Final Exam MATH 26600 FALL 2018

MARK TEST 01 ON YOUR SCANTRON!

Name ______________________

Student ID ______________________

Section Number (see list below) ______________________

052	UNIV 303	1:30pm TR	Wang, Changyou	062	UNIV 101	1:30pm TR	Mariano, Phanuel
063	BRNG B222	10:30am TR	Lipshitz, Leonard	064	REC 123	1:30pm TR	Ma, Zheng
065	UNIV 219	10:30am MWF	Moon, Yong Suk	066	UNIV 103	12:30pm MWF	Han, Jiyuan
067	REC 313	9:00am TR	Alper, Onur	068	BRNG B222	9:00am TR	Lipshitz, Leonard
069	REC 313	10:30am TR	Alper, Onur	070	UNIV 219	11:30am MWF	Moon, Yong Suk
071	UNIV 303	12:00pm TR	Wang, Changyou	072	REC 123	12:00pm TR	Ma, Zheng
073	UNIV 119	9:30am MWF	Phillips, Daniel	074	UNIV 101	4:30pm TR	Mariano, Phanuel
075	UNIV 119	9:00am TR	Wang, Changyou	076	UNIV 117	7:30am MWF	Han, Jiyuan
077	REC 307	3:30pm MWF	VanKoughnett, Paul	078	REC 307	4:30pm MWF	VanKoughnett, Paul
079	UNIV 119	7:30am TR	Indrei, Emanuel				

INSTRUCTIONS:

1. This exam contains 20 problems, worth 10 points each. There is one correct answer for each problem.
2. There is a table of Laplace transforms provided at the end of the exam.
3. Please fill in your name, ID, section number and test number 01 on the scantron.
4. Work only in the space provided, or on the backside of the pages. You must show your work.
5. Mark your answers clearly on the scantron. Also circle your choice for each problem in this booklet.
6. No books, notes, calculators, phones or other electronic devices, please.

ACADEMIC DISHONESTY

Purdue University faculty and students commit themselves towards maintaining a culture of academic integrity and honesty. The students taking this exam are not allowed to seek or obtain any kind of help from anyone to answer questions on this test. If you have questions, consult only an instructor or a proctor. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you finish your exam and hand it in to a proctor or to an instructor. You may not consult notes, books, calculators, cameras, or any kind of communications devices until after you finish your exam and hand it in to a proctor or to an instructor. If you violate these instructions you will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students. Your instructor and proctors will do everything they can to stop and prevent academic dishonesty during this exam. If you see someone breaking these rules during the exam, please report it to the proctor or to your instructor immediately. Reports after the fact are not very helpful.

Anyone who is seen handling any communication device gets automatically a score of 0 for the exam

and likely an F in the course

I have read and understood the instructions regarding academic dishonesty:

Student name: Signature:

1. Find the **explicit** solution to the initial value problem

$$\frac{dy}{dt} = \frac{t}{y\sqrt{1+t^2}}, \quad y(0) = -2.$$

A. $y = -\sqrt{4\sqrt{1+t^2}}$

B. $y^2 = 2\sqrt{1+t^2} + 2$

C. $y = \pm\sqrt{2\sqrt{1+t^2} + 2}$

D. $y = -\sqrt{2\sqrt{1+t^2} + 2}$

E. $y = \sqrt{4\sqrt{1+t^2}}$

2. Initially, a tank contains 100 L of water with 10 kg of sugar in solution. Water containing sugar flows into the tank at the rate of 2 L/min, and the well-stirred mixture in the tank flows out at the rate of 5 L/min. The concentration $c(t)$ of sugar in the incoming water varies as $c(t) = 2 + \cos(3t)$ kg/L. Let $Q(t)$ be the amount of sugar (in kilograms) in the tank at time t (in minutes). Which differential equation does $Q(t)$ satisfy?

A. $\dfrac{dQ}{dt} = 2(2 + \sin(3t)) - \dfrac{5Q}{100 - 3t}$

B. $\dfrac{dQ}{dt} = 2(2 + \cos(3t)) - \dfrac{5Q}{100 - 3t}$

C. $\dfrac{dQ}{dt} = 2(2 + \sin(3t)) - \dfrac{5Q}{100}$

D. $\dfrac{dQ}{dt} = 2(2 + \cos(3t)) - \dfrac{5Q}{100}$

E. $\dfrac{dQ}{dt} = 2 + \cos(3t) - \dfrac{Q}{100 - t}$

3. Find the implicit solution of the following initial value problem,

$$x\cos\left(\frac{x^2+y^2}{2}\right)+\left[2y+y\cos\left(\frac{x^2+y^2}{2}\right)\right]y'=0, \qquad y(0)=\sqrt{\pi}.$$

A. $\sin\left(\frac{x^2+y^2}{2}\right)=1$

B. $\cos\left(\frac{x^2+y^2}{2}\right)+y^2=\pi$

C. $2\cos\left(\frac{x^2+y^2}{2}\right)+x^2=0$

D. $\sin\left(\frac{x^2+y^2}{2}\right)+x^2=1$

E. $\sin\left(\frac{x^2+y^2}{2}\right)+y^2=\pi+1$

4. Find the implicit solution to the initial value problem:

$$\frac{dy}{dx}=\frac{y}{x}+\frac{x}{2(y+x)} \quad \text{for } x>0,$$
$$y(1)=1.$$

A. $y^2+xy=x^2\ln x+2x^2$

B. $y^2+2x^2y=x^2\ln x+3x^2$

C. $y^2+2xy=x^2\ln x+3x^2$

D. $y^2-xy=x^2\ln x$

E. $y^2+x^2y=x^2\ln x+2x$

5. Find the approximation of $y(3)$ by using Euler's method with a time step $h = 1$, where y solves the initial value problem,

$$y'(t) = \cos(\pi t)y(t) - t, \quad y(0) = 2.$$

A. -3

B. -4

C. -1

D. 2

E. 4

6. Let $y(t)$ be the solution of the initial value problem

$$ty' - 2y = t^4 \text{ for } t > 0,$$

$$y(1) = 0.$$

Find $y(2)$.

A. 1

B. 2

C. 4

D. 6

E. 8

7. Which of the following is the set of unstable equilibrium solution(s) of

$$y' = y(y-1)(y+2)^2$$

A. $y(t) = 0$

B. $y(t) = 1$

C. $y(t) = -2, 1$

D. $y(t) = -2$

E. $y(t) = 0, 1$

8. Find the solution of the initial value problem

$$y'' + y' + \frac{5}{4}y = 0, \quad y(0) = 3,\ y'(0) = 1.$$

A. $y(t) = 3e^{\frac{t}{2}} \cos t - \frac{1}{2}e^{\frac{t}{2}} \sin t$

B. $y(t) = 3e^{t} \cos \frac{t}{2} - 4e^{t} \sin \frac{t}{2}$

C. $y(t) = 3e^{-t} \cos \frac{t}{2} + 8e^{-t} \sin \frac{t}{2}$

D. $y(t) = 3e^{-t} \cos \frac{t}{2} + \frac{5}{2}e^{-t} \sin \frac{t}{2}$

E. $y(t) = 3e^{-\frac{t}{2}} \cos t + \frac{5}{2}e^{-\frac{t}{2}} \sin t$

9. The general solution to the homogeneous differential equation

$$x^2y'' - 3xy' + 4y = 0 \quad \text{ for } x > 0$$

is $y = c_1x^2 + c_2x^2 \ln x$. Applying the method of Variation of Parameters a particular solution of the nonhomogeneous equation

$$x^2y'' - 3xy' + 4y = 5x^2 \ln x, \ x > 0.$$

is $y_p = u_1(x)x^2 + u_2(x)x^2 \ln x$. Find u_1' and u_2'.

A. $u_1' = -5x(\ln x)^2, \quad u_2' = 5x \ln x$

B. $u_1' = 5x \ln x, \quad u_2' = -\frac{(5 \ln x)^3}{x}$

C. $u_1' = -\frac{5(\ln x)^2}{x}, \quad u_2' = \frac{5 \ln x}{x}$

D. $u_1' = -\frac{5(\ln x)^2}{2x}, \quad u_2' = \frac{5(\ln x)^3}{3x}$

E. $u_1' = -5x^2(\ln x)^2, \quad u_2' = 5x^2 \ln x$

10. A spring-mass system set in motion was determined by the initial value problem

$$u'' + 49u = 0, \quad u(0) = 3, \ u'(0) = -7.$$

What is the amplitude of the motion?

A. $\sqrt{10}$.

B. 10

C. 4

D. $\sqrt{58}$

E. 7

11. Find the general solution of

$$y^{(4)} + 5y^{(2)} - 36y = 0.$$

A. $y(t) = c_1 e^{3t} + c_2 e^{-3t} + c_3 \sin 2t + c_4 \cos 2t$

B. $y(t) = c_1 e^{-2t} + c_2 e^{3t} + c_3 e^{2t} + c_4 e^{-3t}$

C. $y(t) = c_1 \cos 3t + c_2 \sin 3t + c_3 \sin 2t + c_4 \cos 2t$

D. $y(t) = c_1 e^{-2t} + c_2 e^{2t} + c_3 \cos 3t + c_4 \sin 3t$

E. $y(t) = c_1 e^{2t} + c_2 e^{-2t} + c_3 t \cos 3t + c_4 t \sin 3t$

12. Which of the following functions is the correct form of a particular solution of the equation

$$y^{(4)} - y = 3e^t + te^{-t} + 4\cos t \ ?$$

A. $y(t) = Ce^t + (Dt + E)e^{-t} + F\cos t + G\sin t$

B. $y(t) = Cte^t + (Dt + E)e^{-t} + F\cos t + G\sin t$

C. $y(t) = Ce^t + (Dt^2 + Et)e^{-t} + F\cos t + G\sin t$

D. $y(t) = Cte^t + (Dt^2 + Et)e^{-t} + F\cos t + G\sin t$

E. $y(t) = Cte^t + (Dt^2 + Et)e^{-t} + Ft\cos t + Gt\sin t$

13. One solution of the differential equation

$$x^3 y'' + xy' - y = 0, \quad x > 0$$

is $y_1 = x$. Another solution is of the form $y_2 = vx$ where $v =$

A. $\ln x$

B. e^{x^2}

C. $x \ln x$

D. $x^3 \ln x$

E. $e^{1/x}$

14.

$$f(t) = \mathcal{L}^{-1}\left\{\frac{4e^{-s}}{s^2+2s-3}\right\}.$$

What is the value of $f(3)$?

A. $e^3 - e^{-9}$

B. $\frac{1}{2}u_1(t)(e^{t-1} - e^{-t+1})$

C. $e^2 - e^{-6}$

D. $e^3 - e^{-4}$

E. $e^2\sin(3) - e^2\cos(3)$

15. The solution of

$$y'' + 4y = g(t)$$

satisfying $y(0) = a$ and $y'(0) = b$ is given by

A. $y(t) = \frac{b}{2}\sin(2t) + a\cos(2t) + e^t\int_0^t \sin(2(t-\tau))g(\tau)d\tau$

B. $y(t) = \frac{b}{2}\sin(2t) + a\cos(2t) + \frac{1}{2}\int_0^t \sin(2(t-\tau))g(\tau)d\tau$

C. $y(t) = \frac{b}{2}\sin(2t) + a\cos(2t) + \int_0^t \sin(2(t-\tau))g(\tau)d\tau$

D. $y(t) = \frac{a}{2}\cos(2t) + \frac{1}{2}\int_0^t \sin(2(t-\tau))g(\tau)d\tau$

E. $y(t) = \frac{b}{2}\sin(2t) + a\cos(2t) + 3\int_0^t \sin(2(t-\tau))g(\tau)d\tau$

16. The function $y(t)$ solves the initial value problem

$$y'' + 2y' + 5y = \delta(t-7), \qquad y(0) = 0, \quad y'(0) = 0.$$

For $t > 7$, $\quad y(t) =$

A. $\frac{1}{2}e^{-(t-7)}\sin(2(t-7))$.

B. $\frac{1}{2}e^{-t}\sin(t-7)$

C. $\frac{1}{2}e^{-2(t-7)}\cos(t-7)$

D. $\frac{1}{2}\sin 2(t-7)$

E. $\frac{1}{7}e^{-(t-7)}\sin(2t)$

17. Find the Laplace transform of

$$f(t) = \begin{cases} 0 & \text{when } t < \pi, \\ t - 2\pi & \text{when } \pi \le t < 2\pi, \\ 0 & \text{when } t \ge 2\pi. \end{cases}$$

A. $e^{-\pi s}\dfrac{1}{s^2} - e^{-2\pi s}\dfrac{1}{s^2} - e^{-\pi s}\dfrac{\pi}{s}$

B. $e^{-\pi s}\dfrac{1}{s^2} - e^{-2\pi s}\dfrac{1}{s^2}$

C. $e^{\pi s}\dfrac{1}{s^2} - e^{2\pi s}\dfrac{1}{s^2} - \pi e^{2\pi s}\dfrac{1}{s^2}$

D. $\dfrac{1}{s}\left(e^{-\pi s} - e^{-2\pi s}\right)$

E. $e^{-\pi s}\dfrac{1}{s^2} + e^{-2\pi s}\dfrac{1}{s^2}$

18. The real 2×2 matrix A has an eigenvalue $\lambda_1 = 3 + 2i$ with corresponding eigenvector $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 2i \end{pmatrix}$. Then the general solution of the system of differential equations

$$\mathbf{x}'(t) = A\,\mathbf{x}(t)$$

is $\mathbf{x}(t) =$

A. $C_1\, e^{3t} \begin{pmatrix} \cos 2t \\ \sin 2t \end{pmatrix} + C_2\, e^{3t} \begin{pmatrix} \sin 2t \\ -\cos 2t \end{pmatrix}$

B. $C_1\, e^{3t} \begin{pmatrix} -2\cos 2t \\ \sin 2t \end{pmatrix} + C_2\, e^{3t} \begin{pmatrix} \sin 2t \\ -2\cos 2t \end{pmatrix}$

C. $C_1\, e^{3t} \begin{pmatrix} \cos 2t \\ -2\sin 2t \end{pmatrix} + C_2\, e^{3t} \begin{pmatrix} \sin 2t \\ 2\cos 2t \end{pmatrix}$

D. $C_1\, e^{3t} \begin{pmatrix} \cos 2t \\ 2\sin 2t \end{pmatrix} + C_2\, e^{3t} \begin{pmatrix} -\sin 2t \\ 2\cos 2t \end{pmatrix}$

E. $C_1\, e^{3t} \begin{pmatrix} \cos 2t \\ \sin 2t \end{pmatrix} + C_2\, e^{3t} \begin{pmatrix} \sin 2t \\ 2\cos 2t \end{pmatrix}$

19. In the phase portrait of the system

$$X' = \begin{pmatrix} -3 & -1 \\ 2 & -1 \end{pmatrix} X,$$

the origin $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ is a (an)

A. asymptotically stable spiral point

B. unstable spiral point

C. unstable node

D. asymptotically stable node

E. saddle point

20. The matrix $\begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}$ has eigenvalues 1 and -1 with corresponding eigenvectors $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ respectively. If $\mathbf{x} = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$ solves the initial value problem

$$\mathbf{x}' = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} e^{2t} \\ -2e^{2t} \end{pmatrix},$$

$$\mathbf{x}(0) = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$$

then $x_1(t)$ equals

A. $e^{2t} + e^t + 2e^{-t}$

B. $e^{2t} + 2e^t + e^{-t}$

C. $-e^{2t} - e^t + 6e^{-t}$

D. $2e^{2t} + e^t + e^{-t}$

E. $-2e^{2t} + 4e^t + 2e^{-t}$

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\dfrac{1}{s}$
2.	e^{at}	$\dfrac{1}{s-a}$
3.	t^n	$\dfrac{n!}{s^{n+1}}$
4.	$t^p \ (p > -1)$	$\dfrac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sin at$	$\dfrac{a}{s^2+a^2}$
6.	$\cos at$	$\dfrac{s}{s^2+a^2}$
7.	$\sinh at$	$\dfrac{a}{s^2-a^2}$
8.	$\cosh at$	$\dfrac{s}{s^2-a^2}$
9.	$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}$
10.	$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$
11.	$t^n e^{at}$	$\dfrac{n!}{(s-a)^{n+1}}$
12.	$u_c(t)$	$\dfrac{e^{-cs}}{s}$
13.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$
14.	$e^{ct}f(t)$	$F(s-c)$
15.	$f(ct)$	$\dfrac{1}{c}F\left(\dfrac{s}{c}\right), \ c > 0$
16.	$\int_0^t f(t-\tau)\, g(\tau)\, d\tau$	$F(s)\, G(s)$
17.	$\delta(t-c)$	e^{-cs}
18.	$f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - \cdots - sf^{(n-2)}(0) - f^{(n-1)}(0)$
19.	$(-t)^n f(t)$	$F^{(n)}(s)$

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