MA26600 Final Exam

GREEN VERSION 01

NAME: ______________________________

INSTRUCTOR: ______________________ SECTION/TIME: __________

- You must use a **#2 pencil** on the mark-sense answer sheet.
- Fill in the **ten digit PUID** (starting with two zeroes) and your **Name** and blacken in the appropriate spaces.
- Fill in the correct **Test/Quiz number** (GREEN is **01**, ORANGE is **02**)
- Fill in the **four digit section number** of your class and blacken the numbers below them. Here they are:

0010	MWF	10:30AM	Gayane Poghotanyan	0084	MWF	1:30PM	Ping Xu
0011	MWF	9:30AM	Gayane Poghotanyan	0095	MWF	1:30PM	Jiahao Zhang
0022	MWF	11:30AM	Gayane Poghotanyan	0096	MWF	1:30PM	Krishnendu Khan
0034	MWF	3:30PM	Guang Yang	0107	MWF	12:30PM	Krishnendu Khan
0035	MWF	2:30PM	Guang Yang	0108	MWF	2:30PM	Shuyi Weng
0046	MWF	12:30PM	Heejong Lee	0109	MWF	3:30PM	Shuyi Weng
0061	MWF	12:30PM	Jiahao Zhang	0110	MWF	9:30AM	Jakayla Robbins
0062	MWF	9:30AM	Arun Debray	0111	MWF	8:30AM	Jakayla Robbins
0071	MWF	10:30AM	Arun Debray	0112	MWF	11:30AM	Ping Xu
0072	TR	12:00PM	Xingshan Cui	0113	MWF	12:30PM	Ping Xu
0083	TR	10:30AM	Xingshan Cui	0114	TR	1:30PM	Christian Noack

- Sign the mark-sense sheet.
- Fill in your name and your instructor's name and the time of your class meeting on the exam booklet above.
- There are 20 multiple-choice questions, each worth 10 points. **Blacken in** your choice of the correct answer in the spaces provided for questions 1–20 in the answer sheet. Do all your work on the question sheets, in addition, also **Circle** your answer choice for each problem on the question sheets in case your mark-sense sheet is lost.
- Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.
- **No calculators, books, electronic devices, or papers are allowed.** Use the back of the test pages for scratch paper.
- Pull off the **table of Laplace transforms** on the last page of the exam for reference. Do not turn it in with your exam booklet at the end.

1. Find a general solution of the following differential equation:

$$\frac{dy}{dx} = \frac{x+4}{\sqrt{xy}}, \qquad (x > 0,\ y > 0)$$

A. $\frac{1}{2}x^{3/2} + 8x^{1/2} - \frac{2}{3}y^{2/3} = C$

B. $\frac{3}{2}x^{2/3} + 8x^{1/2} + \frac{3}{2}y^{2/3} = C$

C. $y = x + 12x^{-1} + C$

D. $y = (x^{3/2} + 12\sqrt{x})^{2/3} + C$

E. $\frac{2}{3}x^{3/2} + 8x^{1/2} - \frac{2}{3}y^{3/2} = C$

2. Find a general solution to the following differential equation

$$\frac{dy}{dx} + 3y = 3x^2e^{-3x} + 2xe^{-3x}.$$

A. $y = (x^3 + x^2)e^{-3x} + C$

B. $y = (x^3 + x^2 + C)e^{-3x}$

C. $y = (3x^2 + 2x + C)e^{-3x}$

D. $y = -\left(x^2 + \frac{4}{3}x + \frac{4}{9}\right)e^{-3x} + C$

E. $y = (x^3 + x^2)e^{-3x} + Ce^{3x}$

3. Which of the following is an implicit solution to the differential equation

$$(2x + 7y)dx + (7x + 8y)dy = 0?$$

A. $x^2 + 7xy + 4y^2 = C$

B. $x^2 - 7xy - 4y^2 = C$

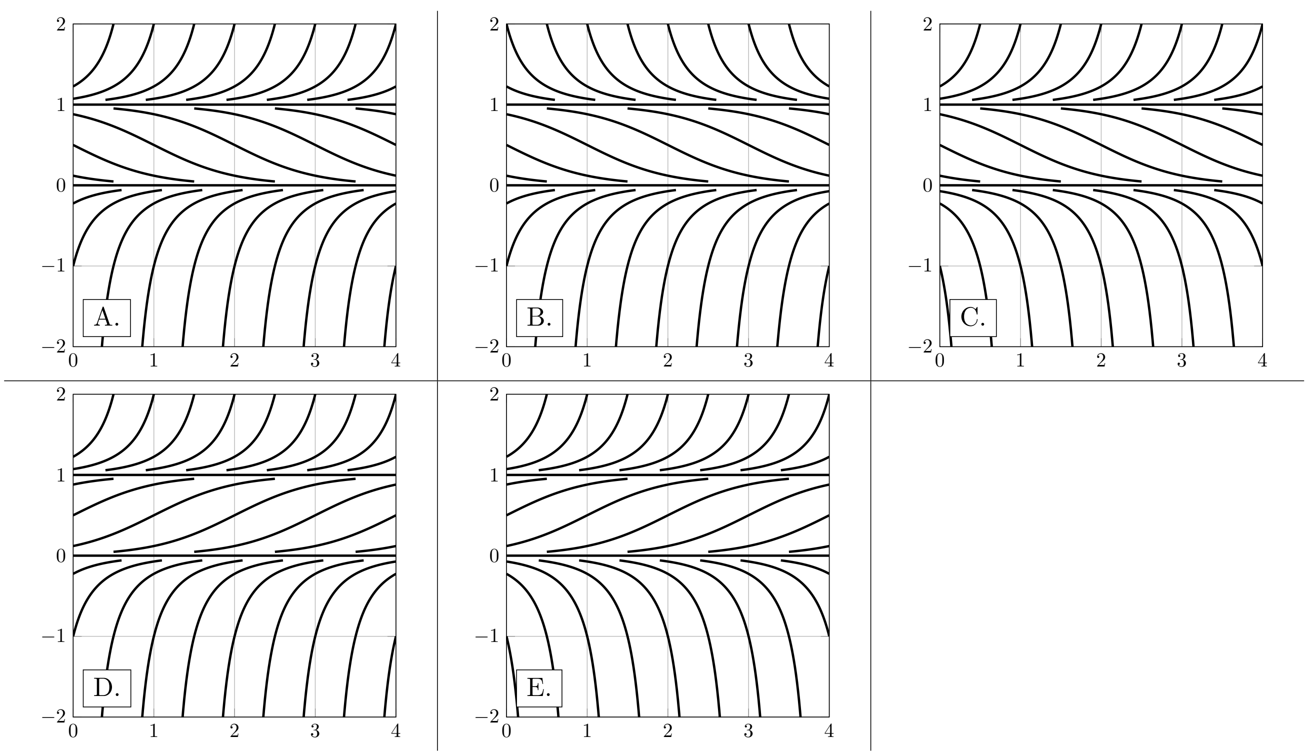
C. $x^2 + 4y^2 = C$

D. $x^2 - 4y^2 = C$

E. $x^3 - 7xy + 4y^2 = C$

4. Which of the following figures sketches typical solution curves of the differential equation

$$\frac{dy}{dx} = 2y^2(y - 1)?$$



5. Solve the initial value problem

$$y'' - 4y' + 4y = 0, \qquad y(0) = 12,\ y'(0) = -3.$$

A. $y(t) = 10e^{2t} - 25te^{2t}$

B. $y(t) = 12e^{2t} - 27te^{2t}$

C. $y(t) = -15e^{2t}$

D. $y(t) = 5e^{2t} + 7e^{-2t}$

E. $y(t) = 27e^{2t} - 15e^{-2t}$

6. The nonhomogeneous differential equation

$$x^2y'' - 4xy' + 6y = x^3, \qquad x > 0$$

has complementary solution $y_c(x) = c_1x^2 + c_2x^3$. Using the method of variation of parameters, the particular solution $y_p(x)$ is given by

A. $y_p(x) = 4x^5 - 6x^4$

B. $y_p(x) = x^5/3 - x^4$

C. $y_p(x) = x^3 - x^2 \ln x$

D. $y_p(x) = x^5/6$

E. $y_p(x) = x^3(\ln x - 1)$

7. Solve the initial value problem

$$y'' - 3y' - 4y = 3e^{2x}, \qquad y(0) = \frac{5}{2},\ y'(0) = 1.$$

A. $y(t) = \frac{3}{2}e^{-x} + 2e^{4x} - e^{2x}$

B. $y(t) = 4e^{x} - e^{-4x} - \frac{1}{2}e^{2x}$

C. $y(t) = e^{-x} + e^{4x} + \frac{1}{2}e^{2x}$

D. $y(t) = 2e^{-x} + e^{4x} - \frac{1}{2}e^{2x}$

E. $y(t) = \frac{7}{2}e^{x} - 2e^{-4x} + e^{2x}$

8. A spring-mass system set in motion is determined by the initial value problem

$$u'' + 100u = 0, \quad u(0) = 2, \quad u'(0) = -10.$$

What is the amplitude of the motion?

A. $\sqrt{5}$

B. $\sqrt{2}$

C. 10

D. $\sqrt{104}$

E. $\frac{1}{10}$

9. A fish tank contains 10 gallons of a salt solution with a concentration of 3 grams of salt per gallon. A salt solution with a concentration of 6 grams/gallon is added to the tank at a rate of 2 gallons per minute. At the same time, the solution is drained from the well mixed tank at a rate of 2 gallons per minute. How many grams of salt are in the tank after 10 minutes?

A. $30 + 12e^{-5}$

B. $30 + 30e^{-10}$

C. $120 - 90e^{-2}$

D. $60 - 30e^{-2}$

E. 60

10. Determine the appropriate form for a particular solution of

$$y^{(3)} + 4y'' + 4y' = 4e^{-2x}x.$$

A. $Ae^{-2x}x^3$

B. $Ae^{-2x}x^3 + Be^{-2x}x^2$

C. $Ae^{-2x}x^3 + Be^{-2x}x^2 + Ce^{-2x}x + De^{-2x}$

D. $Ae^{-2x}x + Be^{-2x} + C$

E. $Ae^{-2x}x^2 + Be^{-2x}x$

11. Which of the following functions is a solution of $y^{(4)} - 16y = 0$?

A. $y = 3e^{2t} - e^{-2t} + 5\cos t - 2\sin t$

B. $y = 16e^{t} - \cos 4t - \sin 4t$

C. $y = 5e^{2t} - e^{-2t} + 7\cos 2t - 3\sin 2t$

D. $y = (2t+1)e^{2t} + (3t-4)e^{-2t}$

E. $y = (t^3 + 3t^2 - 5t + 1)e^{2t}$

12. Consider the linear system

$$\mathbf{x}' = \begin{bmatrix} 4 & 1 \\ 6 & -1 \end{bmatrix} \mathbf{x}.$$

What is the phase portrait at the origin?

A. Saddle point

B. Spiral sink

C. Spiral source

D. Nodal source

E. Center

13. Solve the initial value problem

$$\begin{cases} x'(t) = 6x(t) - 3y(t) \\ y'(t) = 2x(t) + y(t) \end{cases}$$

where $x(0) = 5$ and $y(0) = 3$

A. $x(t) = 6e^{-4t} - e^{-3t}$ and $y(t) = 4e^{-4t} - e^{-3t}$

B. $x(t) = \frac{24}{5}e^{4t} + \frac{1}{5}e^{3t}$ and $y(t) = \frac{16}{5}e^{4t} - \frac{1}{5}e^{3t}$

C. $x(t) = 6e^{4t} - e^{3t}$ and $y(t) = 4e^{4t} - e^{3t}$

D. $x(t) = \frac{24}{5}e^{-4t} + \frac{1}{5}e^{-3t}$ and $y(t) = \frac{16}{5}e^{-4t} - \frac{1}{5}e^{-3t}$

E. $x(t) = 16e^{4t} - 11e^{3t}$ and $y(t) = -24e^{4t} + 11e^{3t}$

14. If $\mathbf{A}$ is a 2 x 2 real-valued matrix with eigenvalues $\lambda_1 = -3$ and $\lambda_2 = 4$ with corresponding eigenvectors $\mathbf{v_1} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\mathbf{v_2} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$, find the matrix exponential $e^{\mathbf{A}t}$.

A. $\begin{bmatrix} 4e^{4t} - 3e^{-3t} & 2e^{4t} - 2e^{-3t} \\ 3e^{-3t} - 3e^{4t} & 4e^{-3t} - 4e^{4t} \end{bmatrix}$

B. $\begin{bmatrix} e^{-3t} & 3e^{4t} \\ -e^{-3t} & -2e^{4t} \end{bmatrix}$

C. $\begin{bmatrix} e^{-3t} & 0 \\ 0 & e^{4t} \end{bmatrix}$

D. $\begin{bmatrix} 3e^{4t} - 2e^{-3t} & 3e^{4t} - 3e^{-3t} \\ 2e^{-3t} - 2e^{4t} & 3e^{-3t} - 2e^{4t} \end{bmatrix}$

E. $\begin{bmatrix} -2e^{-3t} & -3e^{-3t} \\ e^{-3t} & e^{4t} \end{bmatrix}$

15. Determine the appropriate form for a particular solution $\mathbf{x}_p(t)$ to the nonhomogeneous linear system:

$$\mathbf{x}'(t) = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} -e^{-t} \\ 2e^{t} \end{bmatrix}$$

A. $\mathbf{x_p}(t) = \mathbf{a}te^{-t} + \mathbf{b}e^{t}$

B. $\mathbf{x_p}(t) = (\mathbf{a}t + \mathbf{b})e^{-t} + (\mathbf{c}t + \mathbf{d})e^{t}$

C. $\mathbf{x_p}(t) = \mathbf{a}e^{-t} + \mathbf{b}e^{t}$

D. $\mathbf{x_p}(t) = \mathbf{a}e^{-t} + (\mathbf{b}t + \mathbf{c})e^{t}$

E. $\mathbf{x_p}(t) = (\mathbf{a}t + \mathbf{b})e^{-t} + \mathbf{c}e^{t}$

16. After applying the Laplace transform to the differential equation

$$x'' + ax = b\,t^3, \quad x(0) = 1,\ x'(0) = 0,$$

one obtains that $X(s) = \mathcal{L}\{x(t)\}$ is given by the algebraic formula

$$X(s) = \frac{1}{s} + \frac{1}{s^6}.$$

Find the values of a and b.

A. $a = 0,\ b = \dfrac{1}{3}$

B. $a = 0,\ b = \dfrac{1}{6}$

C. $a = -1,\ b = \dfrac{1}{2}$

D. $a = 1,\ b = \dfrac{1}{6}$

E. $a = 1,\ b = 3$

17. Find the inverse Laplace transform of $F(s) = \dfrac{4s-3}{s^2-4s+29}$.

A. $e^{2t}\cos(5t) + 4e^{2t}\sin(5t)$

B. $4e^{5t}\cos(2t) + \dfrac{17}{2}e^{5t}\sin(2t)$

C. $e^{5t}\cos(2t) - 4e^{5t}\sin(2t)$

D. $4e^{2t}\cos(5t) + e^{2t}\sin(5t)$

E. $4e^{2t}\cos(5t) - \dfrac{3}{5}e^{2t}\sin(5t)$

18. Solve the initial value problem

$$x'' + x = t + \delta_{2\pi}(t), \quad x(0) = 0, \quad x'(0) = -1.$$

A. $x(t) = t$

B. $x(t) = t - \sin(t)$

C. $x(t) = t - \sin(t) + u_{2\pi}(t)\sin(t)$

D. $x(t) = t + u_{2\pi}(t)\sin(t)$

E. $x(t) = t - 2\sin(t) + u_{2\pi}(t)\sin(t)$

19. Find the Laplace transform of $f(t) = \begin{cases} t, & \text{if } 3 \le t < 5, \\ 0, & \text{otherwise.} \end{cases}$

A. $e^{-3s}\left(\frac{1}{s^2}+\frac{3}{s}\right) - e^{-5s}\left(\frac{1}{s^2}+\frac{5}{s}\right)$

B. $e^{-3s}\left(\frac{1}{s^2}+\frac{3}{s}\right) + e^{-5s}\left(\frac{1}{s^2}+\frac{5}{s}\right)$

C. $\frac{e^{-3s}}{s^2} - \frac{e^{-5s}}{s^2}$

D. $\frac{e^{-3s}}{s^2} + \frac{e^{-5s}}{s^2}$

E. $\frac{e^{-2s}}{s^2}$

20. Let $F(x) = \mathcal{L}\{f(t)\}$ be the Laplace transform of the function

$$f(t) = \int_0^t e^{-\tau}\cos(2\tau)\sin(t-\tau)d\tau.$$

What is the value of $F(1)$?

A. $F(1) = \frac{3}{16}$

B. $F(1) = \frac{1}{10}$

C. $F(1) = \frac{1}{8}$

D. $F(1) = 4$

E. $F(1) = \frac{1}{32}$

Table of Laplace Transforms

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\dfrac{1}{s}, \quad s > 0$
2.	e^{at}	$\dfrac{1}{s-a}, \quad s > a$
3.	$t^n, \quad n = \text{positive integer}$	$\dfrac{n\,!}{s^{n+1}}, \quad s > 0$
4.	$t^p, \quad p > -1$	$\dfrac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$
5.	$\sin at$	$\dfrac{a}{s^2+a^2}, \quad s > 0$
6.	$\cos at$	$\dfrac{s}{s^2+a^2}, \quad s > 0$
7.	$\sinh at$	$\dfrac{a}{s^2-a^2}, \quad s > \lvert a \rvert$
8.	$\cosh at$	$\dfrac{s}{s^2-a^2}, \quad s > \lvert a \rvert$
9.	$e^{at} \sin bt$	$\dfrac{b}{(s-a)^2+b^2}, \quad s > a$
10.	$e^{at} \cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}, \quad s > a$
11.	$t^n e^{at}, \quad n = \text{positive integer}$	$\dfrac{n\,!}{(s-a)^{n+1}}, \quad s > a$
12.	$u(t-c)$	$\dfrac{e^{-cs}}{s}, \quad s > 0$
13.	$u(t-c)f(t-c)$	$e^{-cs}F(s)$
14.	$e^{ct}f(t)$	$F(s-c)$
15.	$f(c\,t)$	$\dfrac{1}{c}F\left(\dfrac{s}{c}\right), \quad c > 0$
16.	$\displaystyle\int_0^t f(t-\tau)\,g(\tau)\,d\tau$	$F(s)\,G(s)$
17.	$\delta(t-c)$	e^{-cs}
18.	$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - sf^{(n-2)}(0) - f^{(n-1)}(0)$
19.	$t^nf(t)$	$(-1)^nF^{(n)}(s)$