, Min
- 0015 meets MWF 11:30-12:20 UNIV 017 Scheiblechner, Peter

5. Leave the TEST/QUIZ NUMBER blank.

6. Fill in the 10-DIGIT PURDUE ID and blacken in the appropriate spaces.

7. Sign the mark–sense sheet.

8. Fill in your name and your instructor's name on the question sheets (above).

9. There are 20 questions, each worth 5 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–20 in the answer sheet. Do all your work on the question sheets. Turn in both the mark–sense sheets and the question sheets when you are finished.

10. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.

11. NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

12. The Laplace transform table is provided at the end of the question sheets.

1. If $y = t^r$ is a solution to the given equation

$$t^2 y'' - bty' + by = 0, \quad t > 0$$

then

A. $r = -b$ or $r = -1$

B. $\boxed{r = b \text{ or } r = 1}$

C. $r = b + 1$ or $r = 1$

D. $r = \frac{b \pm \sqrt{b^2 - 4b}}{2}$

E. $r = \pm 1$

2. Identify the differential equation that produces the given direction field.

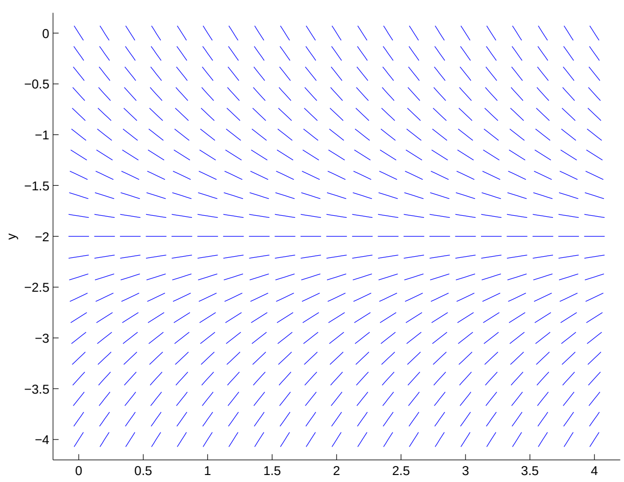
A. $y' = 1 - 2y$

B. $y' = y + 2$

C. $\boxed{y' = -(y + 2)}$

D. $y' = y - 2$

E. $y' = 1 + 2y$



3. Let y be a solution of the initial value problem

$$y' - \frac{1}{2x}y = x, \quad x > 0, \quad \text{and} \quad y(1) = \frac{1}{3},$$

then $y(4)$ is

A. 2

B. 4

C. 6

D. 8

E. $\boxed{10}$

4. The general solution of the differential equation

$$y' = \frac{1}{x}\frac{y}{1-y} - (1-x)\frac{y}{1-y}$$

in implicit form is

A. $y + \ln|y-1| = \ln|x| - x - \frac{1}{2}x^2 + C$

B. $y + \ln|y-1| = -\frac{1}{x^2} - 1 + C$

C. $\boxed{y - \ln|y| = \ln|x| + x - \frac{1}{2}x^2 + C}$

D. $y - \ln|y| = -\frac{1}{x^2} - 1 + C$

E. $y - \ln|y| = \ln|x| - x + \frac{1}{2}x^2 + C$

5. Classify the stability of the equilibrium solutions of the autonomous differential equation

$$\frac{dy}{dt} = \frac{1}{2}y^3 - \frac{1}{2}y^2 - y.$$

A. -2 unstable, 0 stable, 4 unstable

B. $\boxed{-1 \text{ unstable, } 0 \text{ stable, } 2 \text{ unstable}}$

C. -1 stable, 0 unstable, 2 stable

D. -1 stable, 0 stable, 1 unstable

E. -2 unstable, 0 stable, 1 stable

6. If $y = y(x)$ is a solution of

$$y' = \frac{y}{x} + \frac{x}{y}, \quad x > 0 \quad \text{and} \quad y(1) = 2,$$

then $y(e) =$

A. 0

B. $6\sqrt{e}$

C. $\boxed{e\sqrt{6}}$

D. $\sqrt{6}$

E. $\frac{\sqrt{6}}{e}$

7. Solve the initial value problem.

$$y'' - y = \cos(t), \qquad y(0) = 0, \;\; y'(0) = 0.$$

A. $\frac{1}{4}e^t - \frac{1}{4}e^{-t}$

B. $\boxed{\frac{1}{4}e^t + \frac{1}{4}e^{-t} - \frac{1}{2}\cos(t)}$

C. $\frac{1}{4}e^t - \frac{1}{4}e^{-t} - \frac{1}{2}\sin(t)$

D. $\frac{1}{4}e^t + \frac{1}{4}e^{-t} - \frac{1}{2}\cos(t) - \frac{1}{2}\sin(t)$

E. $\frac{1}{2}e^t + \frac{1}{2}e^{-t} - \cos(t)$

8. The function $y_1 = t^2$ is a solution of the equation

$$t^2y'' - 3ty' + 4y = 0, \quad t > 0.$$

Choose a function y_2 from the list so that $\{y_1, y_2\}$ form a fundamental set of solutions to the differential equation.

A. t

B. t^{-1}

C. $\boxed{t^2\ln(t)}$

D. $t^3\ln(t)$

E. t^2e^{2t}

9. A mass weighing 8 lb stretches a spring 0.5 ft. The mass is pulled down 1 ft from the equilibrium position, and then set in motion with an upward velocity of 2 ft/sec. Assume that there is no damping force and that the downward direction is the positive direction. The gravity constant g is 32 ft/sec^2. Then the function $u(t)$ describing the displacement of the mass from the equilibrium position as a function of time t satisfies the following initial value problem.

A. $8u'' + 0.5u = 0, \quad u(0) = 1, \quad u'(0) = 2$

B. $u'' + 64u = 0, \quad u(0) = 1, \quad u'(0) = 2$

C. $u'' + 16u = 0, \quad u(0) = 1, \quad u'(0) = 2$

D. $64u'' + u = 0, \quad u(0) = 1, \quad u'(0) = -2$

E. $\boxed{u'' + 64u = 0, \quad u(0) = 1, \quad u'(0) = -2}$

10. The general solution of the equation

$$y'' - 2y' + 2y = \frac{e^t}{\cos t}, \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}$$

is

A. $c_1 e^t \cos t + c_2 e^t \sin t + \frac{e^t}{\cos t}$

B. $c_1 \cos t + c_2 \sin t - e^t \ln(\cos t)$

C. $\boxed{c_1 e^t \cos t + c_2 e^t \sin t + e^t \cos t \ln(\cos t) + e^t t \sin t}$

D. $c_1 e^t \cos t + c_2 e^t \sin t - e^t \cos t \ln(\cos t) + e^t t \sin t$

E. $c_1 e^t \cos t + c_2 e^t \sin t + e^t \cos t \ln(\cos t) - e^t t \sin t$

11. According to the method of undetermined coefficients, what is the proper form of a particular solution Y to the following differential equation?

$$y^{(4)} - 4y'' = 24t^2 - 4 - 3te^t.$$

A. $Y(t) = At^2 + Bte^t$.

B. $Y(t) = At^2 + Bt + C + Dte^t + Ee^t$.

C. $Y(t) = At^3 + Bt^2 + Ct + D + Ete^t + Fe^t$.

D. $\boxed{Y(t) = At^4 + Bt^3 + Ct^2 + Dte^t + Ee^t.}$

E. $Y(t) = At^2 + Bt + C + Dt^2e^t + Ete^t$.

12. Find the Laplace transform of

$$f(t) = \begin{cases} 0 & \text{when } t < \pi, \\ t - \pi & \text{when } \pi \leq t < 2\pi, \\ 0 & \text{when } t \geq 2\pi. \end{cases}$$

A. $\boxed{e^{-\pi s}\frac{1}{s^2} - e^{-2\pi s}\frac{1}{s^2} - \pi e^{-2\pi s}\frac{1}{s}}$

B. $e^{-\pi s}\frac{1}{s^2} - e^{-2\pi s}\frac{1}{s^2}$

C. $e^{\pi s}\frac{1}{s^2} - e^{2\pi s}\frac{1}{s^2} - \pi e^{2\pi s}\frac{1}{s^2}$

D. $\frac{1}{s}\left(e^{-\pi s} - e^{-2\pi s}\right)$

E. None of the above.

13. Find the Laplace transform of the function

$$f(t) = \int_0^t \cos(t-\tau)e^{\tau}\sin(\tau)\,d\tau.$$

A. $\boxed{\dfrac{s}{(s^2+1)((s-1)^2+1)},}$

B. $\dfrac{s-1}{(s^2+1)((s-1)^2+1)},$

C. $\dfrac{s}{((s-1)^2+1)^2},$

D. $\dfrac{s-1}{((s-1)^2+1)^2},$

E. $\displaystyle\int_0^t \frac{s}{(s^2+1)^2}ds.$

14. Find the solution of the initial value problem

$$y'' - 3y' + 2y = \delta(t-2); \quad y(0) = 0, \quad y'(0) = 1.$$

A. $-e^{2t} + e^t + e^t\,u_2(t)\,(e^{2t-4} + e^{t-2}),$

B. $e^{2t} + e^t + u_2(t)(e^{2t} - e^t),$

C. $\boxed{e^{2t} - e^t + u_2(t)(e^{2t-4} - e^{t-2}),}$

D. $e^{2t} + e^t + u_2(t)(e^{2t} + e^t),$

E. $-e^{2t} + e^t + u_2(t)\,e^{t-2}.$

15. If $y(t)$ solves the initial value problem

$$y'' + y = \begin{cases} t/2, & 0 \le t < 6 \\ 3, & 6 \le t \end{cases}, \quad y(0) = 0, \quad y'(0) = 0$$

for $t \ge 0$, then

A. $y = \begin{cases} \frac{1}{2}\sin(t) + \frac{t}{2} & t < 6 \\ \frac{1}{2}\sin(t) + 3 & t \ge 6 \end{cases}$

B. $y = \begin{cases} \frac{1}{2}\sin(t) + \frac{t}{2} & t < 6 \\ \sin(t) + 3 & t \ge 6 \end{cases}$

C. $y = \begin{cases} \frac{1}{2}\sin(t) + \frac{1}{2}t & t < 6 \\ \frac{1}{2}\sin(t) + \frac{1}{2}\sin(t-6) + 3 & t \ge 6 \end{cases}$

D. $y = \begin{cases} \frac{1}{2}\cos(t) + \frac{1}{2}t & t < 6 \\ \frac{1}{2}\cos(t) + \frac{1}{2}\sin(t-6) + 3 & t \ge 6 \end{cases}$

E. $\boxed{y = \begin{cases} -\frac{1}{2}\sin(t) + \frac{1}{2}t & t < 6 \\ -\frac{1}{2}\sin(t) + \frac{1}{2}\sin(t-6) + 3 & t \ge 6 \end{cases}}$

16. Which of the following is a second order *non-linear* differential equation.

A. $ty'' = \sin(t)y' - \frac{1}{t}y + t^2$.

B. $y'' = 3y' + 4y - 6$

C. $(y')^2 = 6ty - e^t$.

D. $\boxed{y'' = 3e^t(y')^2 + \sin(y) + 5e^t}$

E. $y'y = 6e^t$

17. The function $x_1(t)$ determined by the initial value problem

$$\begin{cases} x_1' = x_2 \\ x_2' = -x_1 \end{cases}$$

with initial conditions $x_1(0) = 1$ and $x_2(0) = 1$ is given by

A. $x_1(t) = -\sin t + \cos t$

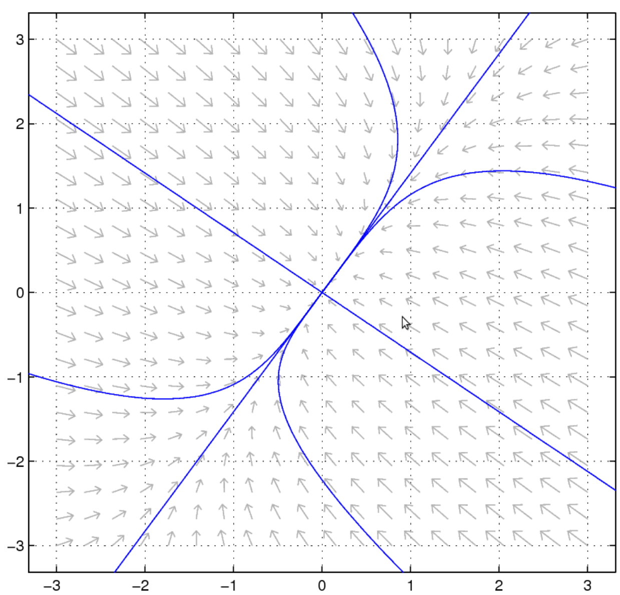
B. $\boxed{x_1(t) = \sin t + \cos t}$

C. $x_1(t) = \frac{1}{2}(e^t + e^{-t})$

D. $x_1(t) = \sin t$

E. $x_1(t) = ie^{it} - ie^{-it}$

18. The phase portrait for a linear system of the form $\mathbf{x}' = A\mathbf{x}$, where A is a 2×2 matrix is as follows. If r_1 and r_2 denote the eigenvalues of A, then what can you conclude about r_1 and r_2 by examining the phase portrait?



A. r_1 and r_2 are distinct and positive

B. $\boxed{r_1 \text{ and } r_2 \text{ are distinct and negative}}$

C. r_1 and r_2 have opposite signs

D. r_1 and r_2 are repeated and positive

E. r_1 and r_2 are complex and have negative real part

19. Consider the system

$$\mathbf{x}' = \begin{pmatrix} \alpha & 1 \\ 1 & \alpha \end{pmatrix} \mathbf{x}.$$

For what values of α is the equilibrium solution $\mathbf{x} = 0$ a saddle point?

A. no values of α

B. $\alpha < -1$

C. $\alpha > 1$

D. $\boxed{-1 < \alpha < 1}$

E. all real α

20. The general solution to the non-homogeneous linear system $\mathbf{x}' = A\mathbf{x} + \mathbf{g}$, where

$$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad \mathbf{g} = \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

is

A. $\boxed{c_1 \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} + c_2 \begin{pmatrix} \sin t \\ -\cos t \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}}$

B. $c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cos t + c_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sin t + \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

C. $c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cos t + c_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sin t + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

D. $c_1 \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} + c_2 \begin{pmatrix} \sin t \\ -\cos t \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

E. $c_1 \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} + c_2 \begin{pmatrix} \sin t \\ \cos t \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \end{pmatrix}$