MA26600 **FINAL EXAM INSTRUCTIONS** **FALL 2017**

GREEN – Version 01

NAME: ______________________ INSTRUCTOR: ______________________

1. You must use a **#2 pencil** on the mark-sense sheet (answer sheet).
2. On the mark-sense sheet, fill in the INSTRUCTOR's name (if you do not know, write down the class meeting time and location) and the COURSE NUMBER which is MA266.
3. Fill in your NAME and blacken in the appropriate spaces.
4. Fill in the SECTION NUMBER boxes with section number of your class (see below if you are not sure) and blacken in the appropriate spaces.

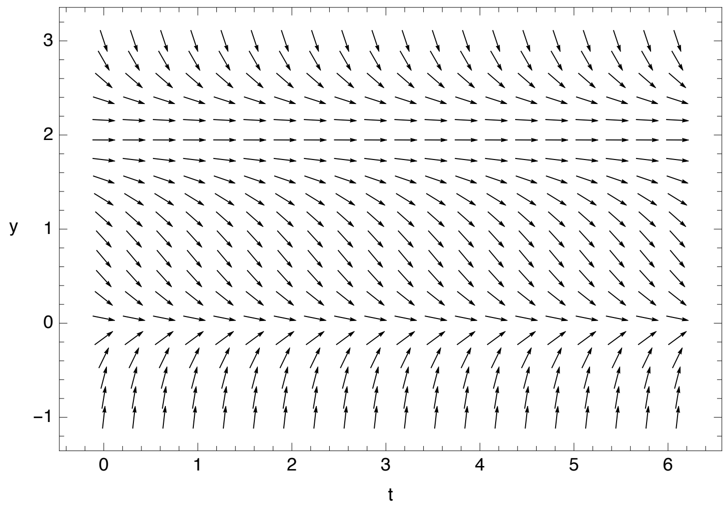
0052	MWF	11:30am	Petrosyan, Arshak	0070	MWF	8:30am	Jin, Long
0062	MWF	9:30am	Yeung, Sai Kee	0071	TR	4:30pm	Liu, Yanghui
0063	MWF	3:30pm	Luo, Tao	0072	TR	3:00pm	Hora, Raphael
0064	TR	4:30pm	Hora, Raphael	0073	MWF	12:30pm	Ma, Zheng
0065	MWF	9:30am	Jin, Long	0074	TR	9:00am	Torres, Monica
0066	MWF	2:30pm	Luo, Tao	0075	TR	3:00pm	Liu, Yanghui
0067	TR	9:00am	Wang, Changyou	0076	TR	9:00am	Alper, Onur
0068	MWF	1:30pm	Ma, Zheng	0077	TR	10:30am	Alper, Onur
0069	MWF	10:30am	Zhang, Ying	0078	MWF	11:30am	Zhang, Ying

5. Fill in the correct TEST/QUIZ NUMBER (GREEN is **01**).
6. Fill in the **10-DIGIT PURDUE ID** and blacken in the appropriate spaces.
7. Sign the mark-sense sheet.
8. Fill in your name and instructor's name on the question sheets (above).
9. There are 20 questions, each worth 10 points. **Blacken in** your choice of the correct answer in the spaces provided for questions 1–20 in the answer sheet. Do all your work on the question sheets, in addition, also **Circle** your answer choice for each problem on the question sheets in case your mark-sense sheet is lost.

   Turn in both the mark-sense sheets and the question sheets when you are finished.
10. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.
11. **NO CALCULATORS, BOOKS, NOTES, PHONES, OR CAMERAS ARE ALLOWED**.

    Turn off or put away all electronic devices. Use the back of the test pages for scrap paper.
12. **The Laplace transform table is provided at the end of the question sheets.**

1. Identify the differential equation corresponding to the direction field below.



A. $y' = -y(y-2)^2$

B. $y' = -y^2(y-2)$

C. $y' = y(y-2)$

D. $y' = y(y-2)^2$

E. $y' = -y(y-2)$

2. Let $y = y(t)$ be the solution to the initial value problem

$$t\frac{dy}{dt} + 2y = \sin t, \quad y(\pi) = 0.$$

Find the value of $y(2\pi)$.

A. 0

B. $-\frac{1}{4\pi}$

C. $-\frac{3}{4\pi}$

D. $-\frac{1}{\pi}$

E. Does not exist

3. Find the solution of the initial value problem:

$$y' = \frac{1-2x}{y}, \quad y(1) = -2$$

in **explicit** form.

A. $y = 2x - 2x^2 - 2$

B. $y = -\ln|2x - 2x^2 - 1|$

C. $y^2/2 = x - x^2 + 2$

D. $y = -\sqrt{2x - 2x^2 + 4}$

E. $y = \sqrt{2x - 2x^2 + 4}$

4. Initially, a tank contains 500 L (liters) of pure water. Water containing 0.3 kg of salt per liter is entering at a rate of 2 L/min, and the mixture is allowed to flow out of the tank at a rate of 1 L/min. Let $Q(t)$ be the amount of salt at time t measured in kilograms (kg). What is the right formulation of the differential equation for $Q(t)$?

A. $\dfrac{dQ}{dt} = 0.3 - \dfrac{Q(t)}{500+t}$

B. $\dfrac{dQ}{dt} = \dfrac{0.3}{500} - \dfrac{2Q(t)}{500-t}$

C. $\dfrac{dQ}{dt} = 0.6 - \dfrac{Q(t)}{500+t}$

D. $\dfrac{dQ}{dt} = 0.6 - \dfrac{Q(t)}{500}$

E. $\dfrac{dQ}{dt} = 0.3 - \dfrac{2Q(t)}{500-t}$

5. Use the Euler method with $h = 0.5$ to approximate $y(1)$ for $y' = y(1 - ty)$, $y(0) = 2$.

A. 3

B. 2.25

C. 2

D. 3.75

E. 0

6. Which of the following is an implicit solution to the initial value problem

$$(e^x \sin y - 2y \cos x + x^2) + (e^x \cos y - 2 \sin x + e^y)y' = 0, \quad y(0) = 0 \quad ?$$

A. $e^x \sin y - 2y \sin x = 0$

B. $e^x \sin y + 2y \sin x - \frac{1}{3}x^3 - e^y = -1$

C. $e^x \sin y + y^2 \sin x + x^2 + e^y = 1$

D. $e^x \sin y - 2y \sin x + \frac{1}{3}x^3 + e^y = 1$

E. $-e^x \cos y - y^2 \cos x + \frac{1}{2}x^3 + e^y = 1$

7. Find the solution of the initial value problem

$$y'' + y' - 6y = 0, \quad y(0) = 0, \quad y'(0) = 5.$$

A. $5e^{2t} - 5e^{-3t}$

B. $e^{3t} - e^{-2t}$

C. $e^{2t} - e^{-3t}$

D. $e^{5t} - 1$

E. $\frac{1}{5}\left(e^{3t} - e^{-2t}\right)$

8. Which of the following is the largest interval in which the solution of the initial value problem

$$(t^2 - 1)y'' + (\sin t)y' - \frac{y}{t-3} = e^t \ln t, \quad y(2) = 0, \quad y'(2) = 1$$

is guaranteed to exist by the theorem of existence and uniqueness?

A. $(0, \infty)$

B. $(1, \infty)$

C. $(1, \pi)$

D. $(1, 3)$

E. $(0, 3)$

9. The function $y_1 = e^x$ is a solution of

$$xy'' + (2 - 2x)y' + (x - 2)y = 0.$$

If we seek a second solution $y_2 = e^x v(x)$ by reduction of order, then $v(x) =$

A. x^{-2}

B. x^{-1}

C. $x^{-1}e^{-x}$

D. x

E. $x^{-2}e^{-x}$

10. A spring-mass system set in motion was determined by the initial value problem

$$u'' + 100u = 0, \quad u(0) = 1, \quad u'(0) = -10.$$

What is the amplitude of the motion?

A. $\dfrac{1}{10}$

B. 10

C. $\sqrt{101}$

D. $\sqrt{5}$

E. $\sqrt{2}$

11. The homogeneous differential equation $t^2y'' - 4ty' + 6y = 0$ $(t > 0)$ has two solutions given by $y_1(t) = t^2$ and $y_2(t) = t^3$. Using the method of Variation of Parameters, find the general solution of the nonhomogeneous equation $t^2y'' - 4ty' + 6y = t^3$.

A. $y = C_1t^2 + C_2t^3 + t^3 \ln t$

B. $y = C_1t^2 + C_2t^3 + t^2 \ln t + t^4$

C. $y = C_1t^2 + C_2t^3 - t^2 \ln t + t^4$

D. $y = C_1t^2 + C_2t^3 + \dfrac{1}{6}t^5$

E. None of the above

12. Find the general solution of
$$y^{(4)} - 10y'' + 9y = 0.$$

A. $y(t) = c_1e^t + c_2e^{-t} + c_3 \cos 3t + c_4 \sin 3t$

B. $y(t) = c_1e^{9t} + c_2te^{9t} + c_3e^t + c_4te^t$

C. $y(t) = c_1e^{3t} + c_2e^{-3t} + c_3 \cos t + c_4 \sin t$

D. $y(t) = c_1 \cos 3t + c_2 \sin 3t + c_3 \cos t + c_4 \sin t$

E. $y(t) = c_1e^{3t} + c_2e^{-3t} + c_3e^t + c_4e^{-t}$

13. Which of the following is the correct form of a particular solution to the equation

$$y^{(4)} - y = 2 + 3te^{-t} + 2\sin t \quad ?$$

A. $A + (B + Ct)e^{-t} + D\cos t + E\sin t$

B. $A + Bt^2e^{-t} + t(C\cos t + D\sin t)$

C. $A + Bte^{-t} + C\sin t$

D. $A + t(B + Ct)e^{-t} + t(D\cos t + E\sin t)$

E. $A + Bte^{-t} + (C + Dt)\sin t$

14. Find the Laplace transform of the function

$$f(t) = \begin{cases} t, & 0 \le t < 2 \\ 2, & t \ge 2 \end{cases}$$

A. $\dfrac{1-e^{-2s}}{s^2}$

B. $\dfrac{1}{s^2} + \dfrac{e^{2s}}{s-2}$

C. $\dfrac{1}{s^2} + \dfrac{s^2 e^{2s}}{s-2}$

D. $\dfrac{1-e^{2s}}{s-2}$

E. $\dfrac{1}{s} + \dfrac{1}{s-2}$

15. If $y(t)$ is the solution of the initial value problem

$$y'' + 4y' + 5y = \delta(t-5), \quad y(0) = 0, \quad y'(0) = 0$$

then $y(t) = ?$

A. $u_5(t)e^{-2t}\sin t$

B. $u_5(t)e^{-2t+10}\sin(t-5)$

C. $\dfrac{1}{2}u_5(t)e^{-2t-5}\sin(2t-10)$

D. $u_5(t)e^{-2t-5}\sin(t-5)$

E. $u_5(t)e^{-2t+10}\cos(t-5)$

16. Find the inverse Laplace transform of

$$F(s) = \frac{s+7}{(s-1)(s^2+2s+5)}.$$

A. $e^t - e^{-t}\cos 2t - e^{-t}\sin 2t$

B. $e^t - e^{-t}\cos 2t - \frac{1}{2}e^{-t}\sin 2t$

C. $e^t + e^{-t}\cos 2t + e^{-t}\sin 2t$

D. $e^t + e^{-t}\cos 2t + \frac{1}{2}e^{-t}\sin 2t$

E. $e^t + e^{-t}\cos 2t - \frac{1}{2}e^{-t}\sin 2t$

17. The Laplace Transform of $f(t) = \int_0^t \tau^2 e^{-\tau} \sin\{\pi(t-\tau)\}d\tau$ is

A. $\dfrac{2\pi}{(s+1)(s^2+\pi^2)}$

B. $\dfrac{2}{(s+1)^3(s^2+\pi^2)}$

C. $\dfrac{6\pi}{(s+1)^3(s^2+\pi^2)}$

D. $\dfrac{2s}{(s+1)^3(s^2+\pi^2)}$

E. $\dfrac{2\pi}{(s+1)^3(s^2+\pi^2)}$

18. Find all values of k for which solutions of the system $\mathbf{x}' = \begin{pmatrix} -2 & -1 \\ k & 0 \end{pmatrix} \mathbf{x}$ spiral into the origin as $t \to \infty$.

A. $k < 1$

B. $k > 1$

C. $k < 2$

D. $k > 2$

E. $k < 4$

19. Find the solution of the following initial value problem

$$\mathbf{x}' = \begin{pmatrix} -4 & 2 \\ 2 & -4 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

A. $\mathbf{x}(t) = \begin{pmatrix} 2 \\ 2 \end{pmatrix} e^{-2t} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-6t}$

B. $\mathbf{x}(t) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-2t}$

C. $\mathbf{x}(t) = \frac{1}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-6t} + \frac{1}{2} \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-2t}$

D. $\mathbf{x}(t) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-6t}$

E. $\mathbf{x}(t) = \frac{1}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-2t} + \frac{1}{2} \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-6t}$

20. Consider the differential system

$$\mathbf{x}' = \begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix} \mathbf{x}.$$

A set of fundamental solutions to this system is given by

A. $\mathbf{x}^{(1)} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} e^{-3t}, \qquad \mathbf{x}^{(2)} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} te^{-3t} + \begin{pmatrix} -1 \\ 0 \end{pmatrix} e^{-3t}$

B. $\mathbf{x}^{(1)} = \begin{pmatrix} -4 \\ -4 \end{pmatrix} te^{-3t} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-3t}, \qquad \mathbf{x}^{(2)} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} e^{-3t}$

C. $\mathbf{x}^{(1)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-3t}, \qquad \mathbf{x}^{(2)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{-3t} + \begin{pmatrix} 1/4 \\ 0 \end{pmatrix} e^{-3t}$

D. $\mathbf{x}^{(1)} = \begin{pmatrix} -4 \\ -4 \end{pmatrix} te^{3t} + \begin{pmatrix} -1 \\ 0 \end{pmatrix} e^{3t}, \qquad \mathbf{x}^{(2)} = \begin{pmatrix} -4 \\ -4 \end{pmatrix} e^{3t}$

E. $\mathbf{x}^{(1)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-3t}, \qquad \mathbf{x}^{(2)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{-3t} + \begin{pmatrix} 0 \\ 1/4 \end{pmatrix} e^{-3t}$

ELEMENTARY LAPLACE TRANSFORMS

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\dfrac{1}{s}, \quad s > 0$
2.	e^{at}	$\dfrac{1}{s-a}, \quad s > a$
3.	$t^n, \quad n = \text{positive integer}$	$\dfrac{n!}{s^{n+1}}, \quad s > 0$
4.	$t^p, \quad p > -1$	$\dfrac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$
5.	$\sin at$	$\dfrac{a}{s^2+a^2}, \quad s > 0$
6.	$\cos at$	$\dfrac{s}{s^2+a^2}, \quad s > 0$
7.	$\sinh at$	$\dfrac{a}{s^2-a^2}, \quad s > \lvert a \rvert$
8.	$\cosh at$	$\dfrac{s}{s^2-a^2}, \quad s > \lvert a \rvert$
9.	$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}, \quad s > a$
10.	$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}, \quad s > a$
11.	$t^n e^{at}, \quad n = \text{positive integer}$	$\dfrac{n!}{(s-a)^{n+1}}, \quad s > a$
12.	$u_c(t)$	$\dfrac{e^{-cs}}{s}, \quad s > 0$
13.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$
14.	$e^{ct}f(t)$	$F(s-c)$
15.	$f(ct)$	$\dfrac{1}{c}F\left(\dfrac{s}{c}\right), \quad c > 0$
16.	$\displaystyle\int_0^t f(t-\tau)\,g(\tau)\,d\tau$	$F(s)\,G(s)$
17.	$\delta(t-c)$	e^{-cs}
18.	$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - sf^{(n-2)}(0) - f^{(n-1)}(0)$
19.	$(-t)^n f(t)$	$F^{(n)}(s)$