MA26600 Final Exam

GREEN VERSION 01

NAME: ______________________ PUID (10 digits): ______________________

INSTRUCTOR: ______________________ SECTION/TIME: ______________________

- You must use a **#2 pencil** on the mark-sense answer sheet.
- Fill in the **ten digit PUID** (starting with two zeroes) and your **Name** and blacken in the appropriate spaces.
- Fill in the correct **Test/Quiz number** (GREEN is **01**, ORANGE is **02**)
- Fill in the **four digit section number** of your class and blacken the numbers below them. Here they are:

0010	TR	10:30A	G Poghotanyan
0011	TR	12:00P	G Poghotanyan
0022	TR	3:00P	Zhaopeng Hao
0023	TR	1:30P	Zhaopeng Hao
0034	TR	1:30P	Ying Liang
0035	MWF	9:30A	Heejin Lee
0046	MWF	1:30P	Ping Xu
0047	TR	3:00P	Ying Liang
0061	MWF	3:30P	Christian Noack
0062	TR	12:00P	Guang Yang
0071	TR	9:00A	Guang Yang
0072	MWF	10:30A	E Birgit Kaufmann
0083	MWF	11:30A	E Birgit Kaufmann
0084	MWF	8:30A	Krishnendu Khan
0095	MWF	7:30A	Krishnendu Khan
0096	MWF	10:30A	Ping Xu
0107	MWF	1:30P	Yilong Zhang
0108	MWF	8:30A	Heejin Lee
0109	MWF	2:30P	Christian Noack
0110	MWF	2:30P	Michelle Michelle
0111	MWF	3:30P	Michelle Michelle
0112	MWF	9:30A	Ping Xu
0113	MWF	11:30A	Yilong Zhang

- Sign the mark-sense sheet.
- Fill in your name and your instructor's name and the time of your class meeting on the exam booklet above.
- There are 20 multiple-choice questions, each worth 10 points. **Blacken in** your choice of the correct answer in the spaces provided for questions 1–20 in the answer sheet. Do all your work on the question sheets, in addition, also **Circle** your answer choice for each problem on the question sheets in case your mark-sense sheet is lost.
- Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.
- **No calculators, books, electronic devices, or papers are allowed.** Use the back of the test pages for scratch paper.
- Pull off the **table of Laplace transforms** on the last page of the exam for reference. Do not turn it in with your exam booklet at the end.

1. What is the largest open interval in which a solution $y(x)$ to the initial value problem

$$(x+2)(x-1)y' + (x-3)y = \frac{2}{x-5}, \quad y(0) = 3$$

is guaranteed to exist?

A. $(-2, 3)$
B. $(5, \infty)$
C. $(-2, 1)$
D. $(-2, 5)$
E. $(-\infty, -2)$

2. Solve the differential equation

$$\frac{dy}{dx} = \frac{4x^3 + 6x^2}{y^2}, \quad y(1) = 3.$$

A. $y = \sqrt[3]{18x^4 + 3x^3 + 6}$
B. $y = \sqrt[3]{3x^4 + 6x^3 + 18}$
C. $y = \sqrt[3]{x^4 + 8x^3 + 18}$
D. $y = \sqrt[3]{3x^4 + 18x^3 + 6}$
E. $y = \sqrt[3]{3x^4 + 9x^3 + 15}$

3. Find the unique solution $y(x)$ to the following initial value problem

$$2xy' + y = 4\sqrt{x}, \quad y(4) = 2.$$

A. $x^{3/2} - 3\sqrt{x}$

B. $\sqrt{x} - \dfrac{2}{\sqrt{x}} + 2$

C. $2x^{3/2} - 7\sqrt{x}$

D. $2\sqrt{x} - \dfrac{4}{\sqrt{x}}$

E. $\sqrt{x} + \dfrac{2}{\sqrt{x}} - 1$

4. A tank initially contains 400 gallons of pure water. Brine containing 2 lb of salt per gallon enters the tank at a rate of 4 gal/min, and the well-stirred mixture leaves the tank at the same rate. How many minutes will it take for the amount of salt in the tank to reach 400 lb?

A. $100 \ln 2$

B. $-400 \ln 3$

C. $200 \ln 5$

D. $100 - 100e^{-2}$

E. $200 - e^{-2}$

5. Which statements below are **TRUE** for this autonomous equation

$$\frac{dy}{dx} = 3y(y-4)^2(y+4)^3 \ ?$$

(I) $y = -4$ is an asymptotically stable equilibrium solution.

(II) $y = 0$ is an asymptotically stable equilibrium solution.

(III) There are precisely three equilibrium solutions.

A. All three are TRUE

B. Only (II) and (III)

C. Only (I) and (III)

D. Only (I) and (II)

E. Only (II)

6. Solve the initial value problem:

$$\begin{aligned} y'' + 4y' + 3y &= 0, \\ y(0) &= 2, \\ y'(0) &= -4. \end{aligned}$$

A. $e^{-x} + \frac{1}{2}e^{-3x}$

B. $e^{-x} + e^{-3x}$

C. $e^{x} - e^{-3x}$

D. $e^{x} - \frac{1}{2}e^{-3x}$

E. $e^{-x} - e^{-3x}$

7. Find **all** values of k for which the general solution to

$$y'' + 2(1-k)y' + k^2 y = 0$$

has the form $y = C_1 e^{ax}\cos bx + C_2 e^{ax}\sin bx$, where $b \neq 0$. (Note: a is allowed to be 0.)

A. $k > 1$

B. $k = b$

C. $0 < k < 1$

D. $k < \frac{1}{4}$

E. $k > \frac{1}{2}$

8. Find the trial solution for a particular solution y_p of the following differential equation using the method of undetermined coefficients.

$$y^{(4)} - 2y''' + 10y'' = t^2 + e^t \cos 2t.$$

A. $At^3 + Bt^2 + Ct + De^t \cos 2t + Ee^t \sin 2t$

B. $At^4 + Bt^3 + Ct^2 + Dt + E + Fe^t \cos 2t + Ge^t \sin 2t$

C. $At^4 + Bt^3 + Ct^2 + Dt^2 e^t \cos 2t + Et^2 e^t \sin 2t$

D. $At^4 + Bt^3 + Ct^2 + De^t \cos 2t + Ee^t \sin 2t$

E. $At^2 + Bt + C + De^t \cos 2t + Ee^t \sin 2t$

9. An object with mass m is attached to a spring with spring constant $k = 1$ lb/ft. If there is no damping and the external force acting on a mass is $F(t) = 5\cos 3t$ pounds, find the value of m for which resonance occurs.

A. $m = 1/9$

B. $m = 20$

C. $m = 9$

D. $m = 1/20$

E. $m = 3$

10. Which of the following differential equations has

$$y(t) = c_1 e^{-t} + c_2 t e^{-t} + c_3 \cos t + c_4 \sin t$$

as a general solution?

A. $y^{(4)} - 2y''' + 2y' - y = 0$

B. $y^{(4)} + 2y''' - 2y' - y = 0$

C. $y^{(4)} - 2y''' - 2y' + y = 0$

D. $y^{(4)} - 2y'' + y = 0$

E. $y^{(4)} + 2y''' + 2y'' + 2y' + y = 0$

11. Given that $y_c(x) = c_1x + c_2x^{-1}$ is a complementary solution of the nonhomogeneous second order differential equation

$$x^2y'' + xy' - y = 600x^5,$$

use the method of variation of parameters to find its particular solution $y_p(x)$.

A. $y_p(x) = 3x^7 \ln x$

B. $y_p(x) = 125x^5$

C. $y_p(x) = 25x^6 \ln x$

D. $y_p(x) = 25x^5$

E. $y_p(x) = \dfrac{45}{2}x^7$

12. Transform the following system into an equivalent system of first-order differential equations by letting $x_1 = x,\ x_2 = x'$ and $y_1 = y,\ y_2 = y'$:

$$x'' + 3x' + 4x - 2y = 0,$$
$$y'' + 2y' - 3x + y = e^{2\pi t}.$$

A. $x_1' = x_2,\quad x_2' = -4x_1 - 2y_1 - 3x_2,\quad y_1' = y_2,\quad y_2' = 3x_1 - y_1 - 2y_2 + e^{2\pi t}$

B. $x_1' = x_2,\quad x_2' = -4x_1 + 2y_1 + 3x_2,\quad y_1' = y_2,\quad y_2' = 3x_1 - y_1 + 2y_2 + e^{2\pi t}$

C. $x_1' = x_2,\quad x_2' = -4x_1 + 2y_1 - 3x_2,\quad y_1' = y_2,\quad y_2' = 3x_1 - y_1 - 2y_2 + e^{2\pi t}$

D. $x_1' = x_2,\quad x_2' = -4x_1 + 2y_1 - 3x_2,\quad y_1' = y_2,\quad y_2' = 3x_1 + y_1 - 2y_2 + e^{2\pi t}$

E. $x_1' = x_2,\quad x_2' = -4x_1 - 2y_1 - 3x_2,\quad y_1' = y_2,\quad y_2' = 3x_1 + y_1 - 2y_2 + e^{2\pi t}$

13. Consider the following systems

$$\mathbf{x}' = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} \mathbf{x} \qquad \text{(I)}$$

$$\mathbf{x}' = \begin{bmatrix} -1 & 2 \\ -2 & -1 \end{bmatrix} \mathbf{x} \qquad \text{(II)}$$

Which of the following statements is true about their phase plots?

A. Both (I) and (II) are saddle points

B. (I) spiral source and (II) spiral sink

C. (I) spiral sink and (II) saddle point

D. (I) spiral source and (II) saddle point

E. (I) spiral sink and (II) spiral source

14. Let $x(t)$ and $y(t)$ be the particular solutions of the following initial value problem:

$$\begin{aligned} x' &= 5x + y, & x(0) &= 1, \\ y' &= 3x + 7y, & y(0) &= -5. \end{aligned}$$

Find $x(2) + y(2)$.

A. $-4e^{16}$

B. $6e^{8}$

C. $6e^{16}$

D. $4e^{8}$

E. $6e^{8} + 7e^{16}$

15. Find the general solution of

$$\mathbf{x}' = \begin{bmatrix} 1 & -4 \\ 4 & 9 \end{bmatrix} \mathbf{x}.$$

A. $\mathbf{x}(t) = c_1 \begin{bmatrix} -4 \\ 4 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{5t}$

B. $\mathbf{x}(t) = c_1 \begin{bmatrix} 4 \\ 0 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} 0 \\ 4t \end{bmatrix} e^{-5t}$

C. $\mathbf{x}(t) = c_1 \begin{bmatrix} -4 \\ 4 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} 1 - 4t \\ 4t \end{bmatrix} e^{5t}$

D. $\mathbf{x}(t) = c_1(1+t) \begin{bmatrix} -4 \\ 4 \end{bmatrix} e^{5t}$

E. $\mathbf{x}(t) = c_1 \begin{bmatrix} -4 \\ 4 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} -4 \\ t+4 \end{bmatrix} e^{-5t}$

16. Find the Laplace transform of the solution to the initial value problem

$$x'' - x = 5\sin 2t, \quad x(0) = 1,\ x'(0) = 1.$$

A. $\dfrac{1}{s-1} + \dfrac{1}{(s-1)^2} + \dfrac{5}{s^2+4}$

B. $\dfrac{1}{(s-1)^2} + \dfrac{5}{2(s^2+4)}$

C. $\dfrac{1}{s+1} - \dfrac{1}{s-1} + \dfrac{2}{s^2+4}$

D. $\dfrac{2}{s+1} + \dfrac{1}{s-1} - \dfrac{1}{s^2+2}$

E. $-\dfrac{1}{s+1} + \dfrac{2}{s-1} - \dfrac{2}{s^2+4}$

17. Find the solution y of the initial value problem

$$4y'' + 4y = 1 + \delta(t-\pi), \quad y(0) = 0,\ y'(0) = 0.$$

A. $\frac{1}{4} - \frac{1}{4}\cos t - \frac{1}{4}u(t-\pi)\sin t$

B. $-\frac{1}{4}\cos t - \frac{1}{4}u(t-\pi)\sin t$

C. $\frac{1}{4} - \frac{1}{4}\sin t - \frac{1}{4}u(t-\pi)\cos t$

D. $\frac{1}{4} - \frac{1}{4}\cos t + \frac{1}{4}u(t-\pi)\cos t$

E. $\frac{1}{4} - \frac{1}{4}u(t-\pi)\sin t$

18. The Laplace transform of

$$f(t) = \int_0^t (t-\tau)e^{t-\tau}\cos(2\tau)d\tau$$

is

A. $F(s) = \dfrac{2}{(s^2+1)(s+4)^2}$

B. $F(s) = \dfrac{s}{(s-4)^2(s-1)}$

C. $F(s) = \dfrac{1}{(s-1)(s^2+4)}$

D. $F(s) = \dfrac{s}{(s-1)^2(s^2+4)}$

E. $F(s) = \dfrac{s}{(s^2+1)(s^2+4)}$

19. Find the Laplace transform of

$$f(t) = \begin{cases} 0, & t < 1, \\ t^2 e^{2t}, & t \geq 1. \end{cases}$$

A. $e^{-s-2}\left(\dfrac{2}{(s-2)^3} - \dfrac{2}{(s-2)^2} + \dfrac{1}{s-2}\right)$

B. $e^{-s+2}\left(\dfrac{2}{(s-2)^3} + \dfrac{2}{(s-2)^2} + \dfrac{1}{s-2}\right)$

C. $e^{-s+2}\left(\dfrac{2}{(s-1)^3} + \dfrac{4}{(s-1)^2} + \dfrac{4}{s-1}\right)$

D. $\dfrac{2e^{-s}}{(s-2)^3}$

E. $e^{-2s+1}\left(\dfrac{2}{(s-1)^3} + \dfrac{2}{(s-1)^2} + \dfrac{1}{s-1}\right)$

20. Find the inverse Laplace transform $\mathcal{L}^{-1}\{F(s)\}$ of the function

$$F(s) = \frac{13}{s(s^2 + 4s + 13)}.$$

A. $2 + \cos 3t + \frac{1}{2} \sin 3t$

B. $3 + e^{-2t} \cos 3t - e^{-2t} \sin 3t$

C. $1 - \frac{2}{3} e^{-2t} \sin 3t$

D. $e^{-2t} \cos 3t + e^{-2t} \sin 3t$

E. $1 - e^{-2t} \cos 3t - \frac{2}{3} e^{-2t} \sin 3t$

Table of Laplace Transforms

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\dfrac{1}{s}, \quad s > 0$
2.	e^{at}	$\dfrac{1}{s-a}, \quad s > a$
3.	$t^n, \quad n = \text{positive integer}$	$\dfrac{n!}{s^{n+1}}, \quad s > 0$
4.	$t^p, \quad p > -1$	$\dfrac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$
5.	$\sin at$	$\dfrac{a}{s^2 + a^2}, \quad s > 0$
6.	$\cos at$	$\dfrac{s}{s^2 + a^2}, \quad s > 0$
7.	$\sinh at$	$\dfrac{a}{s^2 - a^2}, \quad s > \lvert a \rvert$
8.	$\cosh at$	$\dfrac{s}{s^2 - a^2}, \quad s > \lvert a \rvert$
9.	$e^{at} \sin bt$	$\dfrac{b}{(s-a)^2 + b^2}, \quad s > a$
10.	$e^{at} \cos bt$	$\dfrac{s-a}{(s-a)^2 + b^2}, \quad s > a$
11.	$t^n e^{at}, \quad n = \text{positive integer}$	$\dfrac{n!}{(s-a)^{n+1}}, \quad s > a$
12.	$u(t-c)$	$\dfrac{e^{-cs}}{s}, \quad s > 0$
13.	$u(t-c)f(t-c)$	$e^{-cs}F(s)$
14.	$e^{ct}f(t)$	$F(s-c)$
15.	$f(ct)$	$\dfrac{1}{c} F\left(\dfrac{s}{c}\right), \quad c > 0$
16.	$\displaystyle\int_0^t f(t-\tau)\, g(\tau)\, d\tau$	$F(s)\, G(s)$
17.	$\delta(t-c)$	e^{-cs}
18.	$f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0) - \cdots - s f^{(n-2)}(0) - f^{(n-1)}(0)$
19.	$t^n f(t)$	$(-1)^n F^{(n)}(s)$