MA26600 FINAL EXAM INSTRUCTIONS Fall 2019

GREEN - Test Version 01

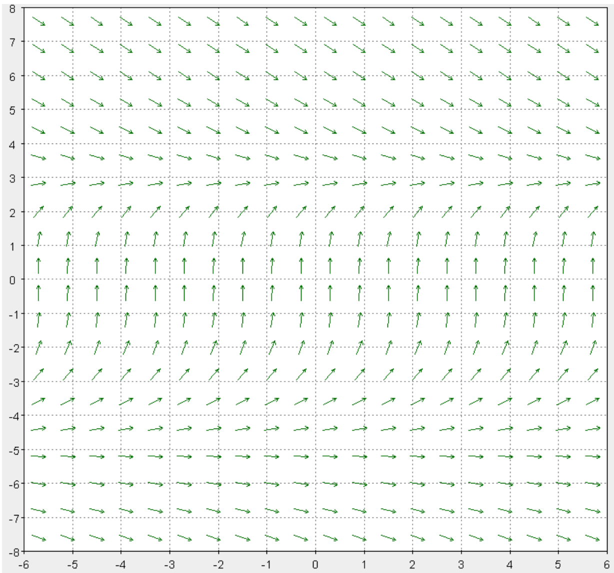
NAME ____________________ INSTRUCTOR ____________________

1. You must use a **#2 pencil** on the mark–sense sheet (answer sheet).
2. On the mark–sense sheet, fill in the **instructor's** name (if you do not know, write down the class meeting time and location) and the **course number** which is **MA266**.
3. Fill in your **NAME** and blacken in the appropriate spaces.
4. Fill in the **SECTION Number** boxes with section number of your class (see below if you are not sure) and blacken in the appropriate spaces.

0052	TR	1:30PM	Ali Mohammad-Nezhad	0071	MWF	10:30AM	Ying Zhang
0062	MWF	8:30AM	Otavio Menezes	0072	TR	3:00PM	Shuddhodan Vasudevan
0063	MWF	2:30AM	Ke Wu	0073	TR	10:30AM	Emanuel Indrei
0064	TR	4:30PM	Shuddhodan Vasudevan	0074	MWF	9:30AM	Otavio Menezes
0065	TR	3:00PM	Zhiguo Yang	0075	TR	1:30PM	Zhiguo Yang
0066	MWF	3:30PM	Ke Wu	0076	MWF	10:30AM	Gayane Poghotanyan
0067	MWF	1:30PM	Rongqing Ye	0077	MWF	11:30AM	Gayane Poghotanyan
0068	TR	12:00PM	Ali Mohammad-Nezhad	0078	MWF	12:30PM	Alaa Haj Ali
0069	MWF	2:30PM	Rongqing Ye	0079	MWF	1:30PM	Alaa Haj Ali
0070	TR	9:00AM	Emanuel Indrei				

5. Fill in the correct TEST/QUIZ NUMBER (**GREEN** is 01).
6. Fill in the 10-DIGIT PURDUE ID and blacken in the appropriate spaces.
7. Sign the mark–sense sheet.
8. Fill in your name and your instructor's name on the question sheets (above).
9. There are 20 questions, each worth 10 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–20 in the answer sheet. Do all your work on the question sheets, in addition, also **CIRCLE** your answer choice for each problem on the question sheets in case your scantron is lost. **Turn in both the mark-sense sheets and the question sheets to your instructor when you are finished**.
10. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.
11. NO CALCULATORS, BOOKS, NOTES, PHONES OR CAMERAS ARE ALLOWED on this exam. Turn off or put away all electronic devices. Use the back of the test pages for scrap paper.
12. **The Laplace transform table is provided at the end of the question sheets**.

1. The field



is a slope field for the differential equation:

A. $y' = \frac{(y-3)(y+5)}{y}$

B. $y' = \frac{(y-3)(y+5)^2}{y}$

C. $y' = \frac{(3-y)(y+5)}{y^2}$

D. $y' = \frac{(y-3)(y+5)}{y}$

E. $y' = \frac{(y-3)(y+5)^2}{y^2}$

2. Find the number of **stable** critical points for the autonomous equation

$$\frac{dx}{dt} = x(x-1)^2(x+3)(x^2-4).$$

A. 1

B. 2

C. 3

D. 4

E. 0

3. Find the explicit solution of the initial value problem

$$y' = \frac{xy^2}{x^2+1} \quad , \quad y(0) = 3.$$

A. $\frac{1}{2}(6 + \ln(1 + x^2))$

B. $\dfrac{6}{2 - 3\ln(1 + x^2)}$

C. $\dfrac{6}{2 + 3\ln(1 + x^2)}$

D. $\frac{1}{2}(6 - 3\ln(1 + x^2))$

E. $\frac{1}{3}(9 - 2\ln(1 + x^2))$

4. A stone is dropped from rest at an initial height $h = 25$ feet above the surface of the earth. Ignoring air resistance, we assume that the acceleration of the stone is $a = -g$ where $g = 32$ ft/s^2 is the gravitational acceleration. How long does the stone need to strike the ground?

A. 2 s

B. 1.5 s

C. 1s

D. 0.5s

E. 1.25 s

5. If the following differential equation is exact, select the implicit solution to the initial value problem

$$(\frac{y}{x} + 6x) + (\ln x - 2y)y' = 0 \qquad y(1) = 2; \qquad x > 0.$$

If it is not exact, select "NOT EXACT".

A. $y^2/x - 6xy + y\ln x = -8$

B. $y\ln x - 3x^2 + y^2 = 1$

C. $y\ln x - 4xy = -8$

D. $y\ln x + 3x^2 - y^2 = -1$

E. NOT EXACT

6. The solution of

$$y' + \frac{y}{x} = \frac{2}{x^2 y}, \qquad x \neq 0$$

is given by

A. $x^2y^2 + 4xy = C$

B. $x^2y^2 + 4x = C$

C. $xy^2 - 2x = C$

D. $x^2y^2 - 4x = C$

E. $xy^2 - 4x = C$

7. Using Euler's method with step size $h = 1$, find the approximate value of $y(3)$, where $y(x)$ solves the initial value problem

$$\frac{dy}{dx} = x + \frac{y}{2}, \qquad y(0) = -8. \tag{1}$$

A. 30.5

B. 28.5

C. 19.5

D. -24.5

E. -23.5

8. Let $y(t)$ be the solution of the initial value problem

$$y''' + y' = 0, \quad y(0) = 2, \quad y'(0) = 1, \quad y''(0) = 1,$$

then $y(\pi) =$

A. 4

B. 3

C. 2

D. 1

E. 0

9. A spring system with external force is represented by the equation

$$y'' + y' + y = -13 \sin 2t.$$

Let $y_p(t)$ be a **particular solution** to the differential equation obtained from the method of **undetermined coefficients,** find $y_p(\frac{\pi}{4})$.

A. 2

B. 3

C. -3

D. -2

E. -1

10. A particular solution of the equation

$$y^{(4)} - y'' = 2 \sin t - 3e^{-t} + 4t$$

is of the form

A. $y_p(t) = A \cos t + B \sin t + Cte^{-t} + t^2(Dt + E)$

B. $y_p(t) = t(A \cos t + B \sin t) + Cte^{-t} + t^2(Dt + E)$

C. $y_p(t) = A \cos t + B \sin t + Ce^{-t} + t^2(Dt + E)$

D. $y_p(t) = t(A \cos t + B \sin t) + Ce^{-t} + t^2(Dt + E)$

E. $y_p(t) = t(A \cos t + B \sin t) + Cte^{-t} + t(Dt + E)$

11. Let $y_p(t)$ be a **particular solution** to the following second-order differential equation obtained from the method of **variation of parameters**

$$y'' - 2y' + y = \frac{e^t}{t}, \qquad t > 0.$$

Then a **general solution** $y(t)$ is

A. $c_1e^{-t} + c_2te^{-t} + t^2e^t\big(\ln(t) - 1\big)$

B. $c_1e^{t} + c_2te^{t} + t^2e^t\big(\ln(t) - 1\big)$

C. $c_1e^{t} + c_2te^{t} + te^t\big(\ln(t) - 1\big)$

D. $c_1e^{-t} + c_2te^{-t} + te^t\big(\ln(t) - 1\big)$

E. $c_1e^{t} + c_2te^{t} + te^t\big(\ln(t) - t\big)$

12. For the system $\mathbf{x}' = \begin{bmatrix} 5 & 5 \\ -8 & -7 \end{bmatrix}\mathbf{x}$, the origin is

A. a saddle point

B. a proper node source

C. a center point

D. a spiral sink

E. a spiral source

13. The 2×2 real matrix $\mathbf{A}$ has eigenvalues $\lambda_1 = -2$, $\lambda_2 = 5$ with corresponding eigenvectors $\mathbf{v_1} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\mathbf{v_2} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, respectively. Find the matrix exponential $e^{\mathbf{A}t}$

A. $\begin{bmatrix} 2e^{-2t} & 3e^{-2t} \\ e^{5t} & 2e^{5t} \end{bmatrix}$

B. $\begin{bmatrix} 2e^{-2t} & 2e^{5t} \\ e^{5t} & 3e^{-2t} \end{bmatrix}$

C. $\begin{bmatrix} 4e^{-2t} - 3e^{5t} & -2e^{-2t} + 2e^{5t} \\ 6e^{-2t} - 6e^{5t} & -3e^{-2t} + 4e^{5t} \end{bmatrix}$

D. $\begin{bmatrix} 4e^{-2t} - 3e^{5t} & 2e^{-2t} + 2e^{5t} \\ 6e^{-2t} + 6e^{5t} & -3e^{-2t} + 4e^{5t} \end{bmatrix}$

E. $\begin{bmatrix} 2e^{-2t} & e^{5t} \\ 3e^{-2t} & 2e^{5t} \end{bmatrix}$

14. Let $\begin{bmatrix} x \\ y \end{bmatrix}$ be the solution of the initial value problem

$$\begin{bmatrix} x \\ y \end{bmatrix}' = \begin{bmatrix} 3 & -1 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}, \quad \begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix},$$

find $x(1)$.

A. $-2e^4$

B. $-5e^4$

C. $5e^4$

D. $3e^4$

E. $2e^4$

15. A particular solution for the nonhomogeneous linear system

$$\mathbf{x}' = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} t \\ 1 \end{bmatrix}$$

is given by

A. $\mathbf{x_p}(t) = \begin{bmatrix} 1-t \\ 1+t \end{bmatrix}$

B. $\mathbf{x_p}(t) = \begin{bmatrix} 1-t \\ 1-t \end{bmatrix}$

C. $\mathbf{x_p}(t) = \begin{bmatrix} 1+t \\ 1+t \end{bmatrix}$

D. $\mathbf{x_p}(t) = \begin{bmatrix} 1+t \\ 1-t \end{bmatrix}$

E. $\mathbf{x_p}(t) = \begin{bmatrix} -1-t \\ 1-t \end{bmatrix}$

16. Use the Laplace transform to find $y(t)$ for the following linear differential system

$$x'' + y' = 0, \quad y'' + x' = 4\sin t, \quad x(0) = y(0) = x'(0) = y'(0) = 0.$$

A. $y(t) = \cos(t) + \sin(t) - e^t$

B. $y(t) = -2\sin(t) + e^t - e^{-t}$

C. $y(t) = \cosh(t) + \sinh(t)$

D. $y(t) = \cos(t) - \sin(t) - e^{-t}$

E. $y(t) = \cosh(t) - \sinh(t) + e^{-t}$

17. Find the Laplace transform of

$$f(t) = \begin{cases} 0, & 0 \le t < 1 \\ e^{2t}t^2, & t \ge 1. \end{cases}$$

A. $e^{s}\left[\frac{2}{(s+2)^3} + \frac{2}{(s+2)^2} + \frac{1}{s+2}\right]$

B. $e^{s+2}\left[\frac{2}{(s+2)^3} + \frac{2}{(s+2)^2} + \frac{1}{s+2}\right]$

C. $e^{s-2}\left[\frac{2}{(s-2)^3} + \frac{2}{(s-2)^2} + \frac{1}{s-2}\right]$

D. $e^{-s}\left[\frac{2}{(s-2)^3} + \frac{2}{(s-2)^2} + \frac{1}{s-2}\right]$

E. $e^{-s+2}\left[\frac{2}{(s-2)^3} + \frac{2}{(s-2)^2} + \frac{1}{s-2}\right]$

18. Find the Laplace transform of $f(t) = te^t \cos 2t$.

A. $F(s) = \dfrac{4s - 4}{(s^2 - 2s + 5)^2}, \quad s > 0$

B. $F(s) = \dfrac{s^2 - 2s + 3}{(s^2 - 2s + 5)^2}, \quad s > 0$

C. $F(s) = \dfrac{s^2 - 2s - 3}{(s^2 - 2s + 5)^2}, \quad s > 0$

D. $F(s) = \dfrac{s^2 - 4}{(s^2 + 4)^2}, \quad s > 0$

E. $F(s) = \dfrac{4s}{(s^2 + 4)^2}, \quad s > 0$

19. Let $y = y(t)$ be the solution of the initial value problem

$$y'' + 4y' + 3y = \int_0^t \cos(\tau)\sinh(t-\tau)\,d\tau; \qquad y(0) = y'(0) = 1.$$

Find $Y(s) = \mathcal{L}\{y(t)\}$.

A. $\dfrac{1}{s^2+4s+3}\left[\dfrac{s}{s^4-1} + s + 5\right]$

B. $\dfrac{s}{(s^4-1)(s^2+4s+3)}$

C. $\dfrac{1}{s^2+4s+3} + \dfrac{s}{s^2+1} + \dfrac{1}{s^2-1}$

D. $\dfrac{s}{s^4-1}\left[\dfrac{1}{s^2+4s+3} - s - 5\right]$

E. $\dfrac{1}{s^2+4s+3}\left[\dfrac{s}{s^4-1} - s - 5\right]$

20. Let $x(t)$ be the solution of the following initial value problem

$$x'' + 2x' + 10x = \delta(t-5), \quad x(0) = x'(0) = 0.$$

For $t > 5$, $x(t) =$

A. $\frac{1}{5}e^{-t+1}\cos(3t-5)$

B. $\frac{1}{3}e^{-t}\sin(3(t-5))$

C. $\frac{1}{3}e^{-t}\cos(3(t-5))$

D. $\frac{1}{3}e^{-t+5}\sin(3(t-5))$

E. $\frac{1}{3}e^{-t+5}\cos(3(t-5))$

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\dfrac{1}{s}$
2.	e^{at}	$\dfrac{1}{s-a}$
3.	t^n	$\dfrac{n!}{s^{n+1}}$
4.	$t^p \ (p > -1)$	$\dfrac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sin at$	$\dfrac{a}{s^2+a^2}$
6.	$\cos at$	$\dfrac{s}{s^2+a^2}$
7.	$\sinh at$	$\dfrac{a}{s^2-a^2}$
8.	$\cosh at$	$\dfrac{s}{s^2-a^2}$
9.	$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}$
10.	$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$
11.	$t^n e^{at}$	$\dfrac{n!}{(s-a)^{n+1}}$
12.	$u_c(t)$	$\dfrac{e^{-cs}}{s}$
13.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$
14.	$e^{ct}f(t)$	$F(s-c)$
15.	$f(ct)$	$\dfrac{1}{c}F\left(\dfrac{s}{c}\right), \ c > 0$
16.	$\displaystyle\int_0^t f(t-\tau)\,g(\tau)\,d\tau$	$F(s)\,G(s)$
17.	$\delta(t-c)$	e^{-cs}
18.	$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - sf^{(n-2)}(0) - f^{(n-1)}(0)$
19.	$(-t)^n f(t)$	$F^{(n)}(s)$