MA26600 FINAL EXAM INSTRUCTIONS December 13, 2010

NAME ______________________ INSTRUCTOR ______________________

1. You must use a #2 pencil on the mark–sense sheet (answer sheet).

2. On the mark-sense sheet, fill in the instructor's name (if you do not know, write down the class meeting time and location) and the course number which is MA266.

3. Fill in your NAME and blacken in the appropriate spaces.

4. Fill in the SECTION NUMBER boxes with section number of your class (see below if you are not sure) and blacken in the appropriate spaces.

0001	MWF 08:30	Albers, Peter	0009	MWF 03:30	Holman, Sean
0002	MWF 09:30	Albers, Peter	0010	MWF 03:30	Eckhardt, Caleb
0003	MWF 10:30	Li, Fang	0011	MWF 01:30	Barton, Ariel
0004	TR 09:00	Cai, Zhiqiang	0012	TR 12:00	Scheiblechner, Peter
0005	MWF 11:30	Li, Fang	0013	TR 12:00	Petrosyan, Arshak
0006	MWF 01:30	Holman, Sean	0014	TR 01:30	Tohaneanu, Mihai
0007	MWF 02:30	Eckhardt, Caleb	0015	TR 03:00	Tohaneanu, Mihai
0008	MWF 02:30	Barton, Ariel			

5. Leave the TEST/QUIZ NUMBER blank.

6. Fill in the 10-DIGIT PURDUE ID and blacken in the appropriate spaces.

7. Sign the mark–sense sheet.

8. Fill in your name and your instructor's name on the question sheets (above).

9. There are 20 questions, each worth 5 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–20 in the answer sheet. Do all your work on the question sheets. Turn in both the mark–sense sheets and the question sheets when you are finished.

10. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.

11. NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

12. The Laplace transform table is provided at the end of the question sheets.

1. For which value of a is $y = a\,t^2 e^{2t}$ a solution of the following ODE?

$$ty' - 2y = -2t^3 e^{2t}$$

A. $\boxed{a = -1}$

B. $a = -1/2$

C. $a = 1$

D. $a = \ln 2$

E. $a = e^2$

2. The general solution of $y' + 3y = e^{-2x}$ is

A. $y = e^{-2x} + c\,e^{3x}$

B. $y = c\,e^{2x} + e^{-3x}$

C. $y = e^{-2x}$

D. $y = c\,e^{-2x} + e^{-3x}$

E. $\boxed{y = e^{-2x} + c\,e^{-3x}}$

where c is some number.

3. What is the largest open interval in which a solution to the initial value problem

$$(t-1)y' + \sqrt{t+2}\,y = \frac{3}{t-3}, \quad y(2) = -5$$

is guaranteed to exist?

A. $(-2, 1)$

B. $(2, 3)$

C. $(-2, 3)$

D. $\boxed{(1, 3)}$

E. $(-2, \infty)$

4. Suppose that $\dfrac{dy}{dx} = (x-y)^2+1$. What is the general solution to this differential equation? (*Hint*: use the substitution $v(x) = y(x) - x$.)

A. $y = -\dfrac{1}{x+c}$

B. $\boxed{y = x - \dfrac{1}{x+c}}$

C. $\ln|1 + (y-x)^2| = x + c$

D. $y = \tan(x+c)$

E. $y = x + \tan(x+c)$

5. Which of the following contains all the asymptotically stable equilibrium solution(s) for the autonomous differential equation $y' = (y^2 - 1)(4 - y^2)$?

A. $\boxed{y = -1 \text{ and } y = 2}$

B. $y = -1$ and $y = 1$

C. $y = -2$ and $y = 1$

D. $y = -2$ and $y = 2$

E. $y = 1$ and $y = 2$

6. The solution to the problem $(2xy + x^4)dx + (x^2 + y^3)dy = 0$ and $y(0) = 1$ is

A. $x + \frac{1}{4}y^4 = \frac{1}{4}$

B. $\boxed{x^2y + \frac{1}{5}x^5 + \frac{1}{4}y^4 = \frac{1}{4}}$

C. $x^2y + 4y^4 = 4$

D. $x^2y + 5x^5 + 4y^4 = 4$

E. $x^2y + x^5 + \frac{1}{4}y^4 = \frac{1}{4}$

7. A tank contains 200 gal of liquid. Initially, the tank contains pure water. At time $t = 0$, brine containing 3 lb/gal of salt begins to pour into the tank at a rate of 2 gal/min, and the well-stirred mixture is allowed to drain away at the same rate. How many minutes must elapse before there are 100 lb of salt in the tank?

A. $\boxed{100 \ln \frac{6}{5}}$

B. $600 - 600e^{-1}$

C. $600 - e^{-1}$

D. $600 + 600e$

E. $-100 \ln(400)$

8. Which of the following statements is true about *every* solution of $y'' = -y$?

A. $\lim_{t\to\infty} y(t) = +\infty$

B. $\boxed{y \text{ is a periodic function}}$

C. $\lim_{t\to\infty} y(t) = 0$

D. $-1 \le y(t) \le 1$

E. $\lim_{t\to 0} y(t) = 0$

9. Find the general form of the solutions to $y'' + 4y' + 5y = 0$.

A. $c_1e^{2t}\cos(t) + c_2e^{2t}\sin(t)$

B. $c_1e^{t}\cos(-2t) + c_2e^{t}\sin(-2t)$

C. $c_1e^{5t}\cos(4t) + c_2e^{5t}\sin(4t)$

D. $\boxed{c_1e^{-2t}\cos(t) + c_2e^{-2t}\sin(t)}$

E. $c_1e^{4t}\cos(5t) + c_2e^{4t}\sin(5t)$

10. Given that a general solution of the homogeneous equation $t^2y'' + 3ty' - 3y = 0$ on the domain $t > 0$ is $y = c_1t + c_2t^{-3}$, a particular solution of the second order equation

$$t^2y'' + 3ty' - 3y = 16\,t$$

is which of the following:

A. $\frac{8}{3}t^3 + t + \frac{7}{3}t^{-3}$

B. $-4t\ln t + 2t$

C. $\boxed{4t\ln t - t}$

D. $\frac{4}{3}t^3$

E. $5t + t^{-3}$

11. A mass of 3 kg hangs from a spring with spring constant of 12 kg/sec^2. Suppose you pull the mass down an additional 5 cm from its equilibrium position, and then release it with an initial velocity of 2 cm/sec upwards. There is no damping. Which of the following give the amplitude and period of the resulting oscillatory motion:

A. 5 cm and 2 sec

B. $\sqrt{26}$ cm and 2 sec

C. $\boxed{\sqrt{26} \text{ cm and } \pi \text{ sec}}$

D. $\sqrt{29}$ cm and π sec

E. $\sqrt{29}$ cm and 2 sec

12. The fundamental set of solutions of $y^{(4)} - 16y = 0$ is

A. $\boxed{\{e^{2t}, e^{-2t}, \cos 2t, \sin 2t\}}$

B. $\{e^t, \cos t, \sin t\}$

C. $\{e^{2t}, e^{-2t}, te^{2t}, te^{-2t}\}$

D. $\{e^{2t}, te^{2t}, t^2e^{2t}, t^3e^{2t}\}$

E. $\{e^{2t}, e^{-2t}, \cos t, \sin t\}$

13. The suitable form of a particular solution $Y(t)$ of $y^{(4)} - y''' - y'' + y' = t^2 + 4 + te^t$ is (*Hint*: $r^4 - r^3 - r^2 + r = r(r-1)^2(r+1)$.)

A. $At^2 + Bt + C + Dt^3e^t$

B. $At^2 + Bt + C + Dt^2e^t + Ete^t$

C. $At^3 + Bt^2 + Ct + Dt^2e^{-t} + Ete^{-t}$

D. $\boxed{At^3 + Bt^2 + Ct + Dt^3e^t + Et^2e^t}$

E. $At^3 + Bt^2 + Ct + Dt^3e^t$

14. Find the inverse Laplace transform of $F(s) = \dfrac{2s+1}{s^2 - 4s + 5}$.

A. $e^t(\cos 2t + \sin 5t)$

B. $2\cos 2t + \sin 2t$

C. $e^{2t}(2\cos t + \sin t)$

D. $-\frac{3}{2}e^t + \frac{7}{2}e^{3t}$

E. $\boxed{e^{2t}(2\cos t + 5\sin t)}$

15. Find the Laplace transform of the function $f(t) = \begin{cases} \sin t, & 0 \le t < \pi \\ \cos t, & t \ge \pi \end{cases}$.

(*Hint*: Recall that $\sin(t-\pi) = -\sin t$ and $\cos(t-\pi) = -\cos t$.)

A. $\dfrac{1 - e^{\pi s}s}{s^2+1}$

B. $\boxed{\dfrac{1 - e^{-\pi s}(s-1)}{s^2+1}}$

C. $\dfrac{1 + e^{-\pi s}(s-1)}{s^2+1}$

D. $\dfrac{1 + e^{-\pi s}s}{s^2+1}$

E. $\dfrac{1 - e^{-\pi s}(s+1)}{s^2+1}$

16. Find the solution of $2y'' - 5y' + 3y = \delta(t-1)$ satifying $y(0) = 0$, $y'(0) = 0$.

A. $y(t) = u_{3/2}(t)e^{3/2t} + u_1(t)e^t$

B. $y(t) = u_1(t)(e^{3t/2} - e^t)$

C. $\boxed{y(t) = u_1(t)(e^{3t/2-3/2} - e^{t-1})}$

D. $y(t) = e^{3t/2} - e^t + 1$

E. $y(t) = u_1(t)(e^{-3t/2+3/2} - e^{-t+1})$

17. The solution of the initial value problem

$$y'' + 9y = g(t); \qquad y(0) = -1, \quad y'(0) = 9$$

is

A. $y = \int_0^t g(\tau)\sin 3(t-\tau)d\tau + 9\sin 3t - \cos 3t$

B. $y = \int_0^t g(\tau)\sin 3(t-\tau)d\tau + 3\sin 3t - \frac{1}{3}\cos 3t$

C. $y = \frac{1}{3}\int_0^t g(\tau)\sin 3(t-\tau)d\tau + 3\sin 3t - \frac{1}{3}\cos 3t$

D. $\boxed{y = \frac{1}{3}\int_0^t g(\tau)\sin 3(t-\tau)d\tau + 3\sin 3t - \cos 3t}$

E. $y = \int_0^t g(\tau)\sin 3(t-\tau)d\tau + 9\sin 3t - \frac{1}{3}\cos 3t$

18. The function $x_1(t)$ determined by the initial value problem

$$\begin{cases} x_1' = x_2 \\ x_2' = -4x_1 \end{cases}$$

with initial conditions $x_1(0) = 1$ and $x_2(0) = -2$ is given by

A. $x_1(t) = \sin 2t + \cos 2t$

B. $x_1(t) = \frac{1}{2}(e^{2t} + e^{-2t})$

C. $x_1(t) = \cos 2t$

D. $\boxed{x_1(t) = -\sin 2t + \cos 2t}$

E. $x_1(t) = ie^{it} - ie^{-it}$

19. Find the general solution of $\mathbf{x}' = \begin{pmatrix} 1 & 3 \\ 1/2 & 1/2 \end{pmatrix} \mathbf{x}$.

A. $\mathbf{x}(t) = c_1 \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{2t} + c_2 \begin{pmatrix} -2 \\ 1 \end{pmatrix} e^{-t/2}$

B. $\mathbf{x}(t) = \begin{pmatrix} -5 \\ 1 \end{pmatrix} e^{-2t} + \begin{pmatrix} 6 \\ -1 \end{pmatrix} e^{t/2}$

C. $\boxed{\mathbf{x}(t) = c_1 \begin{pmatrix} 3 \\ 1 \end{pmatrix} e^{2t} + c_2 \begin{pmatrix} -2 \\ 1 \end{pmatrix} e^{-t/2}}$

D. $\mathbf{x}(t) = c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-2t} + c_2 \begin{pmatrix} 6 \\ -1 \end{pmatrix} e^{t/2}$

E. $\mathbf{x}(t) = c_1 \begin{pmatrix} -3 \\ 1 \end{pmatrix} e^{2t} + c_2 \begin{pmatrix} 3 \\ -2 \end{pmatrix} e^{-t/2}$

20. A particular solution $\mathbf{x}_p(t)$ to the nonhomogeneous linear system $\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{g}$, where

$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}, \qquad \mathbf{g} = t \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

is

A. $\mathbf{x}_p(t) = -\begin{pmatrix} 1/2 \\ 0 \end{pmatrix}$

B. $\mathbf{x}_p(t) = -(t+1) \begin{pmatrix} 1/2 \\ 0 \end{pmatrix}$

C. $\mathbf{x}_p(t) = -t \begin{pmatrix} 1/2 \\ 0 \end{pmatrix}$

D. $\mathbf{x}_p(t) = -(e^t - e^{2t}) \begin{pmatrix} 1/2 \\ 0 \end{pmatrix}$

E. $\boxed{\mathbf{x}_p(t) = -t \begin{pmatrix} 1/2 \\ 0 \end{pmatrix} - \begin{pmatrix} 1/4 \\ 0 \end{pmatrix}}$

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\dfrac{1}{s}$
2.	e^{at}	$\dfrac{1}{s-a}$
3.	t^n	$\dfrac{n\,!}{s^{n+1}}$
4.	$t^p \ (p > -1)$	$\dfrac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sin at$	$\dfrac{a}{s^2+a^2}$
6.	$\cos at$	$\dfrac{s}{s^2+a^2}$
7.	$\sinh at$	$\dfrac{a}{s^2-a^2}$
8.	$\cosh at$	$\dfrac{s}{s^2-a^2}$
9.	$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}$
10.	$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$
11.	$t^n e^{at}$	$\dfrac{n\,!}{(s-a)^{n+1}}$
12.	$u_c(t)$	$\dfrac{e^{-cs}}{s}$
13.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$
14.	$e^{ct}f(t)$	$F(s-c)$
15.	$f(ct)$	$\dfrac{1}{c}F\left(\dfrac{s}{c}\right), \quad c > 0$
16.	$\displaystyle\int_0^t f(t-\tau)\,g(\tau)\,d\tau$	$F(s)\,G(s)$
17.	$\delta(t-c)$	e^{-cs}
18.	$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - sf^{(n-2)}(0) - f^{(n-1)}(0)$
19.	$(-t)^nf(t)$	$F^{(n)}(s)$